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Indeterminacy and directed search

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Abstract

The directed search approach assumes each seller posts a fixed price and, ex post, randomly allocates the good should more than one buyer desire the good. This paper assumes sellers can post prices which are contingent on ex post realized demand; e.g. an advertisement might state the Bertrand price should there be more than one buyer, which corresponds to an auction outcome. Competition in fixed prices and ex post rationing describes equilibrium behavior. There is also real market indeterminacy: a *continuum* of equilibria exists which are not payoff equivalent. Sellers prefer the equilibrium in auctions.

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1. Introduction

There is a large literature on trade in decentralized markets with matching frictions. Typically, search frictions are assumed where potential traders are not fully informed about each others' locations, and so it takes time to meet each other and trade (see [14] for a survey). A recent useful variation on this approach is the so-called directed search approach to decentralized trade. That literature assumes an advertising medium exists through which traders on one side of a market become fully informed on the prices and locations of traders on the other side. For example in a labor market context, firms might advertise vacancies and wages in a situations

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vacant column (e.g. [1,2,13]) while in a goods market context, sellers advertise their price and location (e.g. [5,17]). The central friction in, say, the goods market context, is that each buyer can only visit one seller, while each seller has only one unit of an indivisible good. This generates a coordination problem where some sellers might attract several potential buyers, while others attract none. As a result, when choosing his/her advertised price, each seller trades off ex post profit against attracting at least one potential buyer. The resulting price competition between sellers generates a non-Walrasian equilibrium outcome.

An interesting feature of the directed search approach is that ex post, each seller may be contacted by several potential buyers. A central assumption in this approach is that sellers announce one price and that they commit to the announced prices. The commitment rules out ex post opportunism, in which sellers, despite the announced price, encourage Bertrand competition once several buyers have turned up. While this is a restrictive assumption, in many environments such opportunistic behavior is not observed.[1] For example in the goods market, sellers frequently advertise 'sale' prices and a clause that 'the price holds while stocks last', i.e. the goods are rationed ex post and there is no opportunistic pricing should there be excess demand. Job auctions, where job applicants bid against each other for a vacancy, are particularly rare.

In this paper, we consider the case assumed in the directed search literature—that there is commitment by the sellers—but allow for more general ex ante mechanisms, i.e. sellers are not necessarily restricted to announcing a unique, fixed price. We assume that the number of buyers who visit a given seller is observable to the seller and those buyers. As trade between one seller and two buyers constitutes a different market to one where only one buyer shows, we assume the seller can advertise a price schedule, one where the price charged depends on the number of buyers who show up. This pricing 'mechanism' admits an auction scenario—the announcement can state the Bertrand price should there be more than one buyer. But it also admits the standard directed search assumption—that the seller precommits to a single price and chooses randomly one buyer from the set of buyers arriving at that location. As the typical directed search approach assumes sellers announce a fixed price, the central issue is whether precommiting to a single price describes equilibrium behavior even when more general ex ante mechanisms are allowed.

Somewhat surprisingly we establish that the standard directed search approach, where sellers post a single price and ration the good ex post, does indeed describe equilibrium behavior and so is robust to the criticism that sellers should advertise an auction. It is also consistent with the view that job auctions are not observed in the labor market. However, most surprisingly, we establish there is real market indeterminacy. A *continuum* of equilibria exists, none of which are payoff equivalent. The most profitable equilibrium for sellers involves competition in auctions where

[1] The reputation of the seller or an environment with repeated interaction is often cited as a motive for commitment by the seller.

sellers compete on reserve price and sellers extract maximal ex post rents should there be more than one buyer.[2]

Trading frameworks which imply real market indeterminacy are relatively uncommon. Of course it is well-known in the sunspot literature that once there is real market indeterminacy, additional 'sunspot' or correlated equilibria may also exist (e.g. [16]). It is also well known that market indeterminacy cannot arise in convex, competitive economies, but may arise when there are non-convexities such as when goods are indivisible (e.g. [20]) or when there are increasing returns to aggregate production (e.g. the endogenous growth literature, see [3,4]). However, although the assumption of indivisible goods is central to the directed search approach, it is not central to our results. Instead indeterminacy arises here as prices are not competitively determined. As in Peck and Shell [16], it is the assumed trading procedure which generates market indeterminacy. In particular the directed search approach implies that each seller's price advertisement plays two roles—ex ante each seller wishes to attract at least one buyer and ex post wishes to extract maximal surplus. But all that determines a buyer's visit decision is the expected payoff by visiting a particular seller. It turns out that each seller has a continuum of best responses given the price advertisements of competitors and the equilibrium search strategies of buyers. In particular, advertising more buyer surplus in some states and offering less in others can leave expected buyer and seller surplus unchanged. However, such switches change the demand elasticities of buyers, which then changes the best response correspondence of competing sellers. A continuum of equilibria exist, which are not payoff equivalent. In the absence of further restrictions on the trading process, this trading framework does not uniquely tie down ex ante buyer and seller surplus.

Section 2 describes the basic framework and Section 3 derives the main results. To clarify the indeterminacy result, Section 4 briefly discusses competition in indirect mechanisms and Section 5 concludes.

2. The directed search model

To illustrate the indeterminacy result most simply, we focus on the two-seller, two-buyer case (the results generalize straightforwardly to the N-buyer, M-seller case). The two sellers are indexed by $y \in \mathscr{Y} = \{1, 2\}$, and each holds one unit of an identical good. Consistent with the standard approach, we assume the good is indivisible but note that the results which follow do not depend on this. The two buyers are indexed by $x \in \mathscr{X} = \{1, 2\}$, are identical and anonymous and each wishes to purchase one unit of the good. If a buyer pays price p for a seller's good, the buyer obtains utility $Q - p$ from consuming the good, and the seller obtains utility p. The seller's utility from

[2] Also see the competing auction literature when buyers have private independent values; e.g. [6,12,18,19] but also [10,11]). In that literature, an auction is a strictly dominant mechanism as, ex post, it allocates the good to the buyer who values it most.

consuming her own good is normalized to zero. $Q > 0$ is common knowledge and all wish to maximize expected utility.

Matching and prices are determined by a two-stage game. In the first stage of the game, the sellers simultaneously post advertisements which describe their location and a pricing mechanism. These mechanisms are restricted to being direct mechanisms (an auction being a simple example) and are described in detail below.

Both buyers costlessly observe the posted advertisements and each chooses simultaneously which seller to visit. Given those decisions, and prior to the mechanism being played, each seller and the buyers observe how many buyers have chosen to visit that seller. At this stage, a buyer can walk away and obtain a payoff of zero, but cannot visit the other seller. Given any buyers who remain, the seller's advertised pricing mechanism is then played and determines the final payoffs; i.e. who gets the good and at what price.

A direct mechanism then determines, conditional on the number of buyers that participate, who receives the good and any side payments. Both buyers have the same valuation Q which is common knowledge.[3] Hence given anonymity, an optimal mechanism is simply a price pair (p_1, p_2), where p_1 is the price charged if only one buyer shows up (who gets the good with certainty), while p_2 is the price charged if both buyers visit (and the good is then randomly allocated to one of the buyers). Clearly, advertising $p_1 > Q$ is a dominated strategy—the buyer simply walks away should only one buyer show up. Similarly, it is a dominated strategy to post $p_2 > Q$.[4] We will also restrict attention to the case where prices are non-negative. Ex ante, and in order to attract buyers, a seller may want to announce one negative price (as long as the expected profits are positive). In an earlier version of this paper, we allowed for negative prices and showed that the set of equilibria remains unaltered. With both $0 \leqslant p_1, p_2 \leqslant Q$, this direct mechanism is ex post fully efficient (the good is sold with probability one should at least one buyer visit the seller).

The class of mechanisms we consider is therefore fully defined by the price pair (p_1, p_2) with $p_1, p_2 \leqslant Q$. There are two cases which are of particular interest (below we characterize the entire set).

Case (a) a *fixed price* advertisement: $p_1 = p_2$. If this describes an equilibrium mechanism, then the seller essentially advertises a single price p and adds a proviso that the 'good is sold at this price while stocks last'; i.e. the good is rationed ex post.

Case (b) an *auction*: $p_2 = Q$ (with reserve price $p_1 \leqslant Q$). A trading price $p_2 = Q$ is consistent with a standard auction outcome with multiple identical bidders, where p_1 is the seller's reserve price should only one buyer visit.

[3] For a discussion of direct mechanisms when buyers are not identical, see [7].

[4] If $p_2 > Q$ and $p_1 \leqslant Q$ then the buyers will play a war of attrition—each wants the other to walk away. Suppose each walks away with probability π and their corresponding expected payoff is $u \geqslant 0$. It follows that posting $p_2 \leqslant Q$, where $u = \frac{1}{2}[Q - p_2]$ dominates. Both buyers obtain the same expected payoff, but the total surplus generated increases as the good is sold with probability one (rather than probability $1 - \pi^2 \leqslant 1$).

3. Indeterminacy

Restricting attention to the case that sellers use pure pricing strategies, let (p_1', p_2') denote the price pair announced by seller 1, and (p_1, p_2) the price pair announced by seller 2. Given those price announcements, let $\sigma_x : (p_1, p_2, p_1', p_2') \to [0, 1]$ denote the probability that buyer $x \in \{1, 2\}$ chooses to visit seller 1.

Definition 1. A perfect (Nash) equilibrium is a quadruple of prices (p_1, p_2, p_1', p_2') and functions σ_x, where

1. given (p_1, p_2, p_1', p_2'), σ_x describes the Nash equilibrium in visit strategies for each buyer x;
2. given the subgame visit strategies $\sigma_x : (p_1, p_2, p_1', p_2') \to [0, 1]$, (p_1, p_2) and (p_1', p_2') describe a Nash equilibrium in pricing strategies for the two sellers.

Much of what follows establishes the following theorem.

Theorem 1. *There is a continuum of symmetric perfect (Nash) equilibria indexed by* $\alpha \in (0, Q]$, *where each seller posts* $p_1 = p_1' = \frac{Q}{2}$ *and* $p_2 = p_2' = \alpha$, *and in the resulting subgame, each buyer visits either seller with equal probability.*

This theorem establishes there is indeterminacy. Not only does an equilibrium exist where sellers compete in auctions (and announce reserve price $p_1 = Q/2$), and another where sellers compete in fixed prices (and announce $p_1 = p_2 = Q/2$) but there exists a continuum of related equilibria. In each case, the posted mechanisms are ex post efficient (the good is always sold should at least one buyer show up). However, ex ante price competition does not uniquely determine how much surplus each seller offers potential buyers. In particular, these equilibria are not payoff equivalent—the sellers would like to coordinate on the auction equilibrium.

The rest of this section formally establishes this theorem. The results are driven by a coordination problem—that neither buyer knows which seller the other buyer will visit. To focus entirely on that case, assume the (identical, anonymous) buyers use the same visit strategy; i.e. $\sigma_x = \sigma(p_1, p_2, p_1', p_2'),\ x \in \{1, 2\}$. Of course that is not to say that equilibria with coordinated strategies do not exist (see [5] for example).[5] However, concentrating on symmetric visit strategies is not only intuitively appealing (without prior communication, how do identical, anonymous buyers coordinate their visit strategies?), it also implies a unique equilibrium in Burdett et al. [5]. The relevant distinction is that here sellers post prices which are contingent on ex post demand (p_1, p_2), rather than a single fixed price $p_1 = p_2 = p$.

[5] Burdett et al. [5] also show that when firms compete in fixed prices ($p_1 = p_2 = p$), a continuum of equilibria exist where buyers switch between coordinated strategies ($\sigma_x = 1, \sigma_{-x} = 0$ for some p) and uncoordinated strategies ($\sigma_x = \sigma$ for other p). In essence, the uncoordinated strategies imply lower profits for the sellers and these strategies can then be used as threats which support a continuum of equilibria where buyers use coordinated visit strategies along the equilibrium path.

The following lemma describes the complete set of (subgame) Nash equilibria in symmetric visit strategies ($\sigma_x = \sigma$).

Lemma 1. *Given* (p_1, p_2, p_1', p_2'), *then a Nash equilibrium in symmetric visit strategies implies*:

(i) $\sigma = 1$ *is a dominant strategy equilibrium if* $Q - p_1' > 0.5[Q - p_2]$ *and* $0.5[Q - p_2'] > Q - p_1$;
(ii) $\sigma = 0$ *is a dominant strategy equilibrium if* $Q - p_1' < 0.5[Q - p_2]$ *and* $0.5[Q - p_2'] < Q - p_1$;
(iii) $\sigma \in (0, 1)$ *is the unique Nash equilibrium if* $Q - p_1 > 0.5[Q - p_2']$ *and* $Q - p_1' > 0.5[Q - p_2]$, *where*

$$\sigma = \frac{[Q - p_1'] - \frac{1}{2}[Q - p_2]}{\{[Q - p_1'] - \frac{1}{2}[Q - p_2]\} + \{[Q - p_1] - \frac{1}{2}[Q - p_2']\}} = \frac{Q + p_2 - 2p_1'}{2Q + p_2 + p_2' - 2p_1 - 2p_1'}; \tag{1}$$

(iv) *if* $Q - p_1' \leqslant 0.5[Q - p_2]$ *and* $Q - p_1 \leqslant 0.5[Q - p_2']$ *then multiple equilibria exist where* $\sigma = 0, 1$ *or* $\sigma \in [0, 1]$ *given by* (1).

Proof. $\sigma = 1$ is an equilibrium only if $0.5[Q - p_2'] \geqslant Q - p_1$, and this is a dominant strategy equilibrium if also $Q - p_1' \geqslant 0.5[Q - p_2]$. The equivalent argument holds for $\sigma = 0$ (both buyers visit seller 2).

Should prices satisfy $Q - p_1' > 0.5[Q - p_2]$ and $Q - p_1 > 0.5[Q - p_2']$, the buyers face a coordination problem—each wants to visit a different seller. The above implies $\sigma = 1, 0$ cannot describe Nash equilibria, and standard arguments imply there is a unique Nash equilibrium in mixed strategies, and that σ is given by (1).[6]

Case (iv) arises when both sellers post prices satisfying $Q - p_1' \leqslant 0.5[Q - p_2]$ (with the equivalent condition for $Q - p_1$). In that case, the buyers prefer to visit the same seller. This coordination problem implies multiple Nash equilibria exist, which are as described in the lemma. This completes the proof of the lemma. □

Having described the set of possible subgame outcomes given (p_1, p_2, p_1', p_2'), we now consider the best response of seller 1, assuming seller 2 posts prices (p_1, p_2). Of course case (iv) implies there may be equilibrium selection issues. However, it turns out that this is not an important problem.

[6] An interior solution to σ_1 requires buyer 1 is indifferent between visiting either seller. This requires

$$(Q - p_1')(1 - \sigma_2) + \tfrac{1}{2}(Q - p_2')\sigma_2 = (Q - p_1)\sigma_2 + \tfrac{1}{2}(Q - p_2)(1 - \sigma_2),$$

where the left-hand side is buyer 1's expected payoff to visiting seller 1 given σ_2 is the probability that buyer 2 also visits seller 1. This equation implies $\sigma_2 = \sigma$ defined by (1). The same argument implies $\sigma_1 = \sigma$.

Given (p_1, p_2), first suppose that seller 1's best response is to post a price pair (p'_1, p'_2) which will attract both buyers with probability one; i.e. $\sigma = 1$ in the subgame. By Lemma 1, this requires $0.5[Q - p'_2] \geqslant Q - p_1$ which can be rewritten as $p'_2 \leqslant 2p_1 - Q$. As $\sigma = 1$ implies seller 1's profit equals p'_2, the highest payoff seller 1 can obtain by inducing $\sigma = 1$ is $2p_1 - Q$.

Now suppose given (p_1, p_2), seller 1 chooses prices (p'_1, p'_2) so that a Nash equilibrium exists in the subgame where $\sigma < 1$. Of course $\sigma = 0$ implies a zero payoff, and so suppose $\sigma \in (0, 1)$. In such a mixed strategy equilibrium, seller 1's expected payoff is

$$\pi' = 2\sigma(1 - \sigma)p'_1 + \sigma^2 p'_2, \tag{2}$$

where $2\sigma(1 - \sigma)$ is the probability one buyer shows (and trade occurs at price p'_1) and σ^2 is the probability that two buyers show. As $\sigma \in (0, 1)$ implies σ is given by (1), we can use that equation to substitute out p'_2 in (2) and so obtain the following reduced form profit function $\tilde{\pi}'$ for seller 1:

$$\tilde{\pi}'(\sigma; p_1, p_2) = \sigma[Q + p_2] - \sigma^2[2Q + p_2 - 2p_1]. \tag{3}$$

Note, $\tilde{\pi}'(1; .) = 2p_1 - Q$ (which by the above is the maximum payoff by inducing $\sigma = 1$) while $\tilde{\pi}'(0; .) = 0$.

Now notice that Eq. (1) for σ can be rearranged as

$$[Q - p_1] - \frac{1}{2}[Q - p'_2] = \frac{1 - \sigma}{\sigma}\left[[Q - p'_1] - \frac{1}{2}[Q - p_2]\right]. \tag{4}$$

Given (p_1, p_2) and any $\sigma \in (0, 1)$, it now follows that any price pair (p'_1, p'_2) satisfying (4) implies a mixed strategy (subgame) equilibrium exists where both buyers randomize with that particular value of σ. Furthermore, seller 1's expected payoff is then given by (3) which depends on the particular choice of (p'_1, p'_2) only through σ, as determined in Eq. (4). This implies that in choosing two prices (p'_1, p'_2), there is one degree of freedom.

This surprising result has two immediate consequences. First, we can ignore the equilibrium selection problem. In particular, suppose $\tilde{\pi}'(\sigma; p_1, p_2) > \max\{(2p_1 - Q), 0\}$ for some $\sigma \in (0, 1)$; i.e. inducing a mixed strategy in the subgame dominates inducing $\sigma = 1$ or 0. By choosing (p'_1, p'_2) satisfying (4), a subgame equilibrium then exists for that value of σ. But by also specifying $[Q - p'_1] - \frac{1}{2}[Q - p_2] > 0$, seller 1 guarantees that this is also the unique subgame equilibrium (see Lemma 1(iii)) and so guarantees his maximal payoff $\tilde{\pi}'(\sigma; p_1, p_2)$. Hence, with only a small loss of generality, we can sidestep the equilibrium selection problem by assuming that should either seller wish to induce a mixed strategy in the subgame, that seller sets prices so that the mixed strategy equilibrium is the unique subgame equilibrium (i.e. case (iii) holds in Lemma 1).

Second given (p_1, p_2), (4) implies the seller's best response does not uniquely tie down (p_1', p_2'). In particular, define σ_1^* as

$$\sigma_1^* = \arg\max_{\sigma \in [0,1]} \tilde{\pi}'(\sigma; p_1, p_2) \tag{5}$$

and note that any (p_1', p_2') satisfying (4) with $\sigma = \sigma_1^*$ then describes a best response for seller 1. Lemma 2 now describes those best responses.

Lemma 2. *Given* (p_1, p_2), *the best response of seller* 1 *implies* (p_1', p_2') *satisfy* (4) *with* $\sigma = \sigma_1^*$, *where*:

(a) *if* $p_2 > 2(p_1 - Q)$, *then*

$$\sigma_1^* = \begin{cases} \frac{1}{2}\left(\frac{Q+p_2}{2Q+p_2-2p_1}\right) & \text{if } p_2 \geqslant 0, \text{ and } p_2 > 4p_1 - 3Q, \\ 1 & \text{if } p_2 \geqslant 0, \text{ and } p_2 \leqslant 4p_1 - 3Q, \end{cases}$$

(b) *if* $p_2 < 2(p_1 - Q)$, *then*

$$\begin{aligned} \sigma_1^* &= 0 && \text{if } p_1 < Q/2, \\ \sigma_1^* &\in \{0, 1\} && \text{if } p_1 = Q/2, \\ \sigma_1^* &= 1 && \text{if } p_1 > Q/2, \end{aligned}$$

(c) *if* $p_2 = 2(p_1 - Q)$, *then*

$$\begin{aligned} \sigma_1^* &= 0 && \text{if } p_1 < Q/2, \\ \sigma_1^* &\in [0, 1] && \text{if } p_1 = Q/2, \\ \sigma_1^* &= 1 && \text{if } p_1 > Q/2. \end{aligned}$$

Proof. In appendix. □

The same argument describes σ_2^*; i.e. the set of best responses for seller 2. Identifying a perfect Nash equilibrium reduces to finding a $\sigma \in (0, 1)$, where $\sigma_1^* = \sigma_2^* = \sigma$ (and both sellers are playing best responses). It now follows that if a perfect Nash equilibrium exists, it implies $\sigma \in (0, 1)$.

Lemma 3. *Any solution for* (p_1, p_2, p_1', p_2') *and* $\sigma \in (0, 1)$ *which satisfies* (1),

$$\sigma = \frac{1}{2}\left(\frac{Q + p_2}{2Q + p_2 - 2p_1}\right), \tag{6}$$

$$1 - \sigma = \frac{1}{2}\left(\frac{Q + p_2'}{2Q + p_2' - 2p_1'}\right) \tag{7}$$

and the inequalities

$$p_2 \geqslant 0, \quad 4p_1 - p_2 < 3Q \quad \text{and} \quad p_1, p_2 \leqslant Q, \tag{8}$$

$$p_2' \geqslant 0, \quad 4p_1' - p_2' < 3Q \quad \text{and} \quad p_1', p_2' \leqslant Q, \tag{9}$$

describes a perfect Nash equilibrium. Further,

$$Q - p_1 > \tfrac{1}{2}[Q - p_2'],$$

$$Q - p_1' > \tfrac{1}{2}[Q - p_2]$$

implies the subgame equilibrium is unique.

Proof. Lemma 2(a) implies that seller 1 is playing a best response if (6) and inequalities (8) hold, where it should be noted that those inequalities guarantee $2Q + p_2 - 2p_1 > 0$. The same argument applies to seller 2, where (7) describes the best response of seller 2 if inequalities (9) hold. As (1) describes the buyers' equilibrium strategies in the subgame (given $\sigma \in (0,1)$ in an equilibrium), any solution to these conditions describes a perfect Nash equilibrium. □

There is a continuum of equilibria as the three equilibrium conditions (1), (6), (7) cannot tie down the five unknowns $\{p_1, p_2, p_1', p_2', \sigma\}$, and the inequalities admit a continuum of such solutions. The simplest to characterize are the symmetric equilibria where $p_1' = p_1$ and $p_2' = p_2$. In that case, (1) implies $\sigma = \frac{1}{2}$ and (6), (7) imply $p_1' = p_1 = \frac{Q}{2}$. But p_2 and p_2' are not tied down. The remaining inequalities are satisfied for $p_2 = p_2' = \alpha$, where $\alpha \in (0, Q]$, and so symmetric equilibria (with a unique subgame) exist for those values. This completes the proof of Theorem 1.

Of course, the indeterminacy result is not restricted to symmetric seller strategies. Lemma 3 demonstrates there exists a continuum of equilibria where sellers use different (i.e. asymmetric) pricing strategies. For example, an equilibrium exists where $p_1' = 0$ and $p_2' = Q$. Seller 1 offers to give the good away if only one buyer shows, but will sell at the monopoly price if two show. This describes a perfect Nash equilibrium when seller 2 announces $p_1 = 2Q/3$, $p_2 = Q/3$ and the corresponding visit strategies imply $\sigma = \frac{2}{3}$. It is interesting to note that all asymmetric equilibria are less efficient than any symmetric equilibrium as the probability that one buyer does not obtain a good increases (as $\sigma \neq \frac{1}{2}$ in all asymmetric equilibria).

4. Indirect mechanisms

The analysis above has restricted competition to direct mechanisms. To illustrate clearly the nature of our indeterminacy result, we consider the set of equilibria when sellers can instead compete in indirect mechanisms.

Given advertisements are public information, assume in the second stage that all observe the advertised mechanisms.[7] Competition in indirect mechanisms implies seller 1 can condition prices on seller 2's prices. For example, seller 1 might advertise he 'will not be undercut' by posting $p_1' = \min\{p_1, Q\}$. More generally, suppose seller 1 advertises a price function $p_1' = p_1'(p_1, p_2)$ and $p_2'(p_1, p_2)$. Assume seller 2 also

[7] Though see Epstein and Peters [8] and Peck [15] who instead assume sellers do not observe each others' advertisements.

competes using a price function and suppose that given those price rules, a solution for the subsequent trading prices (p'_1, p'_2, p_1, p_2) exists. Given those prices, σ is given by Lemma 1 and seller 1's expected profit is given by Eq. (3).

Note for given σ, Eq. (3) implies seller 1's expected profit is strictly increasing in seller 2's prices (p_1, p_2). A central aim of seller 1 is to use a price rule which encourages seller 2 to raise price. Allowing competition in indirect mechanisms implies seller 1 can now post an advertisement of the following form:

$$p'_1 = p'_2 = \begin{cases} Q & \text{if } p_1 = p_2 = Q, \\ 0 & \text{otherwise,} \end{cases}$$

i.e. seller 1 threatens to give away the good unless seller 2 announces the monopoly price. It is straightforward to show that seller 2's best response is to announce the monopoly price, and by using the same price strategy, it follows that the monopoly outcome describes a perfect equilibrium. Obviously, a continuum of related equilibria exists, where such 'threats' can support a variety of equilibrium payoffs for the sellers.

This result is not unlike the insights in the financial literature and in the industrial organization literature (e.g. [9] and see [8] for further references), where sellers compete using supply functions. There, multiplicity arises as competition in supply functions implies sellers can make price threats away from the equilibrium outcome, and so sustain multiple pure strategy equilibria. However, the indeterminacy described in Theorem 1 arises for entirely different reasons. In particular, the restriction to direct price mechanisms rules out price 'threats'. Instead, indeterminacy arises as each seller has a continuum of best responses, where offering more buyer surplus in some states and offering less in others can leave expected buyer and seller surplus unchanged. However, such transfers change the demand elasticities of the buyer demand functions σ which changes the best response correspondences of sellers. Theorem 1 establishes that this implies market indeterminacy.

5. Conclusion

It is straightforward to show that the indeterminacy described in Theorem 1 generalizes to the M-buyer, N-seller case.[8] In particular, a continuum of symmetric equilibria always exists where each seller announces a price pair (p_1, p), where p_1 is the trading price should only one buyer show, and p is the trading price if more than one shows. The equilibrium where $p = p_1$ corresponds to competition in fixed prices, and see Burdett et al. [5] who determine the equilibrium price for the N-buyer, M-seller case. The equilibrium with $p = Q$ corresponds to competition in auctions and see Julien et al. [10] who compute the equilibrium reserve prices for that case. Julien

[8] A proof is available from the authors.

et al. [11] further establish that sellers strictly prefer the equilibrium in auctions, but also find that the equilibria are payoff equivalent in the limiting large economy case.

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Appendix

Proof of Lemma 2. (a) If $2Q + p_2 - 2p_1 > 0$, the profit function (3) is strictly concave in σ. The corner solution $\sigma = 0$ is optimal only for $p_2 \leqslant -Q$, and since prices are restricted to be non-negative, $\sigma = 0$ is never optimal. For $p_2 \geqslant 0$ and $4p_1 - p_2 \geqslant 3Q$, the corner solution $\sigma = 1$ is optimal, and otherwise we have the interior optimum.

(b) If $2Q + p_2 - 2p_1 < 0$, the profit function (3) is strictly convex in σ. If $2p_1 - Q < 0$ then $\sigma = 0$ is optimal, while $2p_1 - Q > 0$ implies $\sigma = 1$ is optimal. When $2p_1 - Q = 0$, then both corners $\sigma \in \{0, 1\}$ are optimal and generate zero profit $\pi' = 0$.

(c) If $2Q + p_2 - 2p_1 = 0$, the profit function (3) is linear in σ. If $2p_1 - Q < 0$ then $\sigma = 0$ is optimal, while $2p_1 - Q > 0$ implies $\sigma = 1$ is optimal. When $2p_1 - Q = 0$, then any $\sigma \in [0, 1]$ is optimal. □

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