Competitive Bargaining Equilibrium

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Can we provide a bargaining foundation for Walrasian Equilibrium in a small economy without price-taking behavior?

Investigate the strategic role of prices even if agents have market power

We propose a simple bargaining procedure:

variation of alternating offer bargaining;

announce price and maximum quantity constraint;

separate the price and quantity decision;
Results

1. Convergence of SSP equilibrium to the Walrasian allocation as discounting frictions vanish;
   \[ \rightarrow \text{price taking is not a necessary requirement for competition} \]

2. Bargaining outcome is determinate, independent of bargaining power or relative impatience;
   \[ \rightarrow \text{implications for applying bargaining models} \]
The Model

Agents $A, B$, goods 1, 2, endowments $e = e^A + e^B$, utility functions $u^A, u^B$, infinite (discrete) time horizon, discount factors $\delta^A, \delta^B$

A two person, two goods exchange economy $\{u^i, e^i\}_{i \in \{A,B\}}$; denote Walrasian equilibrium $\{\bar{x}, \bar{p}\}$

Price $p$: terms of trade of good 1 in terms of good 2; $q$ is the maximum quantity constraint (in terms of first coordinate);

Alternating Offer bargaining:

- alternatingly, offer price and maximum quantity constraint;
- recipient accepts (chooses quantity) or rejects (offers next $t$);

Stationary Subgame Perfect (SSP) equilibrium
Stationary Subgame Perfect (SSP*) equilibrium with immediate acceptance \((p^A, q^A), (p^B, q^B)\) such that:

\[
(p^A, q^A) \in \arg \max_{\tilde{p}^A, \tilde{q}^A} u^A(e - x^B(\tilde{p}^A, \tilde{q}^A))
\]  

\[\text{s.t. } u^B(\tilde{x}^B(\tilde{p}^A, \tilde{q}^A)) \geq \deltaBu^B(e - \tilde{x}^A(p^B, q^B))\]

where

\[
\tilde{x}^B(p^A, q^A) = \arg \max_{x^B} u^B(x^B)
\]

\[
p^A(x^B - e^B) \leq 0
\]
\[
|x_1^B - e_1^B| \leq q^A
\]

and similarly for \(B\).
Subgame perfection: accepted offer will be "inside" offer curve:
\( \tilde{x}^B(p^A, q^A) \)
Characterize SSP* by offers $x^B$ (made by $A$) and $x^A$ (made by $B$) such that:

$$x^B \in \arg \max_{\hat{x}^B} u^A(e - \hat{x}^B)$$

$$Du^B(\hat{x}^B)(\hat{x}^B - e^B) \geq 0$$

$$u^B(\hat{x}^B) \geq \delta^B u^B(e - x^A)$$
Graphical illustration of SSP* equilibrium

\[ u^B = U \]
\[ x^B(p^A) \]
\[ \bar{x} \]
Graphical illustration of SSP* equilibrium
Graphical illustration of SSP* equilibrium

$\bar{x}$ contract curve

$\tilde{x}^B(p^A, q^A)$
Graphical illustration of SSP* equilibrium
Lemma 1. Offering agents extract all rents subject to acceptance

Lemma 2. For every SSP* equilibrium, if the offer accepted by $A$ is not on his offer curve, then it is efficient. Likewise for $B$.

Theorem 1. Whenever $\delta^A = \delta^B = 1$, every SSP* equilibrium allocation is Walrasian.

Theorem 2. Every SSP* equilibrium allocation converges to a Walrasian allocation as the agents become infinitely patient.
SSP equilibria with delay

See Merlo and Wilson (1995)

Lemma 3. Whenever the agents are impatient \((\delta^A, \delta^B < 1)\), there does not exist any SSP equilibrium with delay.

Consider a candidate equilibrium where \(A\) accepts \(x_A\) and \(B\) always rejects
\[ u^A(x^A) \]

\[ u^B(e - x^A) \]
Theorem 3. Every SSP equilibrium allocation converges to a Walrasian allocation as the agents become infinitely patient.

Follows immediately from Lemma 3 and Theorem 2.

Note: When \( \delta^A = \delta^B = 1 \) there exist a continuum of SSP equilibria with delay (cf. Rubinstein alternating offer bargaining)
Bargaining over allocations – Rubinstein (1982), Ståhl (1972)

$z^A$ is consumption offered to $A$ by $B$ (and likewise $z^B$)

Equilibrium offer:

$$u^B(z^B) \geq \delta^B u^B(e - z^A)$$
$$u^A(z^A) \geq \delta^A u^A(e - z^B)$$

Define the profiles:

$$P^A = (u^A(z^A), \delta^B u^B(z^B))$$
$$P^B = (\delta^A u^A(z^A), u^B(z^B))$$

with $z^A + z^B = e$. 
The sequence \( \left\{ (\delta_n^A, \delta_n^B) \right\}_n \) converging to one determines the bargaining outcome.
Nash Bargaining

Selects the feasible allocation \((z^A, z^B)\) that maximizes the Nash product
\[ N(\alpha) = u^A(z^A)^\alpha \cdot u^B(z^B)^{1-\alpha}; \]
the bargaining power \(\alpha\) determines the outcome.
Bargaining over Prices only

Same problem, except for the quantity constraint

Problem:

1. there typically exists an SSP equilibrium that is inefficient
See also Yildiz (2002) and Dávila-Eeckhout (2002))
The profiles of utilities

\[ f_{\delta A}^B(p) = (u^A(x^A(p)), \delta^B u^B(e - x^A(p))) \]
\[ f_{\delta A}^B(p) = (\delta^A u^A(e - x^B(p)), u^B(x^B(p))) \]
The profiles of utilities

\[ f_{\delta B}^A(p) = (u^A(x^A(p)), \delta^B u^B(e - x^A(p))) \]
\[ f_{\delta A}^B(p) = (\delta^A u^A(e - x^B(p)), u^B(x^B(p))). \]
The profiles of utilities

\[ f^A_{\delta^B}(p) = (u^A(x^A(p)), \delta^B u^B(e - x^A(p))) \]
\[ f^B_{\delta^A}(p) = (\delta^A u^A(e - x^B(p)), u^B(x^B(p))) \].
Bargaining over Prices only

Same optimization problem, except for the quantity constraint

Problem:

1. there typically exists an SSP equilibrium that is inefficient

2. SSP equilibrium converging to the Walrasian allocation may not exist (depending on sequence of $\delta$s converging to 1) and if it exists, there is multiplicity
Bargaining over Prices with minimum quantity constraints

SSP* equilibrium converging to the Walrasian allocation exists

But also SSP equilibria with delay exist
Concluding Remarks

Bargaining protocol that obtains convergence to the WE in absence of price-taking for any economy

Driving force: intertemporal competition (infinity of counter-offers)

Quantity constraint is crucial: rules out inefficient equilibria with different terms offered by each agent that obtain same utility and guarantees existence

Outcome independent of path of $\delta^A, \delta^B$ or exogenous bargaining power; therefore no indeterminacy (Edgeworth)

Applications of bargaining (e.g. matching model); Can verify Hosios condition from primitives (independent of bargaining power)
"Bargaining over prices with quantity constraint": Examples of related protocols

the dissolution of Partnerships (see a.o. Moldovanu)

union-wage bargaining (see a.o. Farber)

limit orders for selling stock; commodity futures trading;
Yildiz (2003) shows unique convergence to WE under some Assumptions

We find that generically, the intersection at the WE of $f^A$ and $f^B$ is without crossing

A3 (Yildiz): both monopolistic outcomes are dominated by some allocation attainable along an offer curve

A4 (Yildiz): there is a unique crossing of $f_A$ and $f_B$ within the interval defined by the profiles of utilities attained at the monopolistic outcomes

A3 and A4 are non-generic (not satisfied for an open and dense set of economies)
Observe further:

A3 is robust (if it is satisfied for a given economy, then it is also satisfied for all economies in a neighborhood);

A4 is robust;

A3 and A4 jointly are not robust