Competitive Bargaining Equilibrium

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Can we provide a bargaining foundation for Walrasian Equilibrium in a small economy without price-taking behavior?

Investigate the *strategic* role of prices even if agents have market power

We propose a simple bargaining procedure:

variation of alternating offer bargaining;

announce price and maximum quantity constraint;

separate the price and quantity decision;

Results

1. Convergence of SSP equilibrium to the Walrasian allocation as discounting frictions vanish;

$\longrightarrow$ price taking is not a necessary requirement for competition

2. Bargaining outcome is determinate, independent of bargaining power or relative impatience;

$\longrightarrow$ implications for applying bargaining models

The Model

Agents A, B, goods $1, 2$, endowments $e = e^A + e^B$, utility functions u^A, u^B, infinite (discrete) time horizon, discount factors δ^A, δ^B

A two person, two goods exchange economy $\{u^i, e^i\}_{i \in \{A,B\}}$; denote Walrasian equilibrium $\{\overline{x}, \overline{p}\}$

Price p: terms of trade of good 1 in terms of good 2; q is the maximum quantity constraint (in terms of first coordinate);

Alternating Offer bargaining:

alternatingly, offer price and maximum quantity constraint;

recipient accepts (chooses quantity) or rejects (offers next t);

Stationary Subgame Perfect (SSP) equilibrium

Stationary Subgame Perfect (SSP*) equilibrium with immediate acceptance $(p^A, q^A), (p^B, q^B)$ such that:

$$(p^A, q^A) \in \arg\max_{\tilde{p}^A, \tilde{q}^A} u^A(e - x^B(\tilde{p}^A, \tilde{q}^A)) \tag{1}$$

$$\texttt{s.t.} \quad u^B(\tilde{x}^B(\tilde{p}^A, \tilde{q}^A)) \geq \delta^B u^B(e - \tilde{x}^A(p^B, q^B))$$

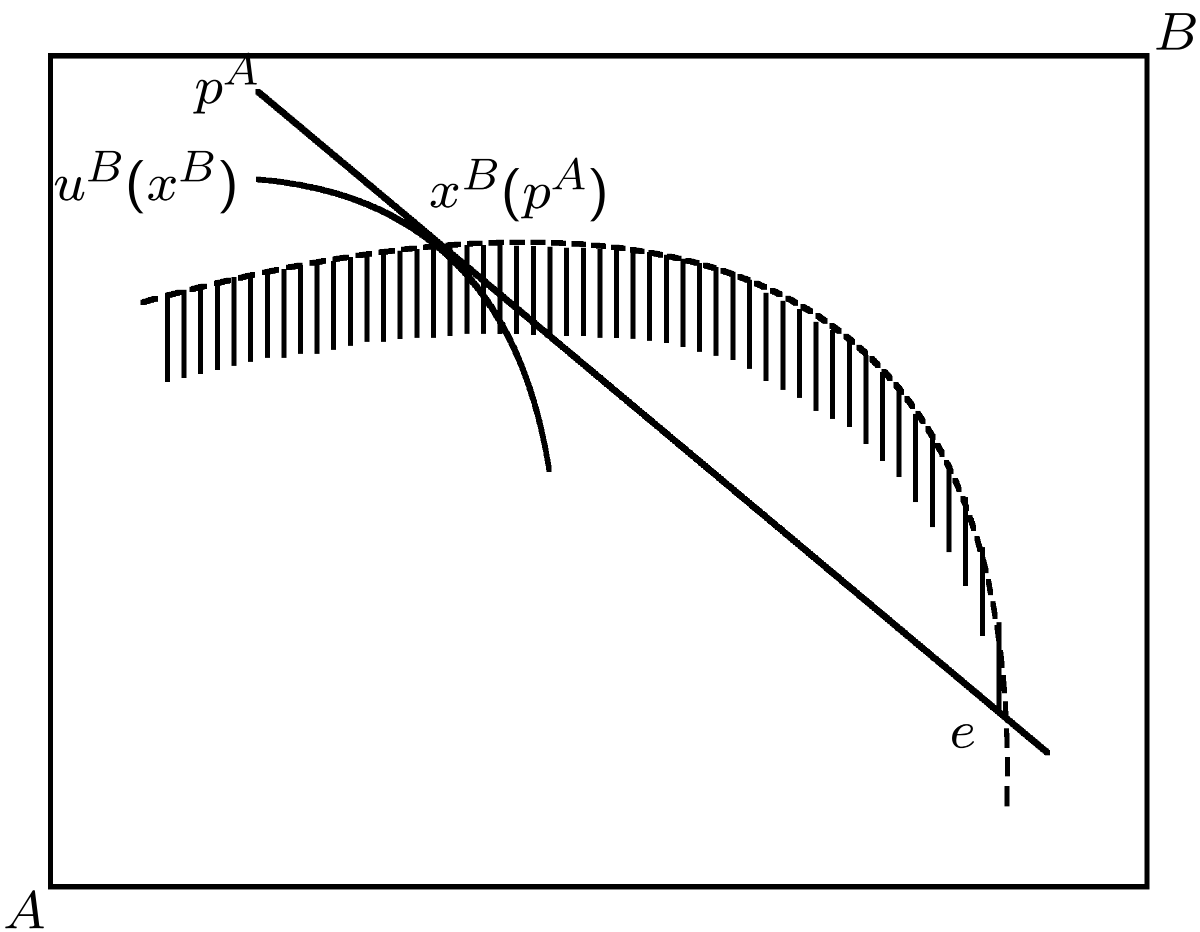
where

$$\tilde{x}^B(p^A, q^A) = \arg\max_{x^B} u^B(x^B) \tag{2}$$

$$\begin{aligned} p^A(x^B - e^B) &\leq 0 \\ \left| x_1^B - e_1^B \right| &\leq q^A \end{aligned}$$

and similarly for B.

Subgame perfection: accepted offer will be "inside" offer curve: $\tilde{x}^B(p^A, q^A)$



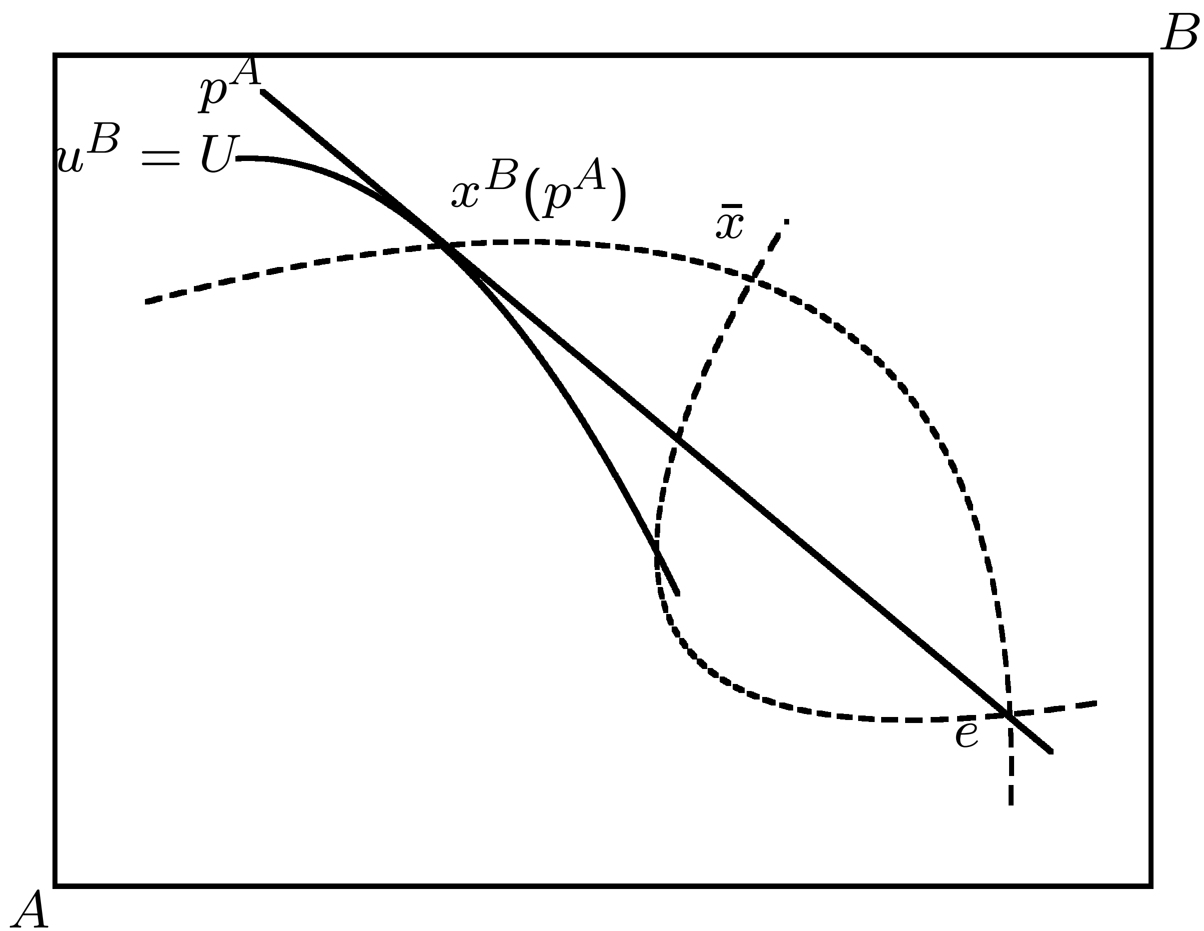
Characterize SSP* by offers x^B (made by A) and x^A (made by B) such that:

$$x^B \in \arg\max_{\hat{x}^B} u^A(e - \hat{x}^B)$$

$$Du^B(\hat{x}^B)(\hat{x}^B - e^B) \geq 0$$

$$u^B(\hat{x}^B) \geq \delta^B u^B(e - x^A)$$

Graphical illustration of SSP* equilibrium



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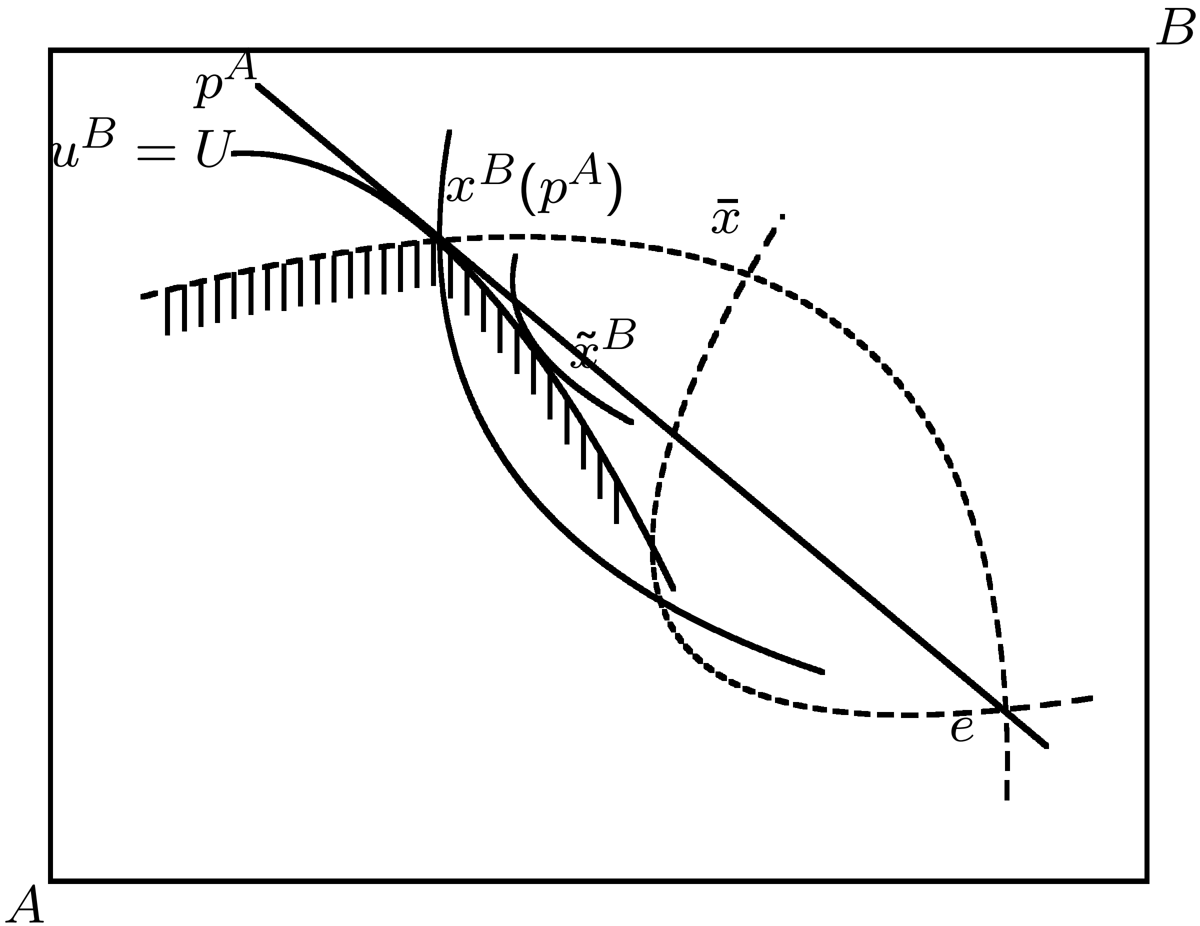

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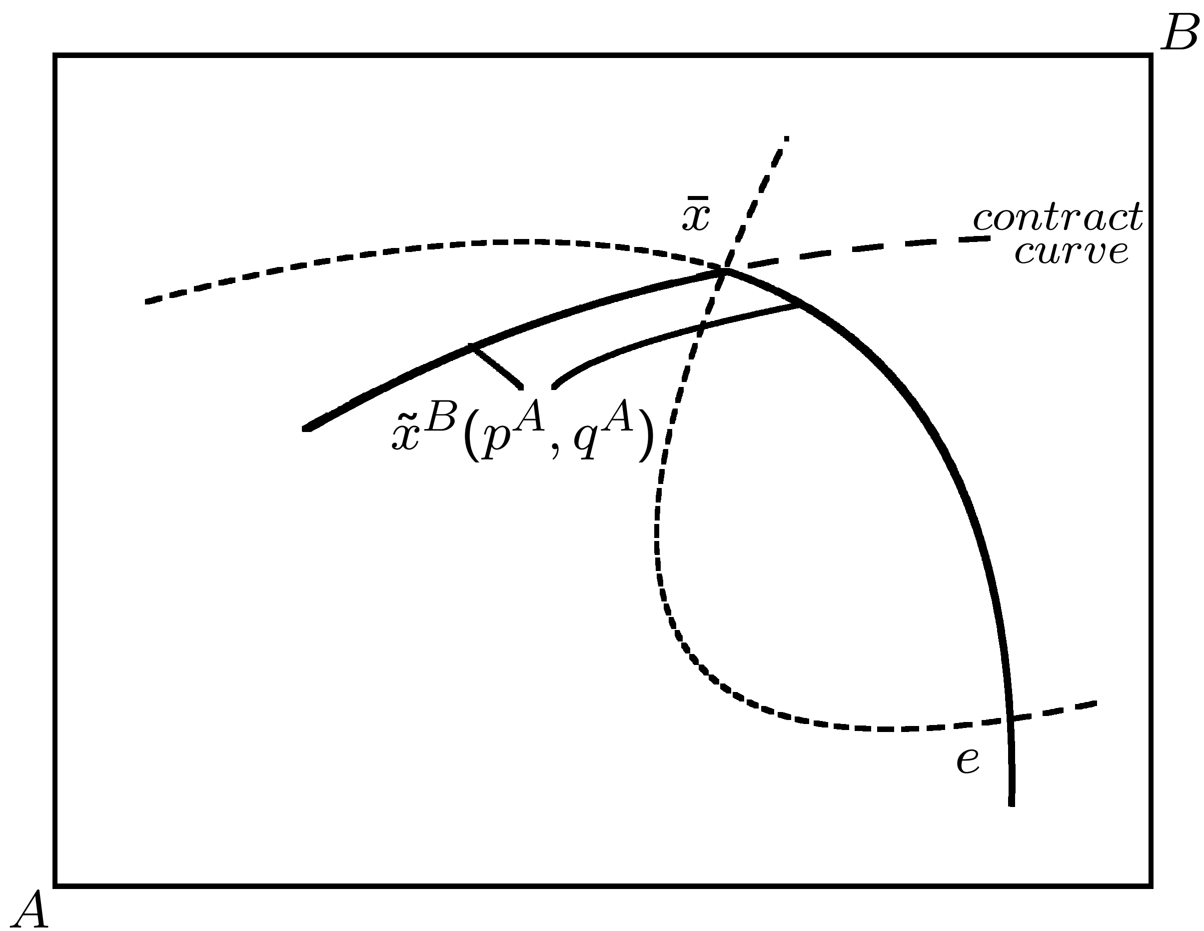
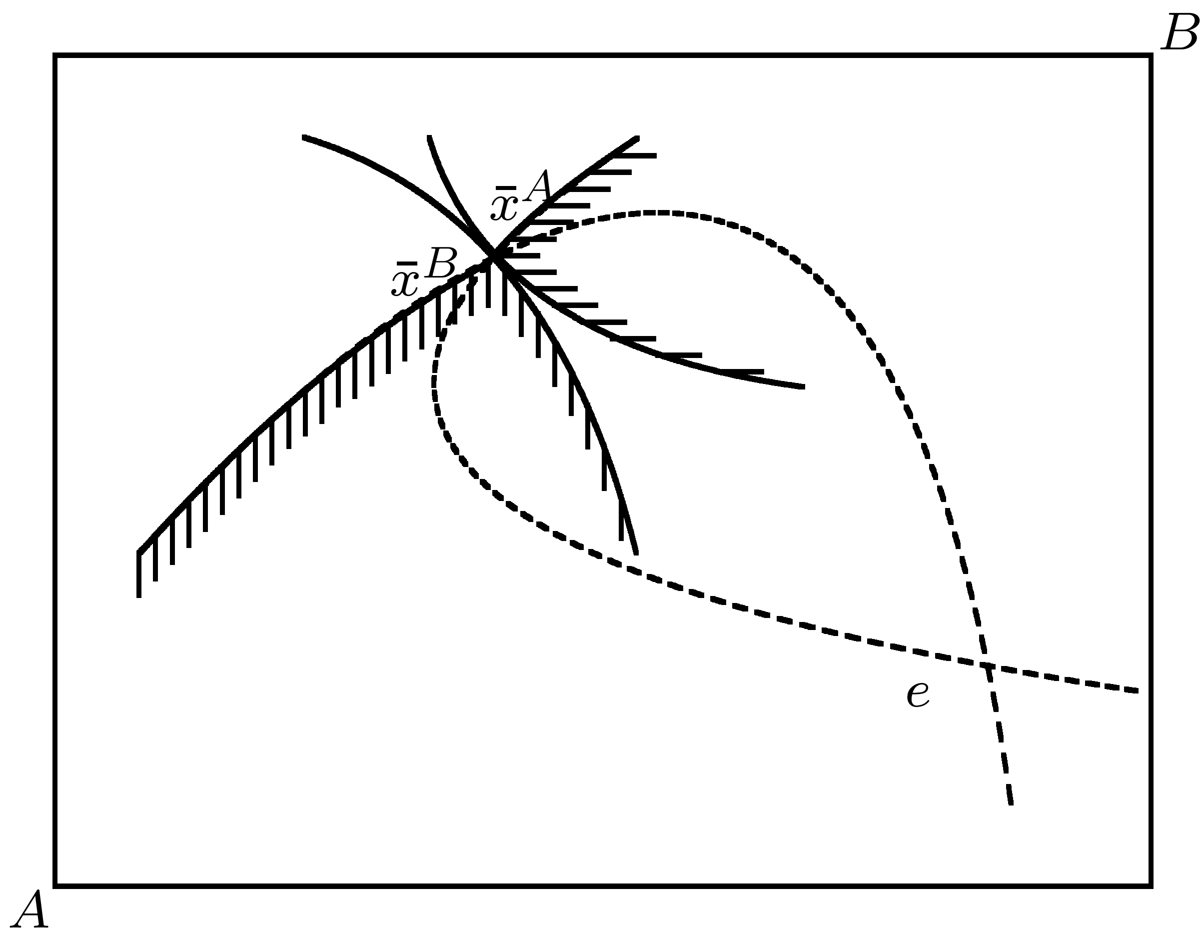


Graphical illustration of SSP* equilibrium



Lemma 1. Offering agents extract all rents subject to acceptance

Lemma 2. For every SSP* equilibrium, if the offer accepted by A is not on his offer curve, then it is efficient. Likewise for B.

Theorem 1. Whenever $\delta^A = \delta^B = 1$, every SSP* equilibrium allocation is Walrasian.

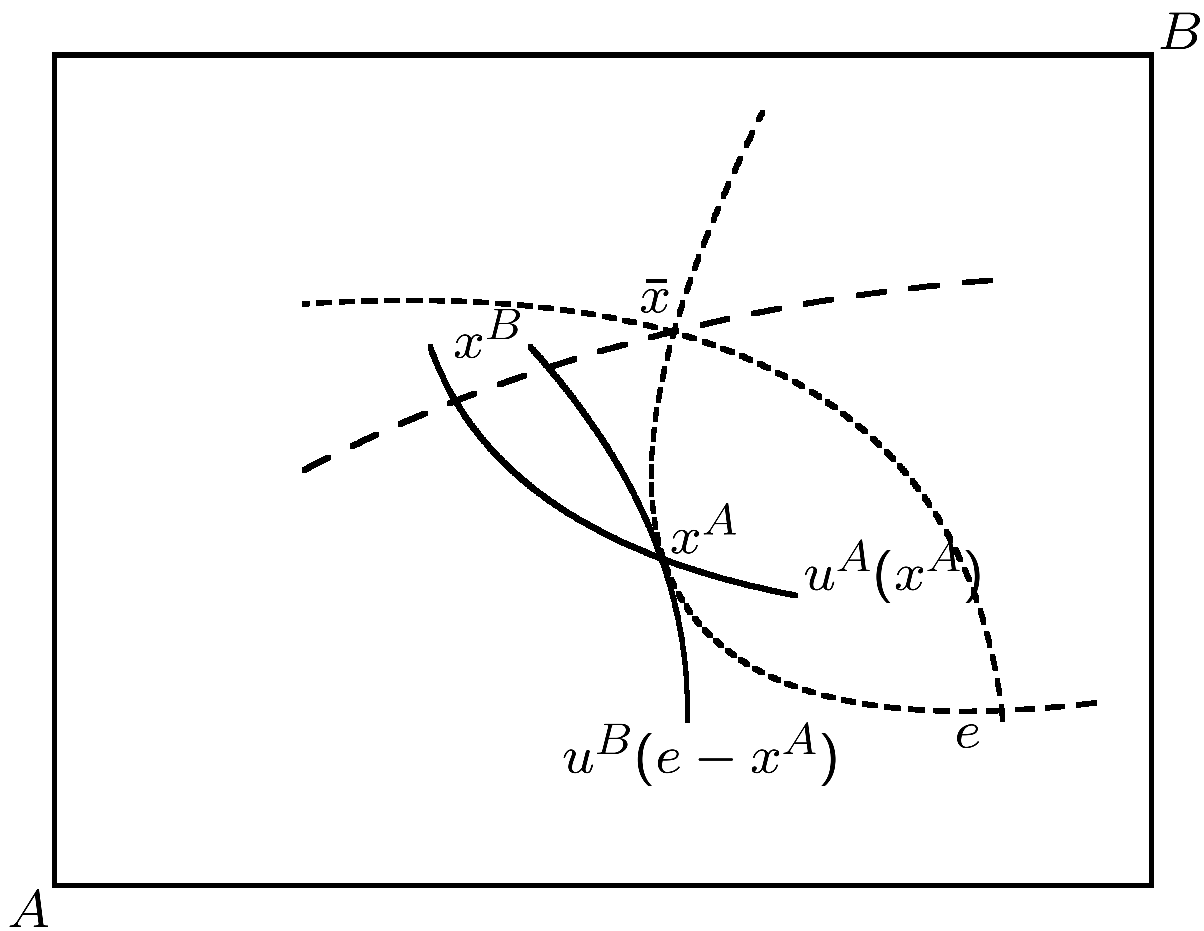
Theorem 2. Every SSP* equilibrium allocation converges to a Walrasian allocation as the agents become infinitely patient.

SSP equilibria with delay

See Merlo and Wilson (1995)

Lemma 3. Whenever the agents are impatient ($\delta^A, \delta^B < 1$), there does not exist any SSP equilibrium with delay.

Consider a candidate equilibrium where A accepts x_A and B always rejects


B
$\bar{x}$
x^B
x^A
$u^A(x^A)$
$u^B(e - x^A)$
e
A

Theorem 3. Every SSP equilibrium allocation converges to a Walrasian allocation as the agents become infinitely patient.

Follows immediately from Lemma 3 and Theorem 2.

Note: When $\delta^A = \delta^B = 1$ there exist a continuum of SSP equilibria with delay (cf. Rubinstein alternating offer bargaining)

Bargaining over allocations – Rubinstein (1982), Ståhl (1972)

z^A is consumption offered to A by B (and likewise z^B)

Equilibrium offer:

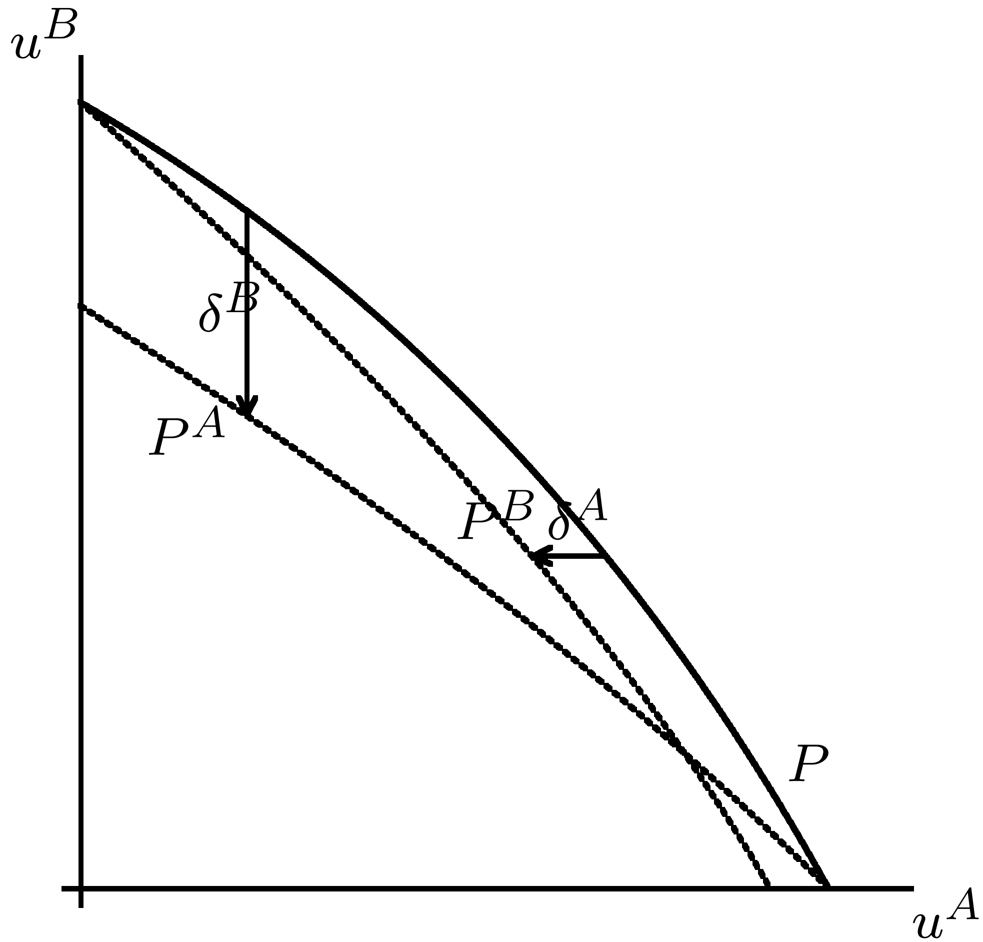
$$\begin{aligned} u^B(z^B) &\geq \delta^B u^B(e - z^A) \\ u^A(z^A) &\geq \delta^A u^A(e - z^B) \end{aligned}$$

Define the profiles:

$$\begin{aligned} P^A &= (u^A(z^A), \delta^B u^B(z^B)) \\ P^B &= (\delta^A u^A(z^A), u^B(z^B)) \end{aligned}$$

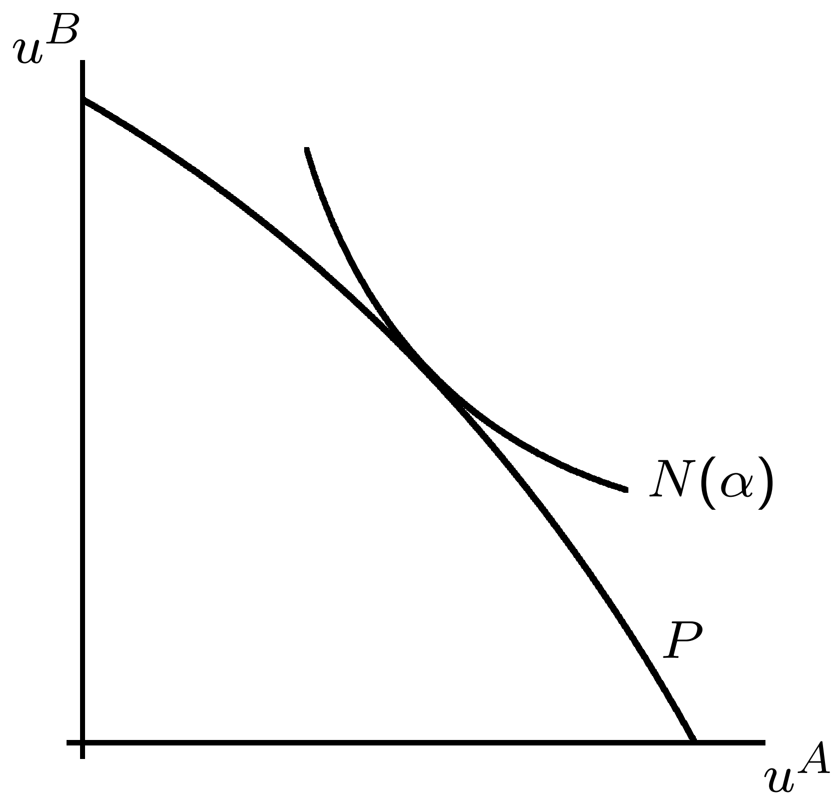
with $z^A + z^B = e$.

The sequence $\left\{(\delta_n^A, \delta_n^B)\right\}_n$ converging to one determines the bargaining outcome



Nash Bargaining

Selects the feasible allocation (z^A, z^B) that maximizes the Nash product $N(\alpha) = u^A(z^A)^\alpha \cdot u^B(z^B)^{1-\alpha}$; the bargaining power α determines the outcome



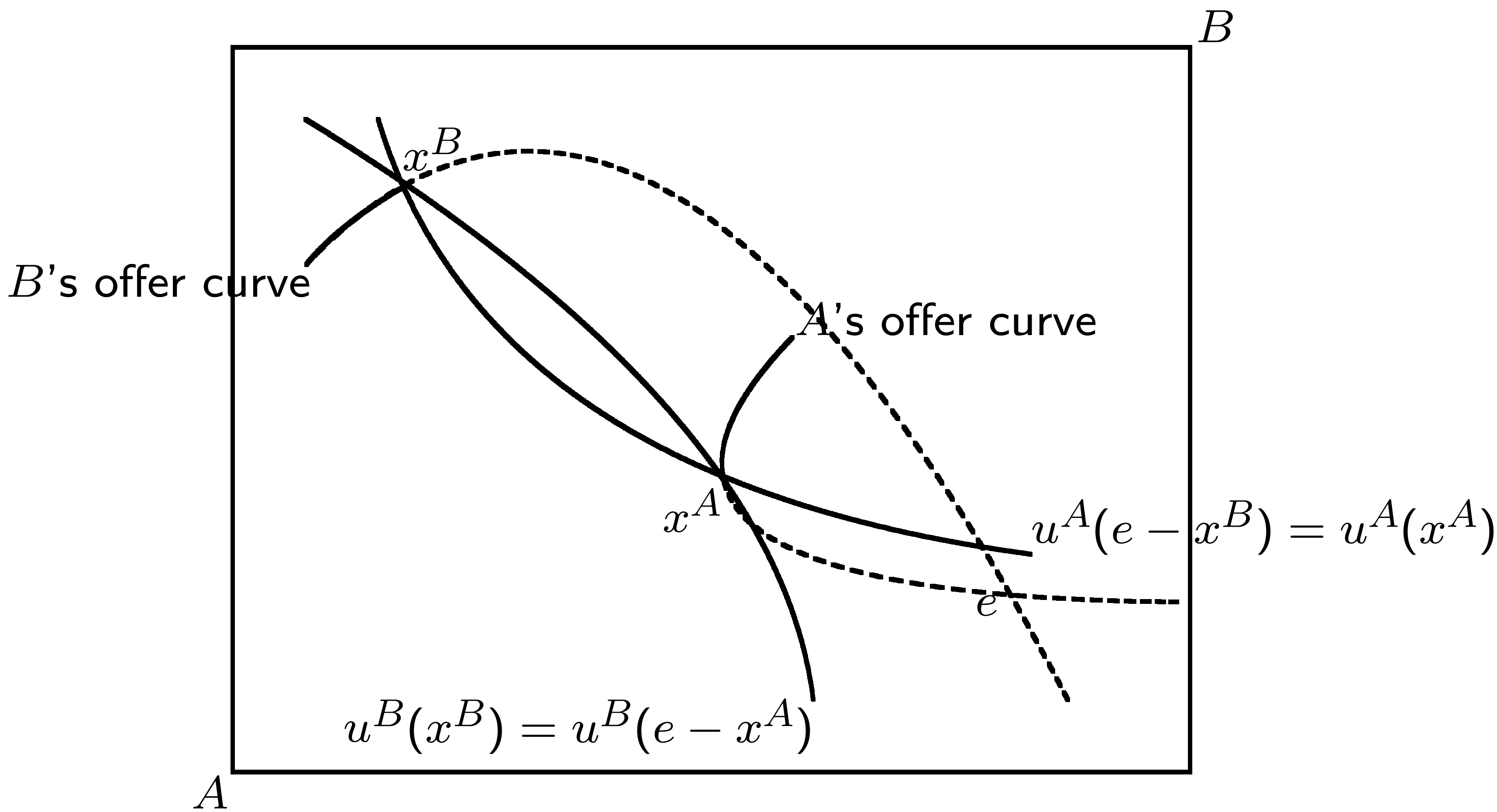
Bargaining over Prices only

Same problem, except for the quantity constraint

Problem:

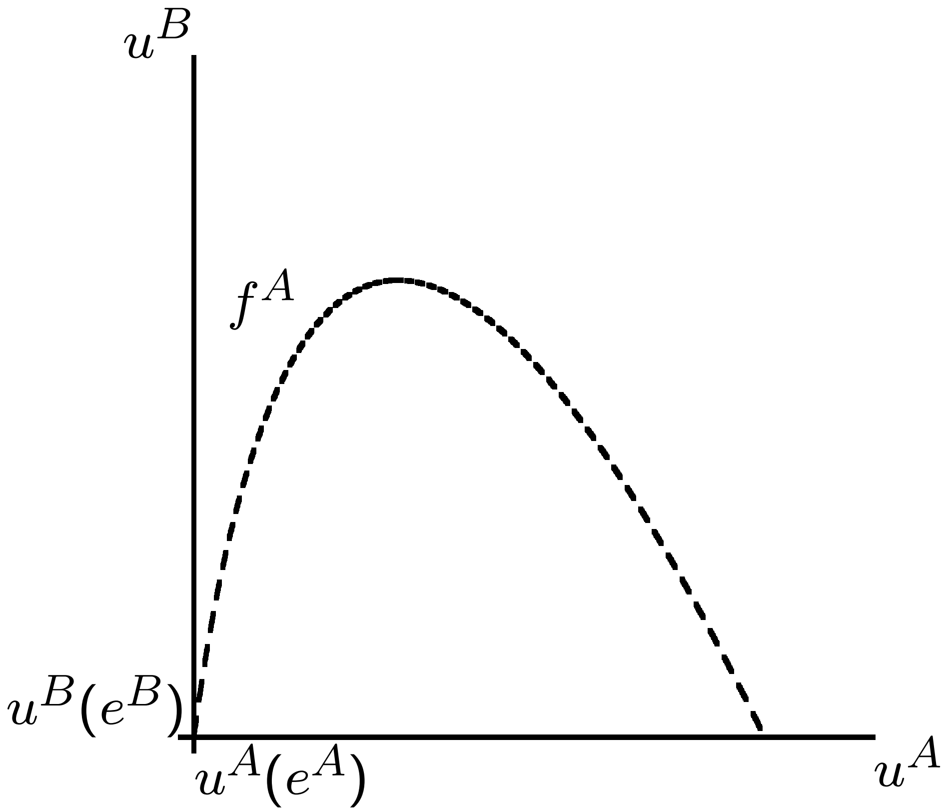
1. there typically exists an SSP equilibrium that is inefficient

See also Yildiz (2002) and Dávila-Eeckhout (2002))



The profiles of utilities

$$f^A_{\delta^B}(p) = (u^A(x^A(p)), \delta^B u^B(e - x^A(p)))$$
$$f^B_{\delta^A}(p) = (\delta^A u^A(e - x^B(p)), u^B(x^B(p)))$$



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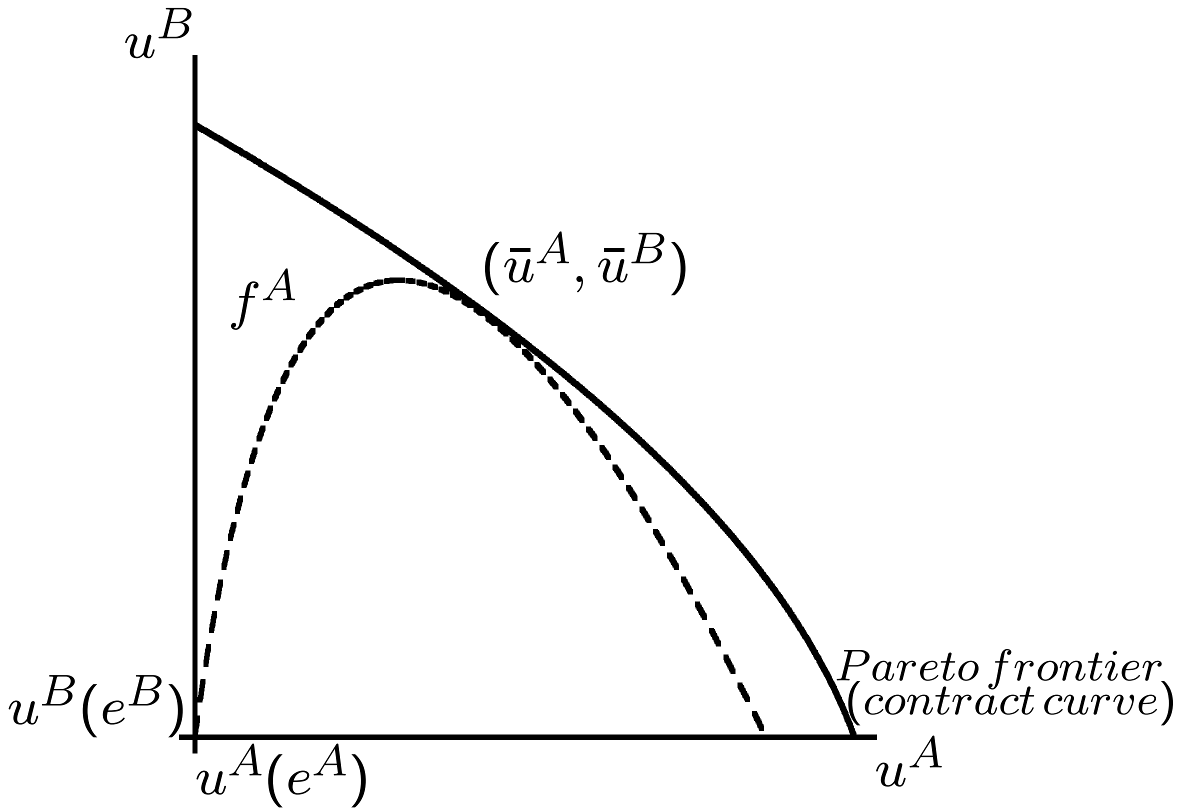

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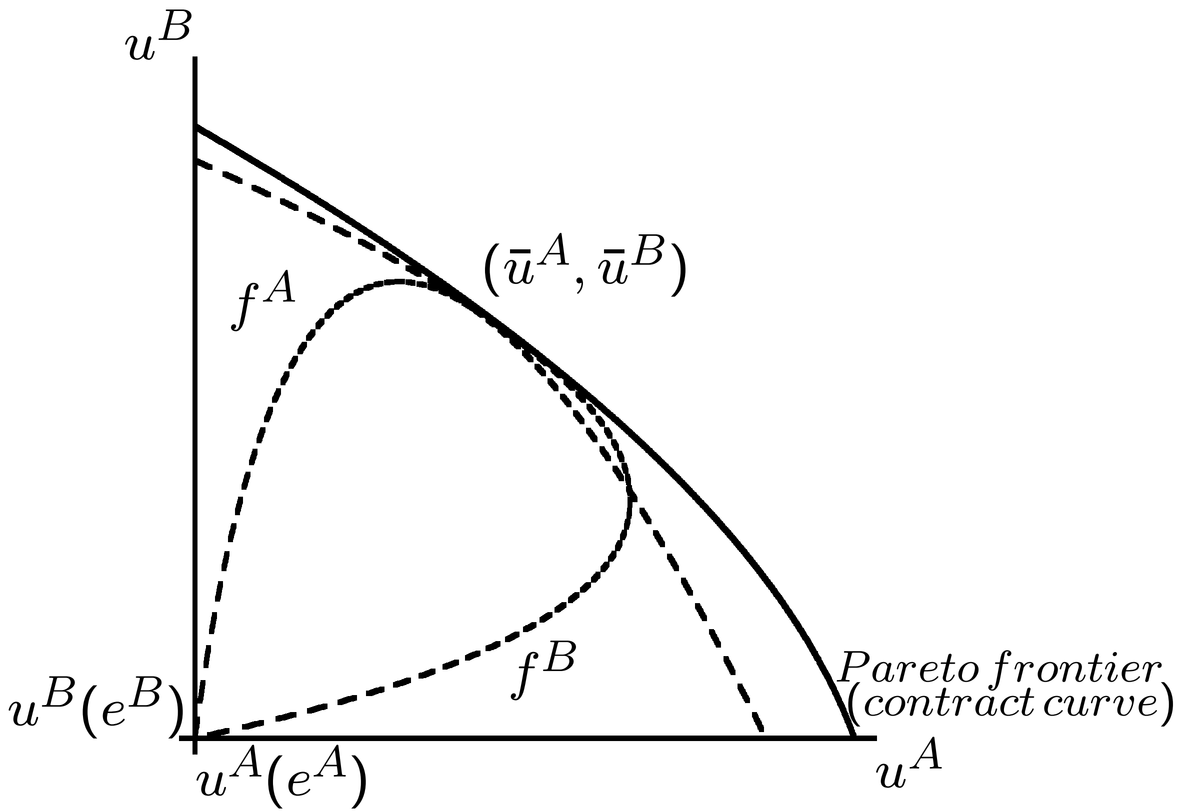


Bargaining over Prices only

Same optimization problem, except for the quantity constraint

Problem:

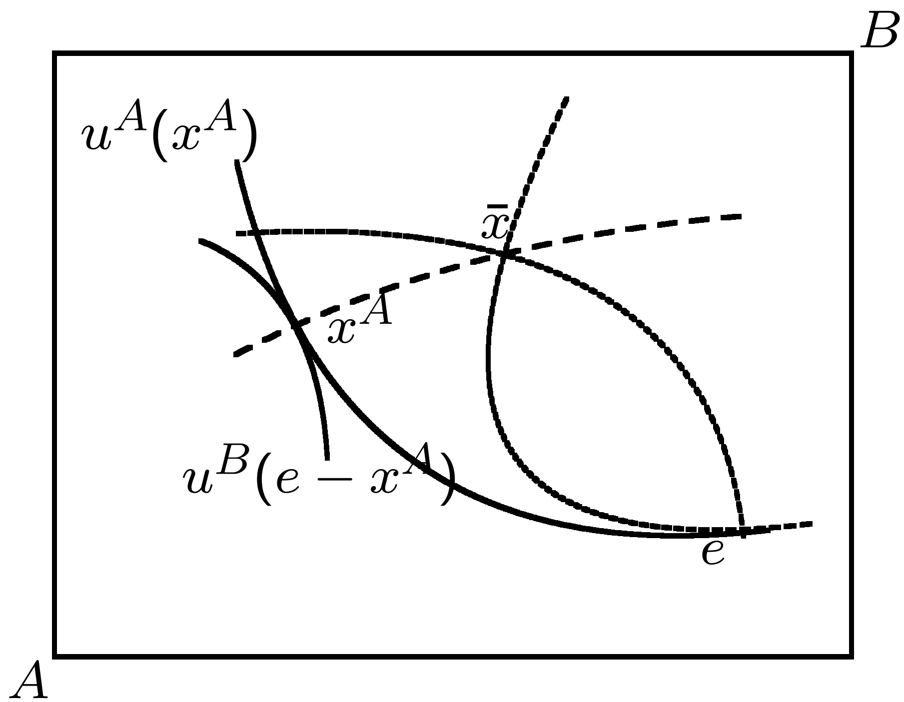
1. there typically exists an SSP equilibrium that is inefficient

2. SSP equilibrium converging to the Walrasian allocation may not exist (depending on sequence of δs converging to 1) and if it exists, there is multiplicity

Bargaining over Prices with minimum quantity constraints

SSP* equilibrium converging to the Walrasian allocation exists

But also SSP equilibria with delay exist



Concluding Remarks

Bargaining protocol that obtains convergence to the WE in absence of price-taking for any economy

Driving force: intertemporal competition (infinity of counter-offers)

Quantity constraint is crucial: rules out inefficient equilibria with different terms offered by each agent that obtain same utility and guarantees existence

Outcome independent of path of δ^A, δ^B or exogenous bargaining power; therefore no indeterminacy (Edgeworth)

Applications of bargaining (e.g. matching model); Can verify Hosios condition from primitives (independent of bargaining power)

"Bargaining over prices with quantity constraint": Examples of related protocols

the dissolution of Partnerships (see a.o. Moldovanu)

union-wage bargaining (see a.o. Farber)

limit orders for selling stock; commodity futures trading;

Yildiz (2003) shows unique convergence to WE under some Assumptions

We find that generically, the intersection at the WE of f^A and f^B is without crossing

A3 (Yilidz): both monopolistic outcomes are dominated by some allocation attainable along an offer curve

A4 (Yildiz): there is a unique crossing of f_A and f_B within the interval defined by the profiles of utilities attained at the monopolistic outcomes

A3 and A4 are non-generic (not satisfied for an open and dense set of economies)

Observe further:

A3 is robust (if it is satisfied for a given economy, then it is also satisfied for all economies in a neighborhood);

A4 is robust;

A3 and A4 jointly are not robust