IDENTIFYING SORTING - IN THEORY

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Motivation

Identifying Sorting: sign and strength

1. Do more productive workers work in more prod. jobs?
   - Positive exercise: learn about production / search process
2. Is sorting important? How big is it?
   - Normative exercise: matters for policy (depends on complementarities)

Motivation

Identifying Sorting: sign and strength

- Constraint: use wage data only (most precise measure of job productivity) and matched employer-employee data
- Objective a minimalist, stylized model (assignment model) that allows us to show:
  1. Identifying the *sign* (1.) is impossible
     Reason: Workers get mainly paid by their marginal product

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  2. Identifying the *strength* (2.) is possible
     Choices reveal how big complementarities/substitutes are.

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     Choices reveal how big complementarities/substitutes are.
  3. Cannot be done with "standard" fixed-effect method

Motivation

The fixed Effects Regression

- Evidence from fixed effects regressions (Abowd, Kramarz, and Margolis (1999), Abowd et al (2004),....):

$$\log w_{it} = a_{it}\beta + \delta_i + \psi_{j(i,t)} + \varepsilon_{it}$$

  where:

  - a_{it}: time varying observables of workers
  - δ_i : worker fixed effect
  - $\psi_{j(i,t)}$: fixed effect of firm (at which i works at t)
  - ε_{it}: orthogonal residual

- Correlation of δ_i and ψ_j between matched pairs is taken as an estimate of the degree of sorting
- Repeatedly established: zero or negative correlation $\Rightarrow$ no complementarities in the production technology?

Motivation

Our approach

- Characterize wages in the frictionless model
- Extend to search frictions $\Rightarrow \exists$ mismatch in equilibrium
- Derive analytically what we can learn from wage data

Relates to recent literature:

- Gautier, Teulings (2004, 2006)
  - Second-order approximation to steady-state; assumes PAM
- Lopes de Melo (2008), Lise, Meghir, Robin (2008), Bagger-Lentz (2008)
  - Simulated search models with strong complementarities give nonetheless small or negative fixed effect estimates

MOTIVATION

OUR FINDINGS

From wage data alone:

1. No identification of sign of sorting from wages:
   - on frictionless equilibrium allocation – Prop 1
   - off-equilibrium set – Prop 2
   - economy with frictions (constant costs) – Prop 3
2. Fixed effects pick up neither *sign* nor *strength* – Prop 4
3. BUT we can identify strength – Prop 5
   This is economically more meaningful than sign
4. Discussion: discounting, type-dependent search costs
   [some, (small) identification], more general technologies...

The Model

Players and Production

- Worker type x, distributed according to Γ (uniform)
- Job type y, distributed according to Υ (uniform)
- Output $f(x, y) \geq 0$
- Common rankings: $f_x > 0$ and $f_y > 0$
- Cross-partials either always positive ($f \in \mathcal{F}^+$ if $f_{xy} > 0$) or always negative ($f \in \mathcal{F}^-$ if $f_{xy} < 0$): monotone matching
- Examples of production functions we will use:

$$
\begin{aligned}
f^+(x, y) &= \alpha x^\theta y^\theta + h(x) + g(y), \\
f^-(x, y) &= \alpha x^\theta (1 - y)^\theta + h(x) + g(y),
\end{aligned}
$$

where $g(\cdot)$ and $h(\cdot)$ are increasing functions.

The Frictionless Model

On the equilibrium path

- Assignment of workers to firms: $\mu(x) = y$ (worker x to firm y)
- Wage schedule: $w(x, y)$
- Profit schedule: $\pi(x, y) = f(x, y) - w(x, y)$

- Equilibrium: (μ, w) such that $\forall x, y$:

$$
\begin{aligned}
w(x, \mu(x)) &\geq w(x, y) \\
\pi(\mu^{-1}(y), y) &\geq \pi(x, y)
\end{aligned}
$$

The Frictionless Model

- Firm maximization:

$$\max_x f(x,y) - w(x,y)$$

- FOC:

$$f_x(x,y) - \frac{\partial w(x,y)}{\partial x} = 0$$

- Let $w^\star(x)$ be the equilibrium wage of worker x

$$w^\star(x) = \int_0^x f_x(\tilde{x}, \mu(\tilde{x}))d\tilde{x} \; + w_0,$$

- Profits: $\pi^\star(y) = \int_0^y f_y(\mu^{-1}(\tilde{y}), \tilde{y})d\tilde{y} \; - w_0$

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- PAM if f supermodular $(f_{xy} > 0) \Rightarrow \mu(x) = x$ (from the SOC)
- NAM if f submodular $(f_{xy} < 0) \Rightarrow \mu(x) = 1 - x$

The Frictionless Model

Cannot identify PAM/NAM

Proposition (1)

For any $f^+ \in \mathcal{F}^+$ that induces PAM there exists a $f^- \in \mathcal{F}^-$ that induces NAM with identical equilibrium wages $w^\star(x)$.

Proof.

$$\begin{aligned} w^{\star,+}(x) &= \int_0^x f_x^+(\tilde{x},\tilde{x})d\tilde{x} \; + w_0 \\ w^{\star,-}(x) &= \int_0^x f_x^-(\tilde{x},1-\tilde{x})d\tilde{x} \; + w_0 \end{aligned}$$

Sufficient: $f_x^+(\tilde{x},\tilde{x}) = f_x^-(\tilde{x},1-\tilde{x})$.

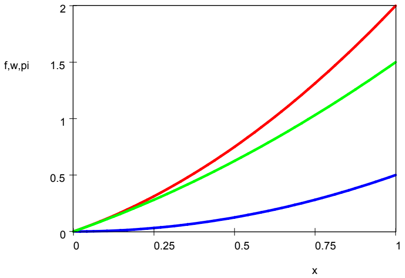
Define: $f^-(x,y) = f^+(x,1-y)$ on $[0,1]^2$

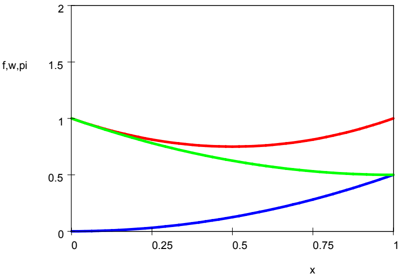
Need: f^- increasing in y. If f_y^- is bounded, add linear term. If not, $g(y)$ increases faster than $-f^+(x,1-y)$ □

The Frictionless Model

Example with $\alpha = +/-1, \theta = 1$

- Wages: $w(x, \mu(x)) = \frac{x^2}{2}$
- Derived from $f^+ = xy + y$ and $f^- = x(1 - y) + y$
- But $\pi^{\star,+}(y) = \frac{y^2}{2}$
  $\pi^{\star,-}(y) = y + \frac{(1-y)^2}{2}$, and $\pi^{\star,-}(x) = 1 - x + \frac{x^2}{2}$





The Frictionless Model

No Identification of PAM/NAM

- Based on wage data alone, we cannot "know" which are the good jobs (higher ranked y)
- The good worker matches with the most attractive firm
- Under NAM, the bad firm is more attractive because it pays higher wages

Mismatch due to Search Frictions

Mismatch in Equilibrium

Two Stage Search Process:

1. First, costless random meeting stage
   - one round of pairwise random meetings
   - if match is formed: wage as split of surplus over waiting
2. Second, if not matched: costly competitive matching
   - pay search cost c each
   - get matched according to the competitive assignment
   - production at end

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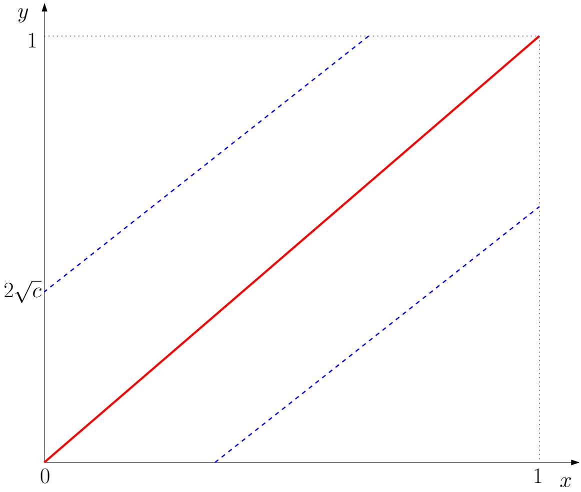
- For simplicity assume symmetry
  - $f_{xy}(x,y) = f_{xy}(y,x)$ for $f \in \mathcal{F}^+$
  - $f_{xy}(x,y) = f_{xy}(1-y,1-x)$ for $f \in \mathcal{F}^-$
- Second stage payoffs: $w(x,\mu(x)) - c$ and $\pi(\mu^{-1}(y),y) - c$
- First stage: Match provided

$$f(x,y) - (w^\star(x) + \pi^\star(y) - 2c) \geq 0$$

Mismatch due to Search Frictions

The Example: $\theta = 1$



Mismatch due to Search Frictions

Wages

$$
\begin{aligned}
w(x,y) &= \frac{1}{2}\left[f(x,y) - w(x,\mu) - \pi(\mu^{-1},y) + 2c\right] + w(x,\mu) - c \\
&= \frac{1}{2}\left[f(x,y) + w(x,\mu(x)) - \pi(\mu^{-1}(y),y)\right]
\end{aligned}
$$

Mismatch due to Search Frictions

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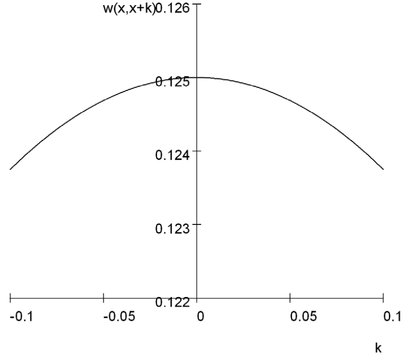
&= \frac{1}{2}\left[f(x,y) + w(x,\mu(x)) - \pi(\mu^{-1}(y),y)\right]
\end{aligned}
$$

- From wages alone we cannot identify the sign of f_{xy}
- Here: we aim to identify the strength of f_{xy} (i.e. $|f_{xy}|$)

Mismatch due to Search Frictions

Bliss Point

Lemma: *(Bliss Point) Wages $w(x, y)$ are non-monotone in y.*



- Example. Mediocre lawyer in top firm: paid less than in mediocre firm. Top firm must forego higher future profit

Mismatch due to Search Frictions

Inconclusive Firm Fixed Effect

Decompose wage process:

$$w(x,y) = \delta(x) + \psi(y) + \varepsilon_{xy}, \tag{1}$$

Unbiased δ and ψ (integrate over y and x, respectively)

$$\delta(x) = \int_{B(x)} [w(x,y) - \psi(y)]\, d\Upsilon(y|x), \tag{2}$$

$$\psi(y) = \int_{A(y)} [w(x,y) - \delta(x)]\, d\Gamma(x|y), \tag{3}$$

Firm fixed effect δ is constant if Ψ is constant:

$$\psi(y) = \underbrace{\int_{A(y)} [w(x,y) - w_{av}(x)]\, d\Gamma(x|y)}_{=:\Psi(y)} + \int_{A(y)} \int_{B(x)} \psi(\tilde{y}) d\Upsilon(\tilde{y}|x) d\Gamma(x|y) \tag{4}$$

Mismatch due to Search Frictions

Inconclusive Firm Fixed Effect

Proposition (4)

The firm fixed effect is ambiguous. It is zero under uniform distributions and $f(x,y) = \alpha xy + h(x) + g(y)$.

- The firm effect Ψ is

$$\Psi(y) = \int_{y-K}^{y+K} \left[w(x,y) - w_{av}(x)\right] d\Gamma(x|y)$$

- Assuming a long panel: $w_{av}(x) = \int_{x-K}^{x+K} w(x,y) d\Upsilon(y|x)$

- Show that $\Psi' \gtrless 0$

$$\begin{aligned} \Psi'(y) \quad = \quad & \int_{y-K}^{y+K} \frac{\partial w(x,y)}{\partial y} \gamma(x|y) dx \\ & + \left(w(y+K,y) - w_{av}(y+K)\right) \gamma(y+K|y) \\ & - \left(w(y-K,y) - w_{av}(y-K)\right) \gamma(y-K|y) \end{aligned}$$

- First effect: change in matched type (Beckerian effect)
- Second effect: change in set of matched partners
- Both effects: ambiguous, often opposite sign, zero under uniform

Identifying the Strength of Sorting

Without Knowing the Sign

Proposition (5)

We can identify strength of sorting, i.e., cross-partial $|f_{xy}|$.

Two parts:

1. Use *wage gap* to identify the cost of search c
2. Use *range* of matched types to identify $|f_{xy}|$

1. Wage Gap

- Maximum wage in panel: identify type (optimal = max):

$$\overline{w}_k = \max_{t \in \{1,\dots,T\}} w_k^t$$

- $\Omega_W(\overline{w})$: distribution of maximum wages ($\Omega_F(\overline{w})$ for firms)
- Identify search by wage gap(where $\underline{w}_x = \min_{t \in \{1,\dots,T\}} w_x^t$):

$$c = \overline{w}_x - \underline{w}_x,$$

Identifying the Strength of Sorting

Without Knowing the Sign

2. Range of Matched Types

- Search loss $L(x,y)$ due to mismatch:

$$\begin{aligned} L(x,y) &= f(x,y) - \int_0^x f_x(\tilde{x},\mu(\tilde{x}))d\tilde{x} - \int_0^y f_y(\mu^{-1}(\tilde{y}),\tilde{y})d\tilde{y} \\ &= -\int_{\mu^{-1}(y)}^x \int_{\mu^{-1}(\tilde{y})}^x |f_{xy}(\tilde{x},\tilde{y})|d\tilde{x}d\tilde{y} \\ &= -\int_y^x \int_{\tilde{y}}^x |f_{xy}(\tilde{x},\tilde{y})|d\tilde{x}d\tilde{y} \quad \text{(for PAM)} \end{aligned}$$

- Search decision: $L(x,\underline{y}(x)) = -2c$.
- This functional equation identifies $|f_{xy}|$: compares variation in matching sets $(x-\underline{y}(x))$ to variation in wage $(2c)$
- If wage variation high, matching sets small $\Rightarrow$ large loss from mismatch, i.e. the cross-partial large

Identifying the Strength of Sorting

Without Knowing the Sign

- More structure (example): constant cross-partial α, then

$$-L(x,y) \quad = \quad |\alpha|(x^{\theta} - \underline{y}(x)^{\theta})^2 = 4c$$

use data on observed pairs x, y to estimate α, θ

Identifying the Strength of Sorting

Without Knowing the Sign

- More structure (example): constant cross-partial α, then

$$\begin{aligned} -L(x,y) &= |\alpha|(x^{\theta} - \underline{y}(x)^{\theta})^2 = 4c \\ &\Leftrightarrow x = \left(2\,(c/|\alpha|)^{1/2} - \underline{y}(x)^{\theta}\right)^{1/\theta} \end{aligned}$$

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Identifying the Strength of Sorting

Without Knowing the Sign

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&\Leftrightarrow \quad x = \left(2\,(c/|\alpha|)^{1/2} - \underline{y}(x)^\theta\right)^{1/\theta}
\end{aligned}
$$

  use data on observed pairs x, y to estimate α, θ

- Total loss from search (mismatch minus perfect matching):

$$
\mathcal{G} = \int_0^1 \int_0^1 L(x,y)dxdy = -|\alpha| \frac{\theta^2}{(2\theta+1)(\theta+1)^2}.
$$

General Costs and Bargaining

Wage equation:

$$w(x,y) = \gamma[f(x,y) - w^\star(x) - \pi^\star(y) + c(x) + k(y)] + w^\star(x) - c(x),$$

where $w^\star(x) - c(x)$ is the outside option. At the cutoff type:

$$\underline{w}(x) = w^\star(x) - c(x),$$

In the second period:

$$f(x^\star, y) = w^\star(x^\star) - \pi^\star(y) \Rightarrow w(x^\star, y) = \gamma[c(x^\star) + k(y)] + \underline{w}(x^\star),$$

which implies

$$c(x) + k(y) = -\frac{w(x^\star, y) - \underline{w}(x^\star)}{\gamma}.$$

We get identification from $L = c(x) + k(y)$ evaluated at $x^\star(\underline{y})$.

Alternative Approaches

- Use of output/profit data.
  But mostly available at firm level: how to attribute profits to an individual (CEO vs. factory worker)? (Haltiwanger et al. (1999), van den Berg and van Vuuren (2003), Mendes, van den Berg, Lindeboom (2007))

  $\Rightarrow$ Need at least a theory of the firm
- Exogenous wage setting: Abowd, Kramarz, Lengermann, Perez-Duarte (2009):
  - "test a simple version of Becker's matching model"
  - *assume* a split of output: $\beta f(x, y)$
  - is *inconsistent* with Becker's (1973) equilibrium wages

CONCLUSIONS

- We cannot identify the sign of sorting from wage data
- We can identify the strength: economically relevant
- Standard fixed effects get neither sign nor strength

- Discussion
  1. Identifying sign: attributing profit or output data
  2. More general technologies: horizontal vs vertical diff
  3. Different reasons for mismatch (e.g. productivity shocks)
  4. Type-Dependent Search Costs (e.g. discounting)
  5. On-the-job Search

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Type-Dependent Search Costs

Discounting – Shimer-Smith (2000)

Result: Non-monotone wages also under discounting

- Discount factor β. Technology $f^{+}(x,y) = xy$
- 1st period wages (surplus matching (split) + value waiting):

$$\begin{aligned} w^{+}(x,y) &= \frac{1}{2}\left[xy - \beta\frac{x^2}{2} - \beta\frac{y^2}{2}\right] + \frac{1}{2}\beta\frac{x^2}{2} \\ &= \frac{1}{2}xy + \beta\frac{x^2}{4} - \beta\frac{y^2}{4} \end{aligned}$$

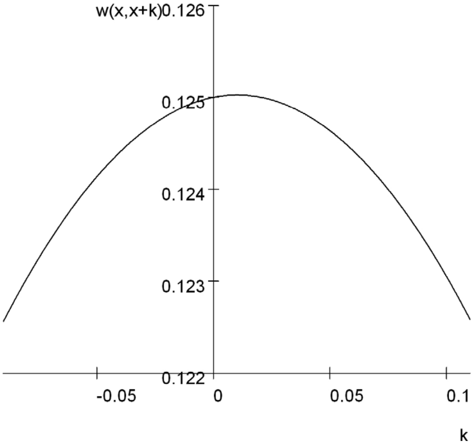
- Match if surplus is positive. [Matching set $A(y) = \left[\underline{K}y, \overline{K}y\right]$, $K = \beta^{-1} \pm \sqrt{\beta^{-2} - 1}$.]
- Under NAM technology, $f^{-}(x,y) = -xy + y$

$$w^{-}(x,y) = \frac{1}{2}x\tilde{y} + \beta\frac{x^2}{4} - \beta\frac{\tilde{y}^2}{4} + \frac{1}{2}(1-\beta)(1-\tilde{y})$$

- $w^{+} \approx w^{-}$ small when $\beta \approx 1$: some, but small sign ident.
- Wage is also inverted U-shaped

Mismatch due to Search Frictions

Non-monotone Wages under Discounting



Non-monotonicities arise generally

General Type-Dependent Search Costs

Non-monotonicities with general search costs:

$$f(x,y) - (w^\star(x) + \pi^\star(y) - c(x) - c(y)) \geq 0.$$

Discounting: $c(y) = (1-\beta)\pi^\star(x)$

Differing arrival rates: $c(y) = (1-\alpha(y)\beta)\pi^\star(x)$

Wages are non-monotonic (whenever $c'(y) \leq y$):

$$w(x,y) = \frac{1}{2}xy + \frac{1}{4}x^2 - \frac{1}{4}y^2 - \frac{1}{2}c(x) + \frac{1}{2}c(y)$$

$$\Rightarrow \qquad \partial w/\partial y = \frac{1}{2}x - \frac{1}{2}y + c'(y)$$

- Non-monotonicities arise *always* when higher types reject some lower types (because then workers obtain their continuation value at the highest and lowest type willing to match).
- Even with OJS (fixed entry cost, then type realized): No opportunity cost for worker, but usually the firm cannot search while matched, and some matches are not formed.