# Diverse Organizations and the Competition for Talent

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# Motivation

## Diversity in Problem Solving

- Problem solving in organizations. Here: focus on diversity
- Groups of agents with mixed ability outperform groups of identical ability (even if all are high skilled)
- Adding lower skilled type to a group of experts can increase productivity more than adding another expert
- Scott Page: Theory, Evidence, Simulations; Casual evidence (Southwest, chess players experiment,...)
- Two interpretations possible: hierarchies/polyarchies

# Motivation

## Diversity in Problem Solving

Objective:

1. Build a simple theory of diversity within the organization
   - Arrival new solutions: non-homogeneous Poisson process
   - Standard aggregation over $\neq$ skills
2. Put the organization in a competitive labor market
   - A continuum of firms/organizations compete for skilled labor
   - Wages are determined competitively
   - Trade off: internal diversity – external prices
3. Tractable General Equilibrium model economy: address role of firm in aggregate economy

# RESULTS

- The firm size is endogenous: increasing in firm TFP
- Skill distribution is endogenous and non-degenerate
- Identically distributed organizations $\iff$ CES
- Diverse Organizations:
  1. First-order Stochastic Dominance of skill distribution: Larger firms have heavier right tails
  2. Large firms hire “more broadly” (larger support)
  3. Predictions about “organigram” of the organization:
     - “taller”: the CEO is more skilled
     - rank: given skill has high rank in small firm; low rank in large
- Evolution of organizations: tech. progress $\Rightarrow$ downsizing
- Investment: endogenous heterogeneity in skill distribution
- Productivity: back out TFP distribution across firms

# The Model

## Set Up

Agents:

- Measure 1 of agents endowed with skill $x$;
- $x$: initially discrete types, later continuous
- $m(x) :=$ measure of workers with skill $x$.

Firms:

- $A :=$ Firm-specific Total Factor Productivity (TFP)
- $\mu(A) :=$ measure of firms with TFP $A$.

# The Model

## The problem-solving technology

- $n(x)$ the measure of workers of skill $x$ in the firm
- Within a skill type $x$
  - Solution probability: a non-homogeneous Poisson process with arrival rate $\lambda(n)$ (assume: $\lambda' < 0$).
  - The expected number of problems solved: $h(n)x$ where

$$h(n) = \int_0^n \lambda(s)\, ds.$$

# The Model

## The problem-solving technology

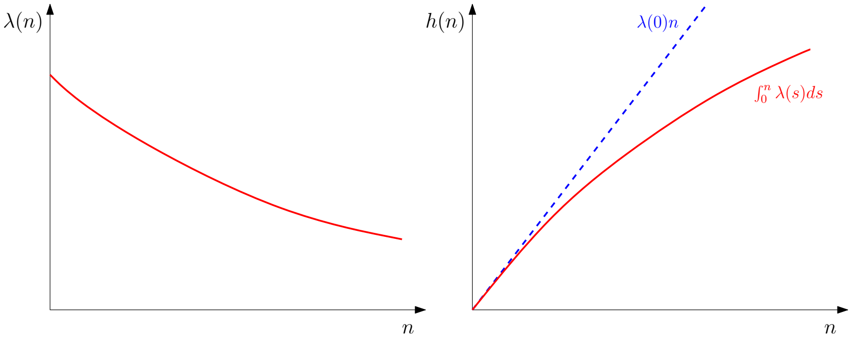


Figure: A. The non-homogeneous Poisson arrival rate; B. The expected number of problems solved.

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$$L(\mathbf{n}) = \left[ \sum_{i=1}^{N} h(n_i) x_i \right]^{\beta}$$

where $\beta > 0$.

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  where $\beta > 0$.
- Firm-level production of output:

$$y = AL(n(x))$$

# The Model

## Continuous Type Distribution

- Let $m(x) = F(x) - F(x - \Delta)$, $\mu(A) = G(A) - G(A - \Delta)$
- Dividing expressions by $\Delta$ and taking $\Delta \to 0$
- Firm's production function becomes:

$$L(\mathbf{n}) = \left[ \int h(n(x)) x dx \right]^{\beta}$$

- Where $x \sim F(x)$, $A \sim G(A)$, with support $[\underline{x}, \overline{x}]$ and $[\underline{A}, \overline{A}]$

# The Model

## The Firm's Problem and Equilibrium

- Markets are competitive; atomless firms are price takers
- Given a vector of wages $w(x)$, firm $A$'s problem is:

$$\pi_A = \max_{n_1,\ldots,n_N} A\left[\sum_{i=1}^{N} h(n_i)\, x_i\right]^{\beta} - \sum_{i=1}^{N} n_i w(x_i)$$

- A competitive equilibrium in this economy:

1. Firms maximize profits $\pi_A$;
2. Workers choose job with the highest wage offered $w(x)$;
3. Markets clear.

# Properties of the Production Technology

## Elasticity of Substitution:

The Elasticity of Substitution between inputs $n_i$ and $n_j$, denoted by $\sigma$, is defined as:

$$\sigma = \frac{d \ln \left( n_j / n_i \right)}{d \ln \left( TRS_{ij} \right)}$$

Then:

$$\sigma = -\frac{h' \left( n_i \right)}{h'' \left( n_i \right)} \frac{1}{n_i}.$$

# Identically Distributed Organizations
## CES

### Lemma

*The following two statements hold for $a, b, \gamma$ constants:*

1. *El. $\sigma$ is constant if and only if $h(n_i)$ is of the form $a + bn_i^\gamma$;*
2. *$L(n)$ is homothetic if and only if $h(\cdot)$ is of the form $a + bn_i^\gamma$.*

- The production function is CES iff

$$L = \left[ \sum_{i=1}^{N} \left( a + bn_i^\gamma \right) x_i \right]^\beta .$$

- Recall: “standard” CES vs. more general CES

$$\left[ \sum_{i=1}^{N} bn_i^\gamma x_i \right]^{1/\gamma} \quad \text{vs.} \quad \left[ \sum_{i=1}^{N} \left( a + bn_i^\gamma \right) x_i \right]^\beta$$

# Identically Distributed Organizations $\iff$ CES

## Proposition

*Firms have the same skill distribution $F_A(x) = F(x) \iff$ the production technology is CES.*

- For CES, from the FOC:

$$\frac{n_i}{n_j} = \left(\frac{w\left(x_j\right)x_i}{w\left(x_i\right)x_j}\right)^{\frac{1}{1-\gamma}}$$

- Imposing market clearing, the demand is given by:

$$n_j\left(A\right) = \frac{A^{\frac{1}{1-\gamma\beta}}m\left(x_j\right)}{\sum_A A^{\frac{1}{1-\gamma\beta}}\mu\left(A\right)}.$$

- Under CES, demand is proportional to total expenditure:

$$\frac{n_j\left(A\right)}{n\left(A\right)} = \frac{m\left(x_j\right)}{m}$$

# Identically Distributed Organizations

## Characterization

### Proposition

*Under CES:*

1. *There is full support of the distribution of all firms; and*
2. *There is no firm size-wage premium (firms of different sizes pay identical average wages)*

- All firms hire "tiny fraction of GE's Jack Welch"
- Necessary (not sufficient): initially, infin. arrival of solutions

$$\lim_{n \to 0} \lambda(n) = \infty.$$

- More productive firms (higher $A$) are larger

# Identically Distributed Organizations

## An Important Caveat

- Technology always quasi-concave, strictly concave: $\beta < \frac{1}{\gamma}$
- Profits are not quasi-concave when $\beta > \frac{1}{\gamma}$
- General: $\beta$ sufficiently large, $\exists$ monopoly power (extreme: all workers should be in the superior technology firm)
- We implicitly assume DRTS: $\beta$ is not too large

# Diverse Organizations

- Assume $\exists$ no infinite problem-solving ability:

$$\overline{h}' = \lim_{n \to 0} h'(n) = \lambda(0) < \infty.$$

- The FOC for $n_i : h'(n_i) \leq \frac{w(x_i)}{Ax_i}, \quad \forall i \in \{1, ..., N\}$
- Demand:

$$n_i(A) = \begin{cases} h'^{-1}\left(\frac{w(x_i)}{Ax_i}\right) & \text{, if } \quad A \geq \underline{A}(x_i) \\ 0 & \text{, otherwise} \end{cases}$$

- $\underline{A}(x_i)$: lowest TFP firm for which FOC is strict
- $\exists$ upper bound on the hired skills and it differs for $\neq$ firms $A$
- Mom-&-pop stores do not hire (fraction of) Jack Welch

# Diverse Organizations

## Size of Firm

### Proposition

*Firms with higher A have a larger labor force of each type*

- True for *all* technologies
- From complementarity TFP–labor

### PROPOSITION

*If $f'(x_i) < 0$, the highest skilled worker $x_{CEO}(A)$ is increasing in $A$ and therefore in the size of the firm.*

- "Taller": the CEO is more skilled
- Higher TFP firms are will "outbid" mom-&-pop store
- $x_{CEO}(A) = \frac{w(x_i)}{\overline{h}' A}$.

### COROLLARY

*Smaller firms hire from a smaller range of skills than larger firms: supp $f_{\underline{A}} \subset$ supp $f_{\overline{A}}$ for all $\underline{A} < \overline{A}$.*

- Large firms hire "more broadly" (larger support)

### Proposition

*There is single-crossing of the densities:* $\frac{d^2\left(\frac{n_i(A)}{n(A)}\right)}{dAdx_i} > 0$

### Proposition

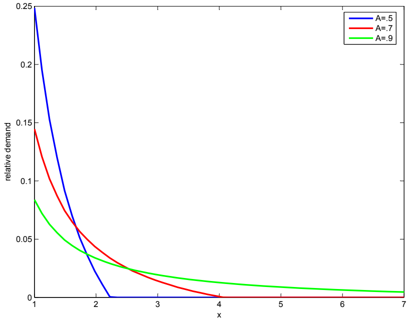
*(Stochastic Dominance). The skill distribution of larger firms stochastically dominates that of smaller firms.*

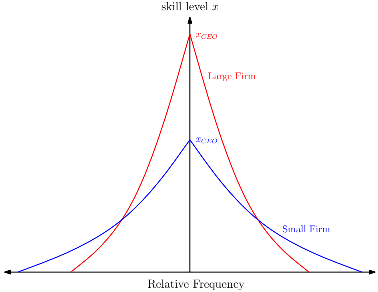
- Larger firms have heavier right tails
- Shape is "leaner": fewer middle managers
- Rank: given skill, high rank in small firm; low in large firm

# Diverse Organizations

## Distribution of Skills – Example

Expon. Decay: $\lambda(n) = e^{-n}$; Skill dist. Pareto. Firms uniform.



# Diverse Organizations

## Firm Size – Wage Premium

### Proposition

*(Firm Size – Wage Premium). Larger firms pay higher wages than smaller firms.*

- Higher average wages: larger *and* more productive firms
- Wage CEO higher in larger/more productive firms

# The Evolution of Diverse Organizations

## Technological Progress ⇒ Downsizing

- Technological Progress: all firms become more productive
  ⇒ First-Order Stochastic Dominance of TFP

### Proposition

*As distribution of TFP First-Order Stochastically Dominates:*

1. *Given A, firms are smaller: $n(x)$ demanded decreases;*
2. *Wages increase;*
3. *The type of the CEO $x_{CEO}$ decreases, given A.*

- Wage pressure from increased competition ⇒ downsizing
- In a more competitive market: accept *worse* CEO
- But: employment size distribution in economy: ambiguous

- Increasing marginal productivity $h'(\cdot)$
- Parameterize: $\frac{dh'(n;a)}{da} < 0$, and $a$ increases, we have:

### Proposition

*As the marginal productivity increases* $\frac{dh'(n;a)}{da} > 0$, *all wages increase.*

- Wages reflect increased productivity
- Demand effect ambiguous: $A \uparrow \Rightarrow$ more demand for skills; but $w \uparrow \Rightarrow$ less demand for skills

Consider an economy with:

- Ex ante identical workers
- Cost $C(x_i) = a + c(x_i)$, $a \geq 0$, $c(x_i)$ convex and $c(0) = 0$.
- Given ex ante identical workers, in equilibrium:

$$w(x_i) = a + c(x_i), \quad \forall x_i \in (0, \overline{x})$$

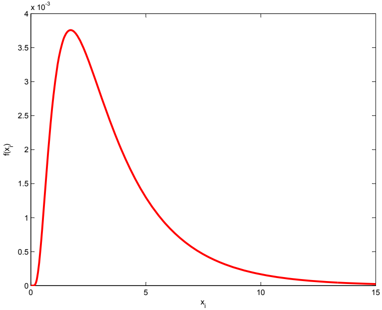
### Proposition

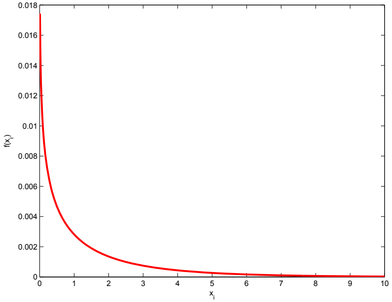
*The equilibrium distribution of skills is always uni-modal and has a long right tail. When there is no fixed cost of investment ($a = 0$), the density is everywhere downward sloping.*

# Investment in Skills

## Example

- Exponential decay in $\lambda$, $c(x) = cx^2$ and $A$ exponentially distributed. Distribution of skills with/without fixed cost ($a > 0$ or $a = 0$)

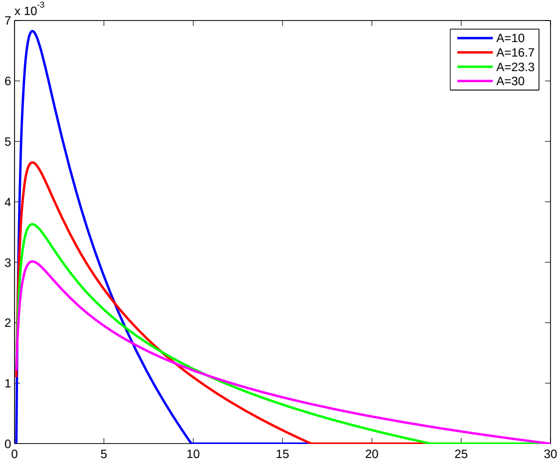


# INVESTMENT IN SKILLS

## EXAMPLE

- Within firm, more unequally distributed skills as $A$ is higher



# Distribution of TFP across Firms

- Productivity: desirable to know, hard to measure directly
- Model: at the skill level of the CEO, $h'(n)$ is evaluated at zero, and common to all firms. Identify $A$ from CEO only:

$$A = \frac{w(x_{CEO})}{h'(0)x_{CEO}}.$$

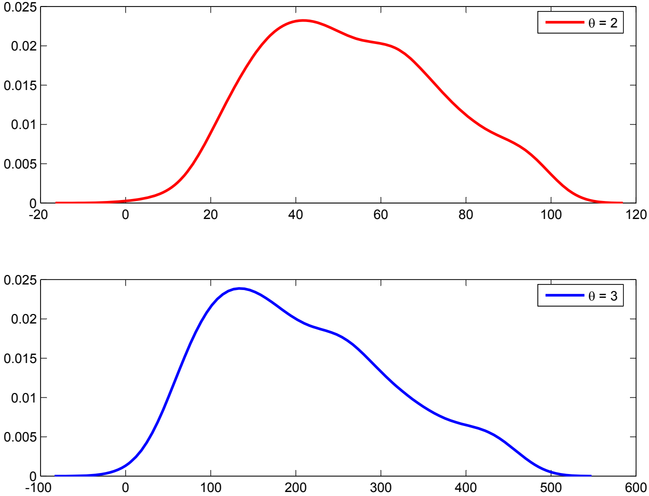
- Instead of using the CEO skill level $x_{CEO}$, we can also use the investment. With cost of investment function $C(x) = bx^{\theta}$, in equilibrium $bx^{\theta} = w(x)$ and we can write

$$A = Kw(x_{CEO})^{1-1/\theta},$$

  where $K = \frac{b^{1/\theta}}{h'(0)}$ is a constant.
- Obtain distribution TFP ($A$) from CEO compensation

# Distribution of TFP across Firms

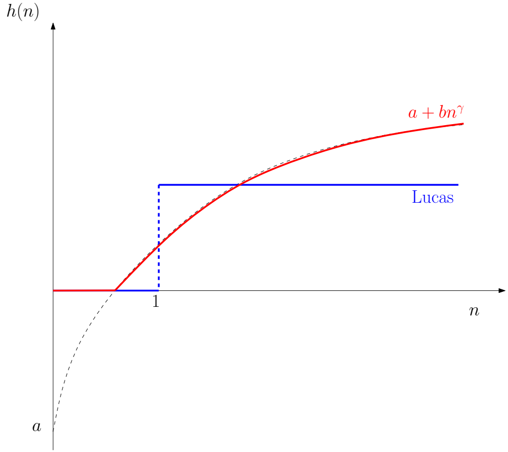
- Using Compustat Executive Compensation Data: Estimated TFP distribution for values $\theta = 2$ and $\theta = 3$.



# Discussion and Extensions

## Lucas (1978) Span of Control

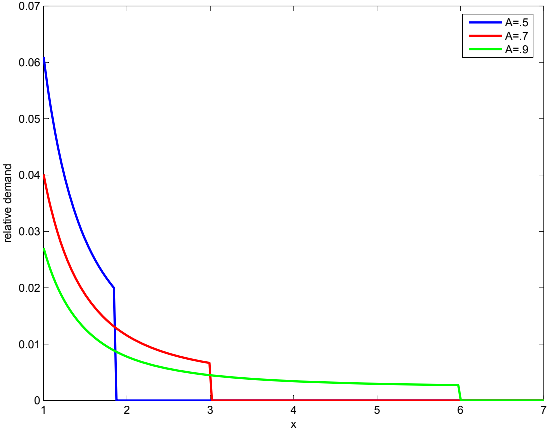
- Instead of 1 manager, CES with fixed cost of employment



# Discussion and Extensions

## Lucas (1978) Span of Control

- Diverse organizations with *truncated* CES
- Equil. Distribution truncated: need *sufficient* CEO skills



# Discussion and Extensions

## Decreasing Elasticity $\sigma$

- $\lambda(0) = h'(0)$ bounded necessary and sufficient for full support
- It is sufficient, not necessary for diverse organizations

### Proposition

*Let $\sigma' < 0$. If the density of $x$ is decreasing then:*

1. *All firms hire workers of all types (full support distributions);*
2. *Average skills and average wages are higher in larger firms than in smaller firms;*
3. *The skill and wage distribution in larger firms First-Order Stochastically dominates those in small firms.*

# Discussion and Extensions

## Productivity of Job from Firm Profits: Needed, a Theory

- Identifying complementarity: do skilled workers produce more in more productive jobs? Evidence on sorting.
- Based on wage data alone: fixed effects regressions conclude: NO complementarities.
- Recent results: fixed effects are not informative; wages are non-monotonic in job productivity
- Why not use profit data as well? Need a theory to *attribute* firm profits to job profits
- Simple attribution rules (e.g. job profits proportional to wages: $\pi_i / \sum \pi_i = w_i / \sum w_i$): strong restrictions on skill distribution

# CONCLUSION

- A simple model of diverse organizations in General Competitive Equilibrium
- Equilibrium: heterogeneity *within* firm and *between* firms
- In terms of the predictions: $\lim_{n \to 0} h'(n) < \infty$ is the most reasonable scenario
- CES is convenient for “representative-organization” models, not for diverse organizations
- Evidence?
    - Employer Size - Wage Effect
    - Skill and salary of CEO is higher in larger firms (Robert’s law (1956), Gabaix and Landier (2008))
    - Firm Productivity – Wage Effect