Assortative Matching with Large Firms

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November 10, 2016

Motivation

- Two cornerstones of analyzing firms in Macro, Labor, IO, Trade,...
  1. Firm size: productive firms are larger and produce more
  2. Sorting of workers: firms compete for skilled workers
- These two aspects are usually treated independently
  1. Firm Size (Lucas 1978, Hopenhayn 1992) $\rightarrow$ intensive margin
  2. Matching: one-to-one (e.g. Becker 1973) $\rightarrow$ extensive margin
- Needed: Trade-off better workers vs. more workers

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$\rightarrow$ Apply theory to technological change: SBTC vs. QBTC

The Model

Intensive and Extensive Margin

- *Population*
  - Workers of type $x \in X = [\underline{x}, \overline{x}]$, distribution $H^w(x)$
  - Firms of types $y \in Y = [\underline{y}, \overline{y}]$, distribution $H^f(y)$
- *Production of firm y* $F(x, y, l_x, r_x)$
  - l_x workers of type x,r_x fraction of firm's resources
  - F increasing in all, concave in last two arguments
  - F constant returns to scale in last two arguments

  $\Rightarrow$ Denote:

$$f(x, y, \theta) = rF\left(x, y, \frac{l}{r}, 1\right), \text{ where } \theta = \frac{l}{r}$$

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  $\Rightarrow$ Total output: $\int F(x, y, l_x, r_x)dx$

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- *Preferences*
  - transferable utility (additive in output goods and numeraire)

The Model

Equilibrium

Hedonic wage schedule $w(x)$ taken as given.

- Optimization:
  - Firms maximize: $\max_{l_x, r_x} \int [F(x, y, l_x, r_x) - w(x) l_x] dx$

  $\Rightarrow$ $r_x > 0$ only if $\left(x, \frac{l_x}{r_x}\right) = \arg\max f(x, y, \theta) - \theta w(x) \quad (\star)$
- Feasible Resource Allocation (market clearing) under PAM:

$$\int_x^{\overline{x}} h_w(s) ds = \int_{\mu(x)}^{\overline{y}} \theta(s) h_f(s) ds$$

- Competitive Equilibrium: optimality + market clearing

Assortative Matching

Proposition (Condition for PAM)

A necessary condition to have equilibria with PAM is that

$$F_{xy}F_{lr} \geq F_{yl}F_{xr}$$

holds along the equilibrium path. The reverse inequality entails NAM.

Assortative Matching

$$F_{xy} F_{lr} \geq F_{yl} F_{xr}$$

- Interpretation ($F_{lr} > 0$ by assumption):
  1. $F_{xy} > 0$: bet. manag. produce more w/ bet. workers (Becker)
  2. $F_{yl} > 0$: bet. manag., larger span of control (as in Lucas)
  3. $F_{xr} > 0$: bet. workers produce more w/ manag. time

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- Quantity-quality trade-off by firm y with resources r:
  1. F_{xy}: better manager manages quality workers better vs.
  2. F_{yl}: better managers can manage more people

  $\Rightarrow$ Marginal increase of better $\gtrless$ marginal impact of more workers

Sketch of Proof of PAM-Condition

Assume PAM allocation with resources on $(x, \mu(x), \theta(x))$. Must be optimal, i.e., maximizes:

$$\max_{x,\theta} f(x, \mu(x), \theta) - \theta w(x).$$

First order conditions:

$$\begin{aligned} f_\theta(x, \mu(x), \theta(x)) - w(x) &= 0 \\ f_x(x, \mu(x), \theta(x)) - \theta(x) w'(x) &= 0 \end{aligned}$$

The Hessian is

$$Hess = \begin{pmatrix} f_{\theta\theta} & f_{x\theta} - w'(x) \\ f_{x\theta} - w'(x) & f_{xx} - \theta w''(x) \end{pmatrix}.$$

Second order condition requires $|Hess| \geq 0$:

$$f_{\theta\theta}[f_{xx} - \theta w''(x)] - (f_{x\theta} - w'(x))^2 \geq 0$$

Differentiate FOC's with respect to x, substitute:

$$-\mu'(x)[f_{\theta\theta} f_{xy} - f_{y\theta} f_{x\theta} + f_{y\theta} f_x / \theta] \quad \geq \quad 0$$

Positive sorting means $\mu'(x) > 0$, requiring $[\ \cdot\ ] < 0$ and after rearranging:

$$F_{xy} F_{lr} \geq F_{yl} F_{xr}$$

Special Cases

Efficiency Units of Labor

- Skill "=" Quantity: $F(x, y, l, r) = \tilde{F}(y, xl, r) \quad \Rightarrow \quad F_{xy}F_{lr} = F_{yl}F_{xr}$

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Multiplicative Separability

- $F(x, y, l, r) = A(x, y)B(l, r)$ sorting if $\frac{AA_{xy}}{A_x A_y}\frac{BB_{lr}}{B_l B_r} \geq 1$
- If B is CES with elast. of substitution ϵ: $\frac{AA_{xy}}{A_x A_y} \geq \epsilon$ (root-sm)

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Becker's one-on-one matching

- $F(x, y, \min\{l, r\}, \min\{r, l\}) = F(x, y, 1, 1) \min\{l, r\}$,
- Like inelastic CES ($\epsilon \to 0$), so sorting if $F_{12} \geq 0$

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Sattinger's span of control model

- $F(x, y, l, r) = \min\left\{\frac{r}{t(x,y)}, l\right\}$; write as CES between both arguments
- Our condition converges for inelastic case to log-supermod. in qualities

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Extension of Lucas' span of control model

- $F(x, y, l, r) = yg(x, l/r)r$, sorting only if good types work less well together ($-g_1 g_{22} \geq -g_2 g_{12}$).

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Spacial sorting in mono-centric city:

- $F(x,y,l,r) = l(xg(y) + v(r/l)) \Rightarrow$ higher earners in center.

Firm Size, Assignment, Wages

Proposition

Under assortative matching (symmetric distributions of x, y)

$$PAM \quad : \quad \theta'(x) = \frac{F_{yl} - F_{xr}}{F_{lr}}; \quad \mu'(x) = \frac{1}{\theta(x)}; \quad w'(x) = \frac{F_x}{\theta(x)},$$

$$NAM \quad : \quad \theta'(x) = -\frac{F_{yl} + F_{xr}}{F_{lr}}; \quad \mu'(x) = \frac{-1}{\theta(x)}; \quad w'(x) = \frac{F_x}{\theta(x)},$$

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Corollary

Under assortative matching, better firms hire more workers if and only if along the equilibrium path

$$F_{yl} > F_{xr} \text{ under PAM}, \quad \text{and} \quad -F_{yl} < F_{xr} \text{ under NAM.}$$

Application: SBTC vs. QBTC

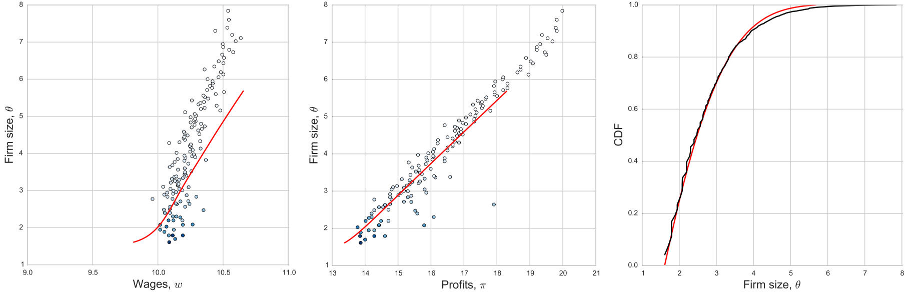
- How has technology changed: 1996 → 2010?
- Estimate technological parameters that affect size and sorting

$$F(x, y, l, 1) = \left(\omega_x x^{\frac{\sigma-1}{\sigma}} + \omega_y y^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} l^{\omega_l}.$$

- Distribution of types x and y assumed log-normal
- Estimate parameters $\omega_x, \omega_y, \omega_l, \sigma$ with parameters of type distributions to match 3 moment conditions:
  1. size-wage
  2. size-profits
  3. size distribution
- German administrative data for matched employer-employees

Results

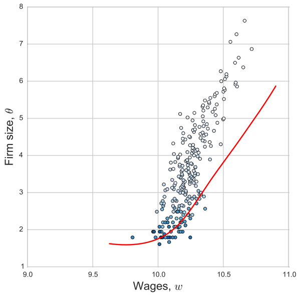
Targeted Moments 1996

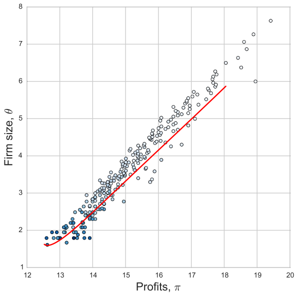


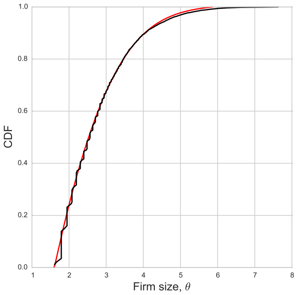
Wages-firm size – Profits-firm size – Firm size distribution

Results

Targeted Moments 2010







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	1996	2010	% change
Technology			
ω_x	0.026	0.060	131.6%
ω_y	0.974	0.964	-1.1%
ω_l	0.123	0.217	76.1%
σ	0.998	0.982	-1.6%
Distributions			
x	$\mathcal{LN}(2.49, 1.35)$	$\mathcal{LN}(2.69, 1.35)$	
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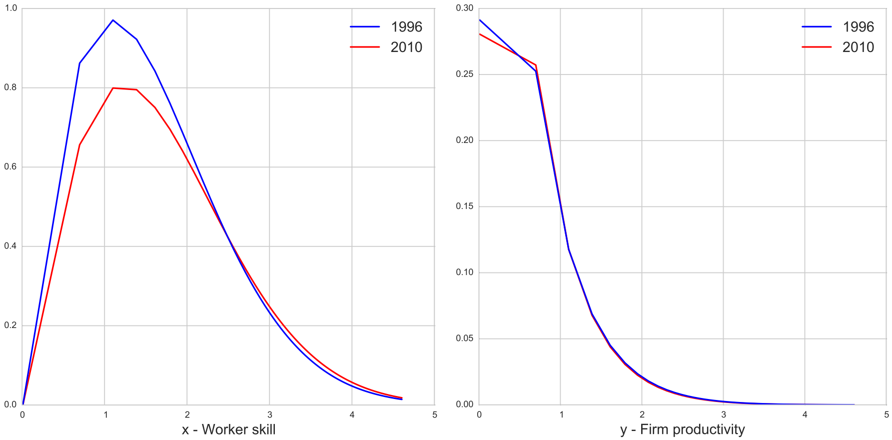

The Distributions of Worker Types x and Firm Types y.

Results

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Technology

- $\sigma < 1 \Rightarrow$ PAM
- $\sigma \approx 1$, technology can be approximated by the Cobb-Douglas

$$F(x, y, l, 1) \approx x^{\omega_x} y^{\omega_y} l^{\omega_l}.$$

but not $\sigma = 1$: No sorting!

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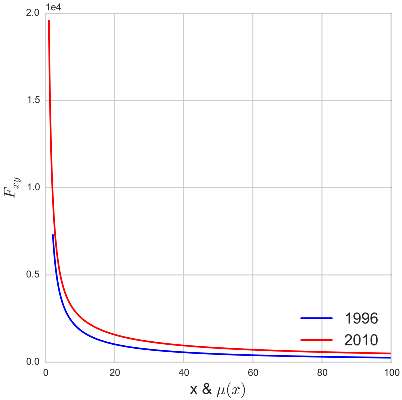
Technological Change

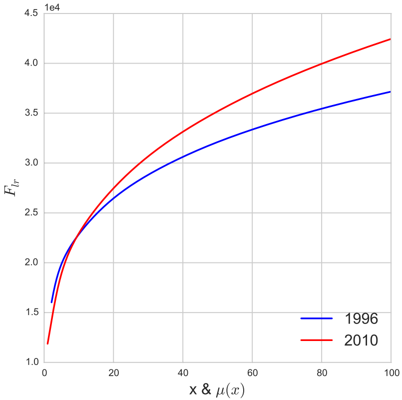
- $\omega_x \uparrow 136\%$: Skill-biased Technological Change (SBTC)
- $\omega_l \uparrow 76\%$: Quantity-biased Technological Change (QBTC)
- ω_y unchanged
- $(1-\sigma) \uparrow 14\times$: Increase in complementarity between x, y

Results

Complementarities



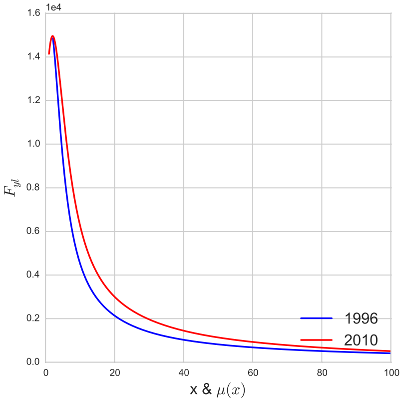
F_{xy}

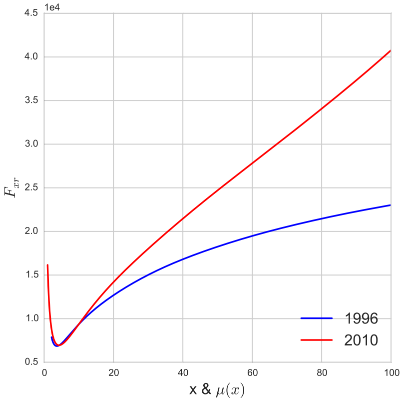


F_{lr}

Results

Complementarities

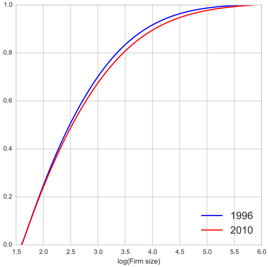


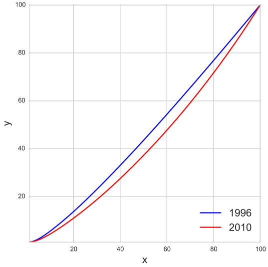


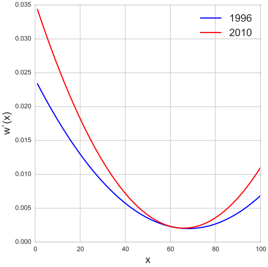
F_{yl} F_{xr}

Results

Firm Size, Allocation, Skill Premium







Size Distribution

Allocation

Skill Premium $w'(x)$

Results

Firm Size, Allocation, Skill Premium

1. There is both SBTC and QBTC
2. FOSD in firm size distribution and shift in allocation
3. Skill premium $\uparrow$, but polarization (Goos-Manning, Autor-Dorn)
4. SBTC and QBTC interact
   - SBTC increases skill premium
   - QBTC decreases skill premium (concave production)

   $\rightarrow$ Skill premium increase dampened by QBTC

Counterfactuals

1996 economy with one 2010 parameter

	Median Firm Size	% change 1996	Average $w'(x)$	% change 1996
1996	11.98		0.019	
2010	12.53	4.60 %	0.027	44.06%
2010 ω_x	14.21	18.66%	0.049	156.90%
2010 ω_y	11.95	-0.21%	0.019	1.90%
2010 ω_l	14.81	23.65%	0.009	-52.04%
2010 σ	12.01	0.24%	0.022	13.68%
2010 Distributions	12.36	3.20%	0.022	13.68%

Conclusion

- Assortative matching with large firms: intensive and extensive margin
- A simple condition for sorting; nests many known models
- Equilibrium allocation: system of 3 differential equations
- Application: Technological Change
  1. both SBTC and QBTC
  2. effect of QBTC on skill premium: negative
  3. effect of SBTC on skill premium would have been 4 times larger