Assortative Matching with Large Firms

Span of Control over More versus Better Workers

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Marseille, April 2012

Motivation

- Background:
  - Matching: one-to-one (e.g. Becker 1973) $\rightarrow$ extensive margin
  - Macro / Labor / Trade / Urban / Devel: intensive margin
  - Intensive Margin $\Rightarrow$ Firm Size
- Trade-Off: better workers vs. more workers
  - managerial time: "span of control": Sattinger 75, Lucas 78
  - assignment of land, of "distance", of assets...

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- Goals:
  1. Capture factor intensity in tractable manner (no peer effects)
  2. Sorting condition: complementarity quality vs. quantity
  3. Characterize firm size, assignment, wages
  4. Introduce frictions: unemployment across skills and firm size

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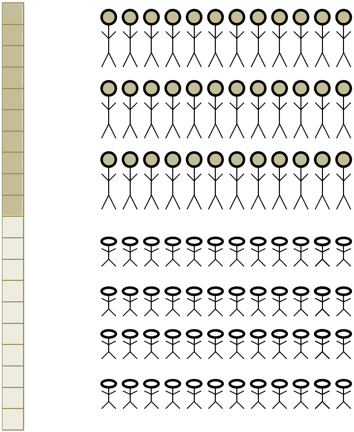
- Economic Relevance
  1. Characterizing production technology across industries: Walmart vs. mom-&-pop store; consulting and law firms;...
  2. Misallocation debate: output difference across economies
     - Firm heterogeneity in productivity $\rightarrow$ differences in K, p, A (Restuccia-Rogerson (08), Hsieh-Klenow (10),...)
     - Intensive margin *and* heterogeneity
     - Also worker heterogeneity $\Rightarrow$ skill (mis)allocation and human capital distribution matter

Motivation

Resources / Firms | Workers

h^f_1 | h^w_1

h^f_2 | h^w_2



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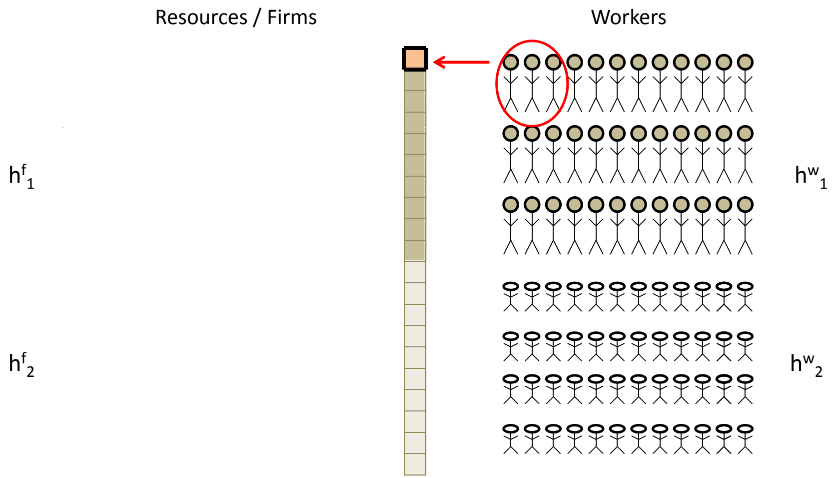

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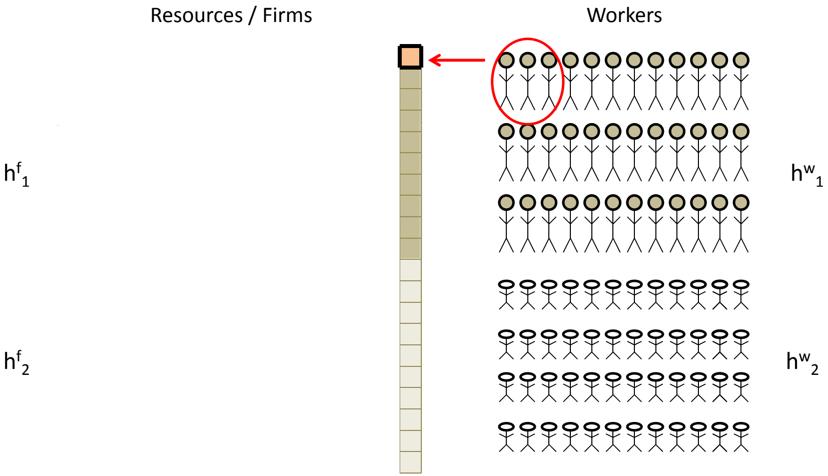

Prod: $f(x_1, y, l_1)$

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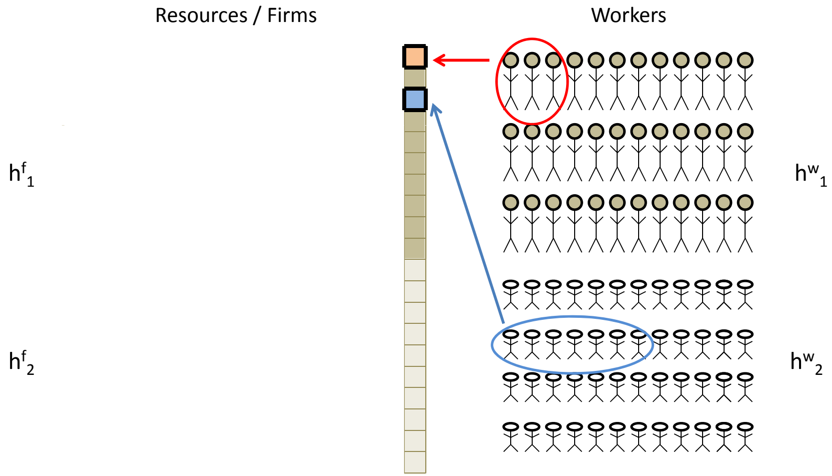

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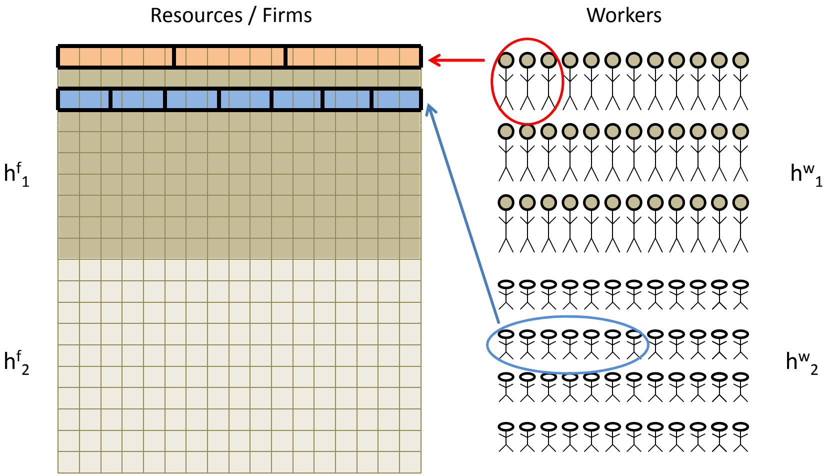

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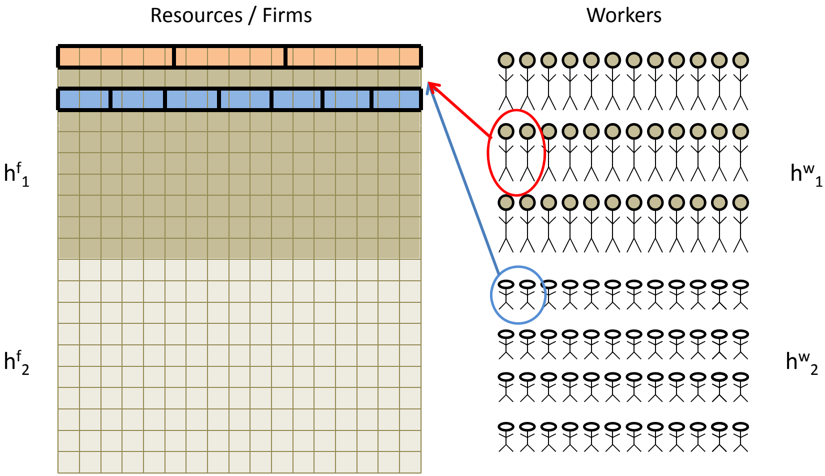

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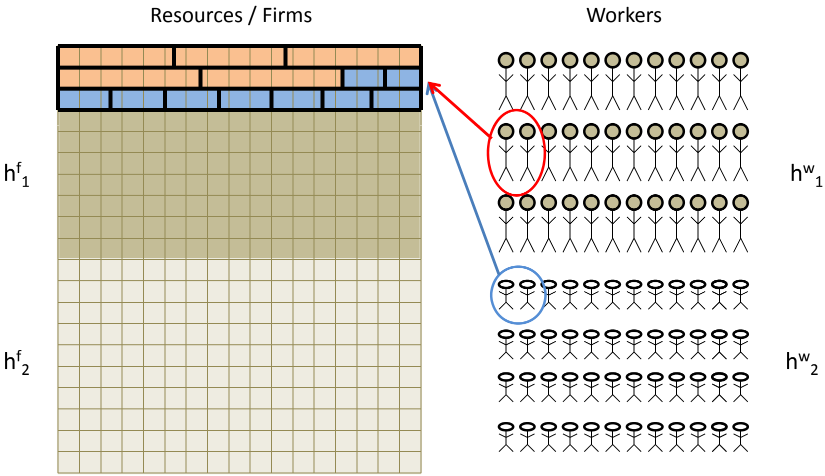

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Prod: $F(\underbrace{x_1, y}_{qualities}, \underbrace{l', r'}_{quantities}) + F(x_2, y, l'', r'')$

The Model

- *Population*
  - Workers of type $x \in X = [\underline{x}, \overline{x}]$, distribution $H^w(x)$
  - Firms of types $y \in Y = [\underline{y}, \overline{y}]$, distribution $H^f(y)$
- *Production of firm y* $F(x, y, l_x, r_x)$
  - l_x workers of type x,r_x fraction of firm's resources
  - F increasing in all, concave in last two arguments
  - F constant returns to scale in last two arguments

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  - Key assumption: no peer effects

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- *Preferences*
  - transferable utility (additive in output goods and numeraire)

Literature

Special Cases

- Becker 73: $l_{ji} = r_{ij} \quad \rightarrow \quad F(x, y, \min\{l, r\}, \min\{l, r\})$
- Sattinger 75: $l_{ji} \leq \frac{r_{ij}}{t(x_i, y_i)} \quad \rightarrow \quad F = \min\left\{l, \frac{r}{t(x,y)}\right\}$
- Garicano 00: $l \leq \frac{r}{t(x)} \quad \rightarrow \quad F = y \min\left\{l, \frac{r}{t(x)}\right\}$
- Lucas 78: Worker input independent of skill $F = yg\left(l\right)$
- Rosen 74: more general; existence

  (also, Kelso-Crawford 82, Cole-Prescott 97, Gul-Stacchetti 99, Milgrom-Hatfield 05)
- Roy 51: $l_{ji} = r_{ij}$ & no factor intensity
- Roy 51+CES: particular functional form for decreasing return
- Frictional Markets: one-on-one matching, competitive search
  (Shimer-Smith 00, Atakan 06, Mortensen-Wright 03, Shi 02, Shimer 05, Eeckhout-Kircher 10)

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Hedonic wage schedule $w(x)$ taken as given.

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- *Feasible Resource Allocation*:
  - $\mathcal{R}(x, y, \theta)$: resources to any $x' \leq x$ by any $y' \leq y$ with $\frac{l_{x'}}{r_{x'}} \leq \theta$.
    1. Resource feasibility $\quad [\mathcal{R}(y|X, \Theta) \leq H^f(y) \; \forall y]$
    2. Worker feasibility $\quad [\int_{\theta \in \Theta} \int_{x' \leq x} \theta d\mathcal{R}(\theta, x'|Y) \leq H^w(x) \; \forall x]$

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- *Competitive Equilibrium* is a tuple $(w, \mathcal{R})$ s.t.
  1. Optimality Cond. $[(x, y, \theta) \in \text{supp}\mathcal{R}$ only if it satisfies $(\star)]$
  2. Market Clearing $[\int \theta d\mathcal{R}(\theta|x, Y) \leq h^w(x),$ "$=$" if $w(x) > 0, \forall x]$

Assortative Matching

Definition (Assortative Matching)

Allocation $\mathcal{R}$ entails positive (negative) sorting if for any x and x' with $x < x'$ it holds that $(x, y, \theta) \in \mathcal{R}$ and $(x', y', \theta') \in \mathcal{R}$ only if $y' \geq y$ (only if $y' \leq y$).

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Main Result:

Proposition (Condition for PAM)

A necessary condition to have equilibria with PAM for any arbitrary distribution of types is

$$F_{12}F_{34} \geq F_{23}F_{14}$$

for all (x,y,l,r). The strict inequality is also sufficient, and guarantees that no other equilibria exist. The reverse inequality is necessary and sufficient for NAM.

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- Interpretation ($F_{34} > 0$ by assumption):

  1. $F_{12} > 0$: bet. manag. produce more w/ bet. workers (Becker)
  2. $F_{23} > 0$: bet. manag., larger span of control (as in Lucas)
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- Quantity-quality trade-off by firm y with resources r:
  1. F_{12}: better manager manages quality workers better vs.
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  $\Rightarrow$ Marginal increase of better $\gtrless$ marginal impact of more workers

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- Examples: technological differences across industries, establishments
  1. Walmart vs. mom-&-pop store: low x, high y, high θ, $\theta' < 0$

     $\Rightarrow$ $F_{23} > 0$, $F_{14} > 0$, F_{12} not too large $\Rightarrow$ NAM
  2. Law firm, Mgt Consulting: high x, high y, low θ, $\theta' > 0$

     $\Rightarrow$ $F_{14} > 0$, $F_{23} > 0$, F_{12} large $\Rightarrow$ PAM

Sketch of Proof of PAM-Condition

Assume PAM allocation with resources on $(x, \mu(x), \theta(x))$. Must be optimal, i.e., maximizes:

$$\max_{x,\theta} f(x, \mu(x), \theta) - \theta w(x).$$

First order conditions:

$$\begin{aligned} f_\theta(x, \mu(x), \theta(x)) - w(x) &= 0 & (1) \\ f_x(x, \mu(x), \theta(x)) - \theta(x) w'(x) &= 0, & (2) \end{aligned}$$

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$$f_x(x, \mu(x), \theta(x)) - \theta(x) w'(x) = 0, \tag{2}$$

The Hessian is

$$Hess = \begin{pmatrix} f_{\theta\theta} & f_{x\theta} - w'(x) \\ f_{x\theta} - w'(x) & f_{xx} - \theta w''(x) \end{pmatrix}.$$

Second order condition requires $|Hess| \geq 0$:

$$f_{\theta\theta}[f_{xx} - \theta w''(x)] - (f_{x\theta} - w'(x))^2 \geq 0. \tag{3}$$

Differentiate (1) and (2) with respect to x, substitute:

$$-\mu'(x)[f_{\theta\theta} f_{xy} - f_{y\theta} f_{x\theta} + f_{y\theta} f_x / \theta] \geq 0$$

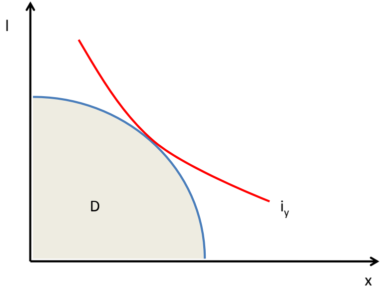
Positive sorting means $\mu'(x) > 0$, requiring $[\,\cdot\,] < 0$ and after rearranging:

$$F_{12} F_{34} \geq F_{23} F_{14}. \tag{4}$$

$F_{12}F_{34} > F_{23}F_{14}$: Graphical

Budget Set: $D = \{(x, l) | lw(x) \leq M\}$

Iso-output Curve: $i_y = \{(x, l) | F(x, y, l, 1) = \Pi\}$



Slope of Iso-output Curve: $\frac{\partial l}{\partial x} = -\frac{F_1(x,y,l,1)}{F_3(x,y,l,1)}$.

Fix $F_{23} > 0$ and consider better firm:

- If $F_{12} \simeq 0$, higher y has flatter slope (numerator is constant).
- If $F_{12} \gg 0$, then higher y will have steeper slope.

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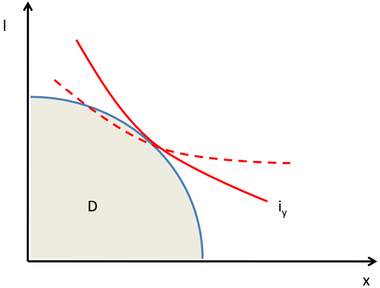

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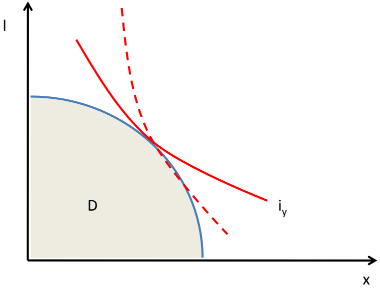

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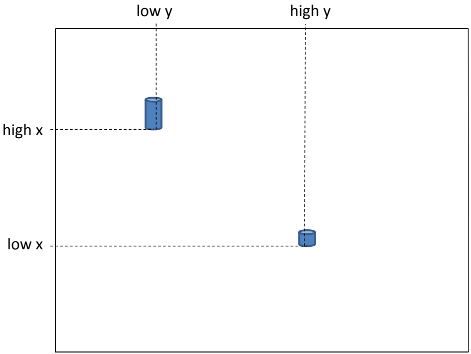

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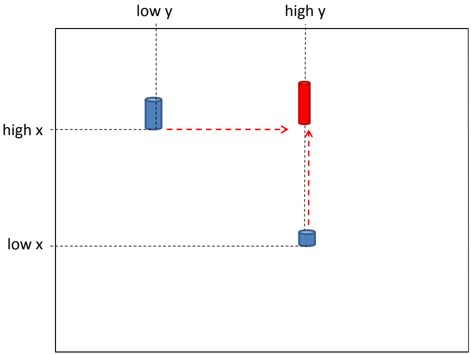

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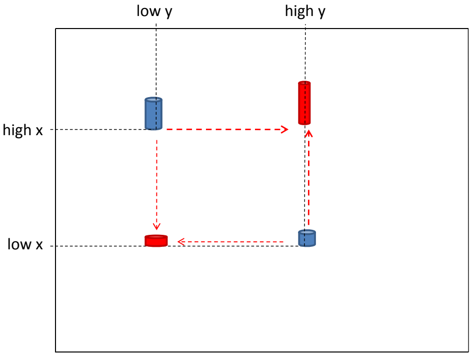

Special Cases

Efficiency Units of Labor

- Skill "=" Quantity: $F(x, y, l, r) = \tilde{F}(y, xl, r) \quad \Rightarrow \quad F_{12}F_{34} = F_{23}F_{14}$

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- $F(x, y, l, r) = A(x, y)B(l, r)$ sorting if $\frac{AA_{12}}{A_1 A_2} \frac{BB_{12}}{B_1 B_2} \geq 1$
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- $F(x,y,\min\{l,r\},\min\{r,l\}) = F(x,y,1,1)\min\{l,r\}$,
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Extension of Lucas' span of control model

- $F(x, y, l, r) = yg(x, l/r)r$, sorting only if good types work less well together ($g_2 g_{12} \geq g_1 g_{22} + g_1 g_2/\theta$).

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Extension of Lucas' span of control model

- $F(x, y, l, r) = yg(x, l/r)r$, sorting only if good types work less well together ($g_2 g_{12} \geq g_1 g_{22} + g_1 g_2/\theta$).

Spacial sorting in mono-centric city:

- $F(x, y, l, r) = l(xg(y) + v(r/l)) \Rightarrow$ higher earners in center.

Firm Size, Assignment, Wages

Proposition

Under assortative matching (symmetric distributions of x, y):

$$PAM \quad : \quad \theta'(x) = \frac{F_{23} - F_{14}}{F_{34}}; \quad \mu'(x) = \frac{1}{\theta(x)} \quad ; \quad w'(x) = \frac{F_1}{\theta(x)},$$

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θ' from FOC: $f_\theta = w(x)$ and $f_x/\theta = w'$, diff. and subst. μ'.

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Corollary

Under assortative matching, better firms hire more workers if and only if along the equilibrium path

$$F_{23} > F_{14} \text{ under PAM, and} \qquad -F_{23} < F_{14} \text{ under NAM.}$$

Firm Size, Assignment, Wages

Proposition

Under assortative matching $\mathcal{H}(x) = \frac{h_w}{h_f}$

$$PAM \quad : \quad \theta'(x) = \frac{\mathcal{H}(x)F_{23} - F_{14}}{F_{34}}; \quad \mu'(x) = \frac{1}{\theta(x)}\mathcal{H}(x); \quad w'(x) = \frac{F_1}{\theta(x)},$$

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$$\mathcal{H}(x)F_{23} > F_{14} \text{ under PAM, and } \quad -\mathcal{H}(x) \quad F_{23} < F_{14} \text{ under NAM.}$$

Firm Size under PAM

$F_{23} > F_{14}$

- Firm size increasing depends on relative strength of
  1. F_{23}: span of control
  2. F_{14}: resource intensity of labor
- If marginal impact of output from firm y' span of control is larger than worker x's marginal impact of resources $\Rightarrow$ high productivity firms are larger

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- Special case: Lucas 78

General Capital, Monopolistic Competition

- General Capital:
  - $F(x, y, l, r) = \max_k \hat{F}(x, y, l, r, k) - ik$; Sorting cond. on max

    $\hat{F}_{12}\hat{F}_{34}\hat{F}_{55} - \hat{F}_{12}\hat{F}_{35}\hat{F}_{45} - \hat{F}_{15}\hat{F}_{25}\hat{F}_{34} \geq \hat{F}_{14}\hat{F}_{23}\hat{F}_{55} - \hat{F}_{14}\hat{F}_{25}\hat{F}_{35} - \hat{F}_{15}\hat{F}_{23}\hat{F}_{45}$

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- Monopolistic Competition in the Output Market:
  - consumers have CES preferences with substitution ρ
  - sales revenue of firm y: $\chi F(x,y,l,1)^{\rho}$
  - Sorting condition

$$\left[\rho\tilde{F}_{12} + (1-\rho)(\tilde{F})\frac{\partial^2 \ln \tilde{F}}{\partial x \partial y}\right]\left[\rho\tilde{F}_{34} - (1-\rho)l\tilde{F}\frac{\partial^2 \ln \tilde{F}}{\partial l^2}\right]$$
$$\geq \left[\rho\tilde{F}_{23} + (1-\rho)\tilde{F}\frac{\partial^2 \ln \tilde{F}}{\partial y \partial l}\right]\left[\rho\tilde{F}_{14} + (1-\rho)\left(l\tilde{F}_{13} - l\tilde{F}\frac{\partial^2 \ln \tilde{F}}{\partial x \partial r}\right)\right].$$

  - independent of χ
  - our condition under $\rho = 1$, log-sm when production linear in l.

Search, Firm Size and Sorting

- Existing literature on search and firm size: identical workers (Smith 99, Acemoglu-Hawkins 06, Mortensen 09, Kaas-Kircher 10, Helpman-Itskhoki-Redding 10, Menzio-Moen 10,...).
- Vacancy filling prob $m(q)$. Job finding prob $m(q)/q$. Post (x, v_x, ω_x)

$$\max_{r_x, l_x, \omega_x, v_x} \int [F(x, y, l_x, r_x) - l_x \omega_x - v_x c] \, dx$$

$$\text{s.t. } l_x = v_x m(q_x); \quad \text{and} \quad \omega_x m(q_x)/q_x = w(x).$$

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- Two equivalent formulations:
  1. $\max_{s_x, r_x} \int [G(x, y, s_x, r_x) - w(x) s_x] dx$, where

     $G(x, y, s_x, r_x) = \max_{v_x} [F(x, y, v_x m(s_x/v_x), r_x) - v_x c]$.
  2. $\max_{r_x, l_x, v_x} \int [F(x, y, l_x, r_x) - C(x, l_x)] dx$, where

     $C(x, l_x) = \min_{v_x, q_x} c v_x + q_x v_x w(x)$ s.t. $l_x = v_x m(q_x)$.

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- Determine unemployment. FOC

$$w(x) q_x = \frac{\eta(q)}{1 - \eta(q)} c$$

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- Check sorting, compute $w(x)$ as in previous part.
- Determine unemployment. FOC (simple closed form with const. elasticity α)

$$w(x) q_x = \frac{\eta(q)}{1 - \eta(q)} c \quad = \quad \frac{1 - \alpha}{\alpha} c$$

Unemployment, Firm Size and Sorting

Proposition

The unemployment rate is falling in worker skills.

- $\eta(q)$ weakly decreasing $\Rightarrow$ q decreasing in x

Unemployment, Firm Size and Sorting

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- $\eta(q)$ weakly decreasing $\Rightarrow$ q decreasing in x

Proposition

The vacancy rate is ambiguous in firm size.

- Consider PAM (likewise for NAM)
- Vacancies $(1/q)$ increasing in x
- Firm size ambiguous in y : $F_{23} \gtrless F_{14}$

Conclusion

This work:

- Lay out a matching model with factor intensity
- Derive tractable sorting condition ($F_{12}F_{34} \geq F_{14}F_{23}$)
- Characterize equilibrium firm size ($F_{23} > F_{14}$), assignment and wages
- Search frictions: relation unemployment, skill and firm size

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Economic Relevance & Applications in trade/macro/labor...:

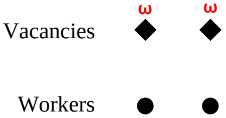
- Mismatch debate: worker heterogeneity matters
- Comparative statics: impact of aggregate fluctuations
- Empirical: How does unemployment change across skills/firm size?

Appendix

Illustration of Extensions

Competitive Search with Large Firms

Wage posting and frictional hiring (Peters 90, Burdett-Shi-Wright 01,...)



- Searchers per vacancy: $q = s/v$
- Vacancy filling prob: m
- Job finding prob: m/q

back

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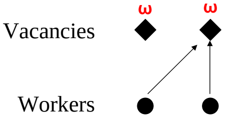

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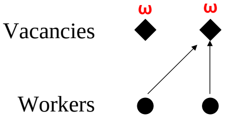

- Searchers per vacancy: $q = s/v$
- Vacancy filling prob: $m = \frac{3}{4}$
- Job finding prob: $m/q = \frac{3}{4}$

back

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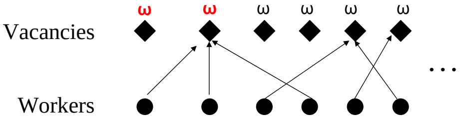

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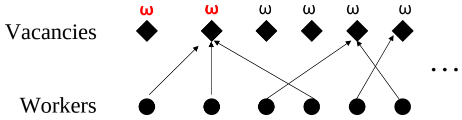

- Searchers per vacancy: $q = s/v$
- Vacancy filling prob: $m(q) \to 1 - e^{-q}$
- Job finding prob: $m/q \to \frac{1-e^{-q}}{q}$

back

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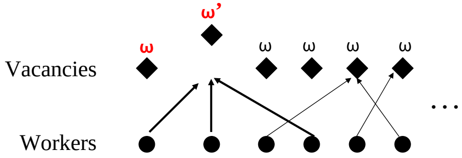

- Searchers per vacancy: $q = s/v$
- Vacancy filling prob: $m(q) \rightarrow 1 - e^{-q}$
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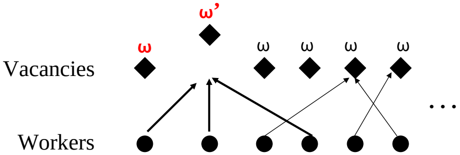

- Searchers per vacancy: $q = s/v$
- Vacancy filling prob: $m(q) \rightarrow 1 - e^{-q}$, m'
- Job finding prob: $m/q \rightarrow \frac{1-e^{-q}}{q}$, $\omega' \frac{m(q')}{q'} = \omega \frac{m(q)}{q} = w(x)$

back