The Effect of Asset Holdings on Worker Productivity*

Jan Eeckhout†
j.eeckhout@ucl.ac.uk

Alireza Sepahsalari‡
alireza.sepahsalari@bristol.ac.uk

October 24, 2018

Abstract

We propose a theory that analyzes how a worker's asset holdings affect their job productivity. In a labor market with uninsurable risk, workers choose to direct their search to jobs that trade off productivity and wages against unemployment risk. Workers with low asset holdings have a *precautionary job search motive*, they direct their search to low productivity jobs because those offer a low risk at the cost of low productivity and a low wage. We show that such sorting occurs under a condition closely related to Decreasing Relative Risk Aversion. We calibrate the infinite horizon economy and find that this mechanism is quantitatively important. We evaluate a tax financed unemployment insurance (UI) scheme and how it affects welfare. We find that in the aggregate across all workers, the insurance effect of a rise in UI dominates the incentive effect of search. And even if higher benefits lead to productivity losses from the decline in firm entry, those losses are compensated by the insurance value of benefits. However, the welfare gains and losses are heterogeneously distributed: the unemployed and those with low asset holdings in particular stand to gain from higher UI benefits. Finally, we compare a one-off severance payment with per period benefits and find that per period benefits generate superior welfare.

Keywords. Unemployment Risk. Precautionary Savings. Precautionary Job Search. Sorting. Unemployment Insurance. Severance Pay.
JEL. C6. E2.

*This paper was circulated before under the title "Unemployment Risk and the Distribution of Assets". We would like to thank Isaac Baley Gaytan, Vasco Carvalho, Hector Chade, Pierre Cahuc, Melvyn Coles, Maria Cristina Di Nardi, Giulio Fella, Chris Flinn, Nezih Guner, Christian Haefke, Gregory Jolivet, Boyan Jovanovic, Per Krusell, Moritz Kuhn, Etienne Lalé, Jeremy Lise, Rory McGee, Espen Moen, Josep Pijoan-Mas, Fabien Postel-Vinay, Morten Ravn, Jose Victor Rios-Rull, Vincent Sterk, Kjetil Storesletten, Hélène Turon, David Wiczer, Ronald Wolthoff and seminar audiences for insightful discussions and comments. This research received financial support from the ERC, Advanced grant 339186, and from ECO2015-67655-P.

†UPF Barcelona (GSE-ICREA) and University College London
‡University of Bristol

1 Introduction

Unemployment risk is arguably the biggest risk a worker faces in their lifetime. Even if there is no market for unemployment insurance, workers can nonetheless self-insure by accumulating assets while employed, in order to run those assets down when unemployed. This allows them to better smooth consumption. But workers simultaneously also use the labor market to self-insure by applying for lower productivity jobs that have a higher job finding probability. We ask a basic question: what is the role of asset holdings and self-insurance for productivity? We then study the welfare implications of government mandated unemployment insurance (UI) benefits. There is of course a tension between the unemployed who receive UI benefits and the employed who pay for them through taxes. But there is now also a tension within the pool of the unemployed. We show that the unemployed with low asset levels benefit considerably more from a UI increases than the rich unemployed. We also evaluate the effect that benefits have on total productivity.

We model the worker's savings and job search decision in a labor market where workers can direct their search towards jobs of different productivity, with firms posting wages to attract applicants. The worker's incentives are thus to trade off wages and job productivity against the probability of finding a job. Asset holdings crucially affect this tradeoff because the worker is less exposed to the consumption risk inherent in joblessness. In addition to the standard *precautionary savings motive* with asset-contingent consumption smoothing à la Bewley-Huggett-Aiyagari, workers now also counter unemployment risk by directing their search to jobs with a high matching probability and low productivity, call it a *precautionary job search motive*. Key in our analysis is the sorting of workers with different assets holdings into different productivity jobs. Our main objective is to analyze how the inequality inherent in a labor market with heterogeneous productivity jobs interacts with the inequality that results from asset accumulation, i.e., how the two precautionary motives interplay and affect the jobs productivity.

The contribution of the paper is double. First, we show theoretically that under a condition related to Decreasing Relative Risk Aversion there is sorting of workers with heterogeneous asset holdings into firms with heterogeneous productivities. The sorting happens despite the fact that there is no technological complementarity (supermodularity) between job productivity and worker skill. There is nonetheless a natural *preference complementarity* between firm productivity and worker assets because risk aversion generates different preferences for self insurance, with high asset holders trading off lower insurance for a higher productivity job. To establish the sorting in this model, we solve this as an allocation problem with risk aversion and therefore imperfectly transferable utility (ITU) as well as search frictions. It is the selection or sorting of workers into different productivity jobs that is responsible for the different matching probabilities of different asset holders. While directed search in the presence of risk aversion has been analyzed in the literature – most notably Acemoglu and Shimer (1999) and more recently Golosov et al. (2012) –, these are representative agent models without a non-degenerate distribution of assets.[1]

This interaction between the distribution of assets and the incentives to search for different productivity jobs as well as smoothing consumption is not merely a theoretical artifact. We show that it is important quantitatively. Our second contribution is the quantitative analysis of the model. We analyze the steady state of an infinite horizon version where workers and firms sort in each period. In the steady state, unemployed workers run down their assets, while at the same time moving their target from high to low productivity jobs. Employed workers run up their assets anticipating the eventual job loss and necessity to insure while unemployed. Workers continuously move up and down the asset distribution, but the aggregate distribution of assets is stationary. We derive the ergodic distribution in this steady state as well as wages, savings, jobs search decisions (and unemployment), and the vacancy posting decision for every asset and productivity level. Unlike most existing work on unemployment insurance, we are able to incorporate the endogenous savings decision of the employed.[2] This is novel in the literature, and we manage to do this computationally using a shooting algorithm to solve for the allocation of unemployed job searchers, and a brute force algorithm to solve for the consumption-savings decision of the employed.

We calibrate our model to the US economy and find that its features are quantitatively important. Workers direct their search towards jobs with different bundles of productivity and job finding probabilities. We find that job finding probability of the low asset holders is 7% higher than that of the high asset holders. This establishes the important role of endogenous job finding rates and their interaction with the distribution of asset holdings.

In this setting we analyze the role of government mandated unemployment benefits. We have no pretense of analyzing a general mechanism design question where agents submit messages about their private asset holdings and receive benefits depending on their and all other agents' messages. This turns out to be a immensely complex problem with an infinite horizon and a continuum of heterogeneous agents. Rather, we analyze a realistic unemployment insurance institution where ex ante homogeneous workers with ex post heterogeneous (but time varying) asset holdings receive a constant benefit while unemployed and pay a constant tax rate on wage income while employed.

There are multiple channels through which benefits affect the equilibrium allocation and therefore welfare. We single out five equilibrium effects that result from an increase in benefits. 1. The unemployed worker is better insured and enjoys smoother consumption. 2. Because of better insurance, workers tend to apply for more productive jobs with higher wages. Both of these affect welfare positively. The next effects are negative. 3. Higher wages reduce the firm's benefits and therefore job

[1] Acemoglu and Shimer (1999) do consider a non-degenerate distribution when analyzing the case of CARA, which, as we show in this paper, is a knife-edge case with no sorting and where the asset distribution is indeterminate.

[2] The standard assumption in the literature is that employed workers values are constant (see for example Hopenhayn and Nicolini (1997), Shimer and Werning (2007) and Shimer and Werning (2008)). This is typically achieved by assuming that once employed, they do not face job separation, in conjunction with the assumption that discounting is exactly proportional to the return on assets. All this implies that workers in each period consume the return on their assets, keeping their asset holdings invariant.

creation. 4. Higher benefits uniformly lead to lower job finding probabilities and therefore higher unemployment. 5. Higher benefits also lead to higher wages and therefore lower dividends. We find that there are conflicts of interest between different workers. Benefits have the strongest positive welfare effects for the unemployed and those with low assets, and have negative effects on the employed with high asset levels. When we aggregate the net welfare gain of moving to different UI economies over the distribution of asset holding, we find that the average net effect of these countervailing forces is positive: higher UI benefits increase aggregate welfare. This is because the positive insurance effect of benefits dominates the negative impact on the job finding rate for unemployed workers and the negative impact on wages for employed workers.

A novel feature of our model is the sorting between workers with different asset holdings and firms with different productivities. This implies that UI benefits affect the productivity of workers in the economy, through the allocation of workers to jobs of different productivities as well as through the firms' entry decision. This is in contrast with models with homogeneous firms where a change in benefits leaves the average firm's productivity unaffected. We find that when UI benefits increase, average productivity first decreases and then increases. Higher benefits result in workers applying to more productive jobs, because they are better insured. But it also reduces the average asset holding, and as a result, there are more unemployed workers with low assets who apply to low productivity jobs. However, a further rise in benefits has a cleansing effect. Low productive firms do not open vacancies when UI goes up and therefore workers get allocated over vacancies with higher average productivity.

We also analyze the alternative policy scheme of severance pay, where a worker who becomes unemployed receives a lump sum payment instead of a per period UI benefit during the unemployment spell. Severance pay offers better incentives to search but less insurance in case the worker is unemployed longer than average. We find that the insurance effect dominates, and as result, per period benefits generate higher welfare than severance pay.

Related Literature. We are intellectually indebted to earlier work that has shaped our thinking on this topic. This paper is related to a large literature on unemployment risk and consumption smoothing. Danforth (1979) is one of the first to analyze search with risk averse workers in a partial equilibrium setting. Hopenhayn and Nicolini (1997), Shimer and Werning (2007) and Shimer and Werning (2008) analyze optimal unemployment insurance in a similar setting. Our paper is a general equilibrium search model with risk averse agents, closely related to Acemoglu and Shimer (1999). They analyze workers with identical asset holdings and focus on the incentives for firms to create jobs. Golosov et al. (2012) consider a similar setup to Acemoglu and Shimer (1999) with identical agents and analyze optimal taxation and benefits. Here, we focus on the distribution of assets and where the distribution of those assets is non-degenerate.

Our model follows in the footsteps of Krusell et al. (2010), who analyze the relation between asset dependent consumption-savings decisions and unemployment risk. Our focus is on directed rather than random search. This is not merely a semantic distinction. Directed search allows for the fact that the asset holdings affect the job finding probability. While Krusell et al. (2010) obtain a welfare function that is decreasing in assets for asset rich workers, we get the opposite. This is because with random search the probability of job finding is exogenous for workers. Therefore when UI goes up, rich workers are disadvantaged: they pay higher taxes yet, they find jobs at lower rates and their consumption smoothing does not change much. Instead in our framework, all workers endogenously adjust their probability of job finding depending on the UI level. This leads to an increase of welfare in benefits. For the same reason of endogenous directed search, we also find that equilibrium job finding rates are increasing in assets and varying considerably, while they are constant with random search.[3]

Our directed search setup is complex – it has risk averse agents, it involves a consumption-savings decision, there is sorting, and the economy is dynamic (infinite horizon) –, and the block-recursivity property (Menzio and Shi (2011)) does not apply because there is two sided heterogeneity, with firm productivity and worker asset holdings. Nonetheless, from the combination of directed search with two-sided heterogeneity (as in Eeckhout and Kircher (2010)), we can solve an assignment problem with risk aversion. We extend the analysis in Legros and Newman (2007) to derive the conditions for sorting. The novelty of our approach allows us to analyze an economy where the asset distribution is endogenous and where both savings and job search decisions depend on the worker's asset holdings. We can thus analyze how unemployment benefits affect workers' asset holdings and in turn the productivities of jobs they search for.

This paper is also related to large literature that looks at the welfare impact of a change in UI in search and matching models with risk averse agents. Merz (1995), Andolfatto (1996) and den Haan et al. (2000) study the macroeconomic implications of search frictions in business cycle models, in an economy where a worker's idiosyncratic income shocks are fully insured. Krusell et al. (2010) nests the Diamond-Mortensen-Pissarides framework with asset dependent consumption savings decisions as in Bewley (1980), Huggett (1993) and Aiyagari (1994). This allows them to analyze the interaction of search frictions with the precautionary savings motive. The contribution of our paper is to take this one step further. We introduce endogenous job search that allows workers to implicitly insure unemployment risk, the precautionary job search motive. We find that this is important quantitatively, and as a result, a change in unemployment insurance changes the workers' welfare by affecting their job search decision as well as the productivity of jobs they choose.

There is direct evidence in the literature for the main mechanism of our model, namely that higher asset holdings leads to prolonged job search. Card et al. (2007) find that a lump sum transfer of two months of salary reduces the job finding rate by 8-12%. These numbers are in line with what we find for our benchmark economy.[4] Chetty (2008) shows that the elasticity of the job finding rate

[3] Other recent models that obtain endogenous wages in an Aiyagari (1994) economy with directed search are Krusell et al. (2018) and Chaumont and Shi (2017), the latter has on-the-job search.

[4] See also Rendon (2006) and Lentz (2009) for related findings.

with respect to unemployment benefits decreases with liquid wealth. And Browning and Crossley (2001) show that unemployment insurance improves consumption smoothing for poor agents, but not for rich ones. Herkenhoff (2013) and Herkenhoff et al. (2015) provide evidence for the effect of better credit access on lower job finding rates. Herkenhoff (2013) shows that through this channel, increased credit access leads to longer recessions and slower recoveries. And Herkenhoff et al. (2015) exploit credit tightening over the business cycle which leads to an increase in employment and a decrease in output and productivity. We believe our model is novel in providing a theoretical framework where this observed relation between asset holdings and job finding rates stems from a precautionary job search motive and firm heterogeneity.

Finally, in an interesting piece, Michelacci and Ruffo (2014) analyze a related question where workers are heterogeneous: how does optimal unemployment insurance vary over the life cycle. Because workers accumulate human capital, young workers have strong incentives to find a job, yet they do not have the means to smooth consumption. Instead, older workers have less incentives and can smooth consumption better. They focus on the role of human capital accumulation and to that end, assume that matching probabilities are exogenous.

This paper is organized as follow. In section 2 we lay out the model. In section 3 we derive the equilibrium allocation and the conditions under which there exists positive (negative) assortative matching. In section 4, we compute and quantitatively analyze the full infinite horizon model. We perform a benchmark calibration, and evaluate the effects of different benefit levels as well the welfare analysis and the comparison of per period benefits with severance payment. We conclude in section 5.

2 The Model

TIMING. This is a T-period economy in which agents make a joint consumption-savings and job search decision. Endowed with assets, in each period $t < T$, unemployed workers choose their consumption-savings level, as well as which job to search for. Our interest is in analyzing the infinite horizon setting $T \to \infty$ (Section 3.2). To gain insights into the mechanism and in order to derive analytical results, we first analyze the two-period model $T = 2$ (section 3.1), in which workers make decisions only once at $t = 1$.

AGENTS. There is a measure one of workers. When they are unemployed they indexed by their heterogeneous asset holdings in period t, $a_t \in \mathcal{A} = [\underline{a}, \overline{a}] \subset \mathbb{R}_+$.[5] Let $F_u(a)$ denote the measure of unemployed workers with asset levels weakly below $a \in \mathcal{A}$ (with positive derivative $f_u(a)$. When they are employed, workers are indexed by both assets a and a wage w. Let $F_e(a, w)$ be the measure of employed workers with asset levels below a and wages below w. We denote the marginal over w by

[5]For much of the paper we will drop the subscript t and in the recursive (two-period) formulation we refer to $a_t = a$ ($a_1 = a$) and $a_{t+1} = a'$ ($a_2 = a'$).

$F_e(a)$ (with positive derivative $f_e(a)$).[6] In order to reduce notation, we denote $F = (F_u(a), F_e(a, w))$. The distribution of asset holdings amongst unemployed and employed workers is endogenous. In the infinite horizon model we derive the ergodic distribution of assets. Each worker supplies her labor and can only apply to one job at a time. Firms are heterogeneous in their productivities y and each has one job. Let $y \in \mathcal{Y} = [\underline{y}, \overline{y}] \subset \mathbb{R}_+$ and assume the firm type is observable. $H(y)$ denotes the measure of firms in the economy and with a type weakly below y. The total measure of firms $H(\overline{y})$ is assumed large. H is assumed C^2 with strictly positive derivative h. Not all firms enter the market, nor are all firms searching for workers. The measure of firms that post vacancies is endogenous and denoted by $G(y)$ (with positive derivative $g(y)$).

Preferences and Technology. Workers are risk averse and their preferences are represented by the Von Neumann-Mergenstern utility function $u(c)$ over consumption level c, where $u : \mathbb{R}_+ \to \mathbb{R}_{++}$. We assume that u is increasing and concave: $u' > 0, u'' < 0$. Agents discount utility with factor $\beta < 1$. Savings can be invested in a risk free bond at a fixed rate $R = 1 + r > 1$. We assume that firms are owned by entrepreneurs who are risk neutral and who do not participate in the labor market.[7] Firms have one job and can post a vacancy at cost k. Output produced at a firm of type y is equal to y.

Search Technology. Job search is directed. Firms post a wage w and there is a search technology that governs the frictions. These frictions crucially depend on the degree of competition for jobs, as captured by the ratio of vacancies to unemployed workers, denoted by $\theta \in [0, \infty]$. This represents the relative supply and demand for jobs, as it determines the probability of a match for an unemployed worker denoted by $m(\theta)$, where $m : [0, \infty] \to [0, 1]$: the higher the value of θ, the easier it is for a worker to find a job, so m is a strictly increasing function: $m' > 0$. In contrast, the higher the ratio of firms to workers, the harder it is for a firm to fill its vacancy. We denote the probability that a firm gets matched by $q(\theta)$, where $q : [0, \infty] \to [0, 1]$ is a strictly decreasing function, $q' < 0$. Since matching is always in pairs, the matching probability of workers must be consistent with those of firms, in particular, it must be the case that $q(\theta) = m(\theta)/\theta$. We also require the standard assumptions hold: m is twice continuously differentiable, strictly concave and has a strictly decreasing elasticity. The fact that we express the matching probability in terms of the ratio of firms to workers θ and not the number of unemployed workers and vacancies effectively means that we assume a matching technology that is constant returns. As the number of workers and firms doubles, the number of matches doubles, yet the matching probabilities remain unchanged.

[6] These are not distributions since their total measure is not equal to one. Because the measure of workers is equal to one and all are either employed or unemployed, it is the case that $F_u(\overline{a}) + F_e(\overline{a}) = 1$ and $F_u(\overline{a})$ is equal to the unemployment rate.

[7] This approach does not affect any of the results since the dividend deterministically increases the workers' asset holdings and merely shifts the asset distribution. However in the infinite horizon version of the model we assume that profits are distributed as the risk free dividend of a mutual fund owned by all workers and that has all firms in its portfolio as in Golosov et al. (2012). This closes the model in order to analyze the welfare implications of changes in UI.

As is inherent in the nature of directed search, there is a separate submarket for each firm-worker type pair. Heterogeneous firms and workers operate in different markets, while identical agents are in a common market. This permits workers to direct their search to those firms that offer the optimal terms (matching probability and wages) and it enables firms with vacancies to influence the search decision of workers by changing the terms of the wage offer. Whenever unemployed, a worker searches to find a job, and once employed she holds the job until the match is separated with exogenous probability λ.

Unemployment Benefits. We assume that there is an unemployment benefit b received by all unemployed workers. The benefit b is financed by a budget balancing proportional tax τ on wages. This requires that the sum of all benefits b over the unemployed agents is equal the sum of all taxes levied on wage income τw. We also assume that the entire income for the unemployed comes from UI. For a given b, the government sets τ to balance its period-by-period budget constraint:

$$ub = \tau \int w(a) f_e(a) da. \tag{1}$$

Actions. In period $t < T$, workers choose their consumption-savings bundle. Given assets a in the current period, each worker chooses the assets a' saved. This period's consumption is contingent on the saved assets but also on the labor market outcome: $c_e = Ra - a' + (1-\tau)w$ when employed and $c_u = Ra - a' + b$ when unemployed. Within the same period t, a two-stage labor search extensive form game determines the labor market outcome. Firms first simultaneously announce wages w' that will be paid starting in the next period: $w' \in \mathcal{W} = [\underline{w}, \overline{w}] \subset \mathbb{R}_+$. The contract space is restricted to invariant wages. After observing all wage-firm type pairs (w', y), the workers then choose which pair to apply to. Denote by $P(y, w')$ and $Q(a, a', y, w')$ the distribution of actions by firms and workers: $P(y, w')$ is the measure of firms that offers a productivity-wage pair below (y, w') and $Q(a, a', y, w')$ is the measure of workers with assets below a who save less than a' and who apply for productivity-wage pairs below (y, w'). We impose that those distributions of actions are consistent with the initial distributions of types $G(y)$ and $F_u(a)$, i.e., that there is market clearing. In particular, it must be the case that $P_{\mathcal{Y}}(\cdot) = G(\cdot)$ and $Q_{\mathcal{A}} = F_u(\cdot)$, where $P_{\mathcal{Y}}$ and $Q_{\mathcal{A}}$ are the marginal distributions. This ensures that the allocation is measure preserving.

Value Functions and Equilibrium. Denote by $U(a)$ the value of being unemployed with asset level

a at period t and by $E(a)$ the value of being employed.[8] We can then write

$$U(a) = \max_{a',\theta} \left\{ u(c_u) + \beta \left[m(\theta) E(a', y, w') + (1 - m(\theta)) U(a') \right] \right\} \tag{2}$$
$$\text{s.t.} \quad c_u = a - a' + b \quad \text{and} \quad a' \geq \underline{a}$$
$$E(a, y, w) = \max_{a'} \left\{ u(c_e) + \beta [\lambda U(a') + (1 - \lambda) E(a', y, w)] \right\} \tag{3}$$
$$\text{s.t.} \quad c_e = a - a' + (1 - \tau) w \quad \text{and} \quad a' \geq \underline{a}$$

A worker carries over last period's assets with return R. If unemployed, her income is thus $Ra + b$ and it is $Ra + (1 - \tau)w$ if employed. Her consumption is equal to this income net of here savings for next period a'. The unemployed worker's decision therefore is to optimally choose next period's assets as well as the submarket θ in which to apply for a job. The employed worker only chooses how much of her assets a' to save. All worker's savings are limited by a borrowing constraint, $\underline{a}$, which measures the incompleteness of the credit market.

The value to the firm that posts a vacancy is:

$$V(y) = -k + \max_{w'} \beta [q(\theta) J(y, w') + (1 - q(\theta)) V(y)]\}, \tag{4}$$

where $V'(y)$ is the continuation value when the job was not filled. In the stationary, infinite horizon allocation, $V'(y) = V(y)$, and in the two period version $V'(y) = 0$. At a cost k, the firm announces a vacancy and commits to a wage w' that it will pay in the next period in the case of a match. Firms also discount the future at rate β. $J(y, w)$ is the value of a filled job for a firm with productivity y when paying a wage w:

$$J(y, w) = y - w + \beta [\lambda V(y) + (1 - \lambda) J(y, w)]. \tag{5}$$

We adopt the equilibrium concept used by Acemoglu and Shimer (1999). To accommodate the two sided heterogeneity of firm productivity and worker assets, we will use the version of their equilibrium adjusted by Eeckhout and Kircher (2010) to allow for two-sided heterogeneity and a continuum of agents. They consider the Acemoglu and Shimer (1999) setup as a large game where each individual's payoff is determined only by her own action and the distribution of actions in the economy, which consists of the optimal choices of each of the individuals in the distribution.[9]

In line with the literature on directed search (see for example McAfee (1993), Acemoglu and Shimer

[8] More precisely, $U(a, a', y, w, P, Q, F)$ is the value of an unemployed worker with assets a who saves a', who applies to a job y with wage w and who anticipates a distribution of offers P and a distribution of jobs Q, and when the asset distributions are given by F. Likewise for $E(a, a', y, w, P, Q, F)$.

[9] The queue length θ is a function of the distribution of offers P and visiting decisions Q. Written explicitly, $\theta_{PQ} : \mathcal{Y} \times \mathcal{W} \to [0, \infty]$ is the expected queue length at each productivity-wage combination (y, w). Then along the support of the firms' wage setting distribution, $\theta_{PQ} = dQ_{\mathcal{YW}}/dP$ is given by the Radon-Nikodym derivative, where $Q_{\mathcal{YW}}$ is the marginal distribution of Q with respect to $\mathcal{Y}$ and $\mathcal{W}$.

(1999)), we impose restrictions on the beliefs about off equilibrium path behavior. In the current setup, beliefs about the queue length corresponding to firm or worker choices that do not occur in equilibrium are not defined. Therefore, we define those off equilibrium path beliefs corresponding to the notion of subgame perfection.[10] Firms expect workers to queue up for jobs as long as it is profitable for them to do so given the options they have on the equilibrium path. Formally, this defines the queue length over the entire domain as: $\theta(a,w) = \sup\{\theta \in \mathbb{R}_+ : \exists a, m(\theta)[y - w \geq \max_{y,w \in \text{supp} P} U(a,y,w,P,Q)]\}$. In all other cases, the queue length is zero.

This now permits us to define equilibrium. When time is finite, the equilibrium can be defined recursively starting from an initial asset distribution. In the infinite horizon economy, we solve for the stationary asset distribution. In each period, an equilibrium is a pair of distributions (P,Q) such that the following conditions hold: 1. Worker optimality: $(a,a',y,w) \in \text{supp } Q$ only if (y,w') maximizes (2) and (14) for a; 2. Firm optimality: $(y,w') \in \text{supp } P$ only if w' maximizes (4) and (5) for y.

This is a matching problem with a non-linear pairwise Pareto frontier. Existence is established in Legros and Newman (2007) and Kaneko (1982). Jerez (2012) establishes the existence of an equilibrium in a directed search model with a continuum of agents and a general matching technology.

The (measure preserving) market clearing condition is particularly transparent when matching is monotone. Then there is one-to-one matching of a to y, which we represent by a function $\mu : \mathcal{A} \to \mathcal{Y}$. Under positive assortative matching (PAM), $\mu'(y)$ is positive and it is negative under negative assortative matching (NAM). Under PAM high asset workers match with high productivity jobs, and the market clearing condition can be written as:

$$\int_a^{\overline{a}} \theta(a) f_u(a) da = \int_{\mu(a)}^{\overline{y}} g(y) dy. \tag{6}$$

3 The Equilibrium Allocation

We first analyze a simple two period model. The objective is to provide us with insights into how the per period allocation of asset holders to firms works. We then turn to the infinite horizon model, where we focus attention on the steady state and where we lay the ground for the calibration and policy exercise. For the purpose of the theory results in this section, we assume benefits and vacancy posting costs are zero: $b = 0, k = 0$. Benefits and vacancy posting costs are important for the calibration in the infinite horizon model, but do not add any insights in understanding the mechanism of the equilibrium allocation.

[10] Peters (1997) and Peters (2000) provide micro foundations for a version of this model where this assumption is indeed justified as the limit of deviations in a finite game.

3.1 The two-period model

We first analyze the decentralized equilibrium allocation in the two period model where all workers are initially unemployed. Let there be an exogenously given initial distribution of assets $G(a)$. With $T = 2$, there is only a consumption-search (a, θ) decision in period 1. In the final period, consumption is determined by period's savings decision and the outcome of the job search. The value of both employment and unemployment are therefore equal to the utility of consumption in the respective states: $E(a', y, w') = u(Ra' + w')$ and $U(a') = u(Ra')$. We can then rewrite (2) (since all are unemployed in the first period, $E(a, y, w)$ is null) as:[11]

$$U(a) = \max_{a',\theta} \left\{ u(a - a') + \beta \left[m(\theta) u(Ra' + w') + (1 - m(\theta)) u(Ra') \right] \right\}, \tag{7}$$

In other words, the consumption is completely pinned down by the choice of a' and the labor market choice θ which determines w' and the matching probability $m(\theta)$. The expected payoff to a firm y posting a vacancy is

$$V(y) = \max_{w'} \beta q(\theta) \left(y - w' \right), \tag{8}$$

from (4) since $J(y, w') = y - w'$, and the continuation value is zero.

The firm's problem is to set wages w to maximize expected profits $V(y)$. The consumer's problem is to maximize expected utility from consumption while simultaneously making an optimal search decision. We can therefore write the equilibrium worker and firm optimization as:

$$\max_{a',\theta} \left\{ u(a - a') + \beta \left[mu(Ra' + w') + (1 - m) u(Ra') \right] \right\}$$
$$\text{s.t. } V = \max_{w'} \beta q \left(y - w' \right),$$

where we use the short hand notation m and q for $m(\theta)$ and $q(\theta)$. Given $w' = y - \frac{V}{\beta q}$ and the optimal choice of wages follows from the optimal choice of queue length θ, we can write this problem as a matching problem with non-linear Pareto frontier denoted by $U(a, y, V)$:

$$U(a, y, V) = \max_{a',\theta} u(a - a') + \beta \left[mu \left(c'_e \right) + (1 - m) u(Ra') \right]$$

where $c'_e = Ra' + y - \frac{V}{\beta q}$. Then the solution to the maximization problem is $a'^{\star}(a, y, V), \theta^{\star}(a, y, V)$ and

[11] Since there is no period 0, the value of the assets in period 1 is set equal to a instead of Ra. This is without loss.

satisfies:

$$-u'(a-a') + \beta R\left[mu'(c_e') + (1-m)\,u'(Ra')\right] = 0 \tag{9}$$

$$\beta m'\left[u\left(c_e'\right) - u(Ra')\right] + \beta u'\left(c_e'\right)\frac{\theta q' V}{\beta q} = 0. \tag{10}$$

The optimal savings behavior and optimal job search simultaneously implies a matching decision. That is, a worker a effectively chooses a firm y. We can now analyze this allocation problem with a non-linear frontier $U(a, y, V)$, where a' and θ are chosen endogenously. We use the standard solution method for an assignment problem. The worker takes the firm payoff $V(y)$ as given (call it the hedonic price schedule) and chooses the firm type y that maximizes her expected utility. From the first order condition, the optimal y therefore satisfies $U_y + U_V \frac{\partial V}{\partial y} = 0$. This implies:

$$\beta m u'(c_e')\left(1 - \frac{V'}{\beta q}\right) = 0. \tag{11}$$

where the effect of y and V on U through a' and θ is zero from the envelope theorem: $U_{a'} = 0, U_\theta = 0$. The details of the derivation of the partial derivatives are in the Appendix.

We want to ascertain under which circumstances there is monotone matching of asset holdings a in job productivities y. This is now a matching problem $U(a, y, V)$ where a type a chooses the optimal y, given optimizing behavior. The allocation is denoted by $a = \mu(y)$. Then the total cross derivative of U with respect to a and y is positive provided

$$\frac{d^2}{dady}U = U_{ay} + U_{Va}\frac{\partial V}{\partial y} = U_{ay} - \frac{U_y}{U_V}U_{aV},$$

where we use the first order condition to substitute for $\frac{\partial V}{\partial y}$. Therefore, there will be Positive Assortative Matching in types a, y provided $U_{ay} > \frac{U_y}{U_V}U_{aV}$ (see also Legros and Newman (2007) and Chade et al. (2017)). The next proposition establishes under which conditions on the primitives (preferences and technology) this is satisfied:

Proposition 1 *Workers with higher initial asset levels a will apply for higher wage jobs provided*

$$\frac{u'(c_e') - u'(Ra')}{u(c_e') - u(Ra')} < \frac{u''(c_e')}{u'(c_e')}. \tag{\textbf{U}}$$

Proof. In Appendix. ■

This Proposition establishes under what conditions of the utility function agents with higher levels of assets will choose more risky jobs. The condition does not immediately allow for a straightforward interpretation, and in the next two results we characterize the properties. First, we show that within the class of Hyperbolic Absolute Risk Aversion (HARA) utility functions, the condition is satisfied whenever absolute risk aversion is decreasing (DARA).

Proposition 2 *Consider the class of utility functions with Hyperbolic Absolute Risk Aversion (HARA):*

$$u(c) = \frac{1-\gamma}{\gamma}\left(\frac{\alpha c}{1-\gamma}+\beta\right)^{\gamma} \qquad \text{where } \alpha > 0, \beta > \frac{\alpha c}{1-\gamma}.$$

*Then condition (***U***) holds whenever there is Decreasing Absolute Risk Aversion (DARA): $\gamma < 1$. It holds with opposite inequality when there is Increasing Absolute Risk Aversion (IARA): $\gamma > 1$.*

Proof. In Appendix. ■

A number of results for special cases of the HARA preferences immediately follow, including CRRA, logarithmic, CARA, risk neutrality and the quadratic.

Corollary 1 *Consider the class of HARA utility functions. Condition (***U***) holds:*

1. *under CRRA $u(c) = \frac{1-\gamma}{\gamma}c^{\gamma}$ $(\alpha = 1-\gamma, \gamma < 1, \beta = 0)$ and Log utility: : $u(c) = \log c$ $(CRRA, \gamma \to 0)$;*
2. *with equality under CARA $u(c) = 1-e^{-\alpha c}$ $(\beta = 1, \gamma \to -\infty)$ and Risk Neutral $u(c) = \alpha c$ $(\gamma = 1)$;*
3. *with opposite inequality under Quadratic utility: $u(c) = -\frac{1}{2}\left(-\alpha c+\beta\right)^2$ $(\gamma = 2)$.*

Proof. In Appendix. ■

The results for HARA may indicate that condition (**U**) holds more generally. The answer is partially true. For small differences between the level of consumption when a job is obtained and the consumption of unemployment ($c_e - Ra' = w$ small), we can indeed completely generalize the characterization: when there is DARA, condition (**U**) is satisfied and high asset types choose high productivity jobs. This is proven in Proposition 3. However, for general utility functions beyond HARA and with wages w large, this characterization does not hold. In Example 1 in the Appendix, we show by counterexample that for w large, Decreasing Absolute Risk Aversion (DARA) is not sufficient for the condition to hold.

Proposition 3 *When w is small, condition (***U***) is satisfied for any utility function that exhibits Decreasing Absolute Risk Aversion (DARA), $-\frac{u''}{u'} < 0$, and thus has positive risk prudence, $u''' > 0$. Likewise, it holds with opposite inequality under IARA.*

Proof. In Appendix. ■

Condition (**U**) establishes that there are complementarities in the match value between a firm type y and a worker with assets a. In other words, the match value $U(a, y, V)$ between types a and y is supermodular, and therefore the equilibrium allocation matches high asset workers with high productivity firms. While there are no technological complementarities (all workers are identically skilled), risk aversion and two-sided heterogeneity generates a *natural preference* complementarity between assets and job productivity.

The implication of this condition is that high asset workers apply for high productivity jobs, they earn higher wages, they have higher unemployment, they consume more and they have higher expected utility. Likewise, high productivity firms post higher wages, they attract higher asset workers, they have higher expected profits and they fill vacancies faster.

3.2 Infinite Horizon

We now consider the stationary equilibrium allocation in the infinite horizon version of the model. The per period allocation problem in the labor market is very similar to the one analyzed for the two period model, with the exception of the continuation value. We derive a condition similar to the (**U**) condition, but now for the infinite horizon economy. Note though that this condition involves value functions, not primitives such as utilities and consumption bundles.

In this section we assume that consumers jointly and equally own the equity of all firms. That is, no consumer holds the claims to the profit of an individual job but she can only hold the claim to a share of the aggregate profit. This is to avoid that an employed worker holds a short position in her own job in order to hedge against the risk of separation. This assumption implies that all workers regardless of their employment status receive a dividend, d, every period. This is to enable the welfare analysis to take into account the impact of a change in unemployment benefits on profitability of firms[12].

An important feature of our model is that we analyze the model for the parameter configuration $\beta R < 1$. This implies that while employed, the consumption-savings decision varies with time, and can thus incorporate precautionary savings by the employed who anticipate the possibility of becoming unemployed.[13] Using the standard technique in directed search, and similar to what we did in the two-period model, we substitute the wage and rewrite the problem as

$$U(a,y,V) = \max_{a',\theta}\left\{u(a-a') + \beta\left[m\frac{1}{1-\beta}u\left((1-\beta)Ra' + y - V\left[-\beta\lambda + \frac{1-\beta(1-q)}{q}\left(\lambda + \frac{1}{\beta} - 1\right)\right]\right) + (1-m)U(Ra')\right]\right\}. \tag{12}$$

We can now repeat the analysis in the proof of Proposition 1, to obtain the infinite horizon version of the result.

Proposition 4 *Workers with higher initial asset levels a will apply for higher wage jobs provided*

$$\frac{E_a(a,y,w) - U_a(a)}{E(a,y,w) - U(a)} < \frac{E_{a,w}(a,y,w)}{E_w(a,y,w)} \tag{$\mathbf{U}_\infty$}$$

[12]Under this assumption dividend is added to the budget constraint of employed $c_e = Ra - a' + (1-\tau)w + d$ and unemployed workers $c_e = Ra - a' + b + d$.

[13]Traditionally, models such as ours with infinitely lived agents have been solved assuming $\beta R = 1$ together with $\lambda = 0$ (see amongst others Acemoglu and Shimer (1999), Shimer and Werning (2008), Hopenhayn and Nicolini (1997); the notable exception is Krusell et al. (2010)). This assumption implies that assets levels when employed are invariant in steady state equilibrium, since the employed workers consume a share of their assets exactly equal to the dividend. In that case, $a' = \frac{a}{R} = \beta a$ and we can explicitly write the value for employment $E(a) = \frac{1}{1-\beta}u\left(w + (1-\beta)a\right)$, which is stationary.

Proof. In Appendix. ■

The following result now follows immediately:

Corollary 2 *Under condition ($\mathbf{U}_\infty$), the job productivity y decreases in the duration of unemployment.*

The result generalizes to the case with an infinite horizon, albeit that we cannot derive conditions on the primitives. However, in the next section we compute the equilibrium allocation and ergodic distributions and we verify whether along the equilibrium allocation Proposition 4 is satisfied.

4 Quantitative Exercise

We will now analyze the full model with ergodic asset and firm productivity distributions but with non-stationary savings by individual workers while unemployed and employed. The objective is to study the impact of unemployment benefits. The key feature of the model is the sorting of unemployed workers into different productivity jobs depending on their assets, just as in the simplified versions in section 2. We solve for the ergodic distribution of assets. Unemployed workers run down their assets while searching for a job to smooth consumption. In the process, as their assets decrease, they apply for lower productivity jobs. When on the job, they face a probability of exogenous separation. Anticipating the possibility of unemployment, they accumulate assets while working. This gives rise to a pattern of individual asset fluctuations to endogenously insure against unemployment risk.

Computationally, we derive the ergodic distribution of assets. It should be pointed out that a major technical innovation of our computation is the fact that the employed have a non-stationary policy function that reflects their precautionary savings behavior, i.e. $\beta R < 1$, unlike much of the exiting literature.[14] This is realistic as it captures both observed behavior by the employed and the fact that the asset distribution of the employed is endogenous. Broadly speaking, it works as follows. An efficient algorithm for any given level of benefit is to make a guess on (i) the dividend, (ii) the tax rate, (iii) the distribution of workers' assets (both employed and unemployed) and firms posting vacancies, (iv) the value of employment and unemployment and (v) the labor market clearing condition. Then we take the following six steps: 1. Given the distribution of unemployed workers and vacancies, we develop an algorithm to first sort the top workers and firms in the first submarket and find the job finding rate and wage for this submarket; then we find the value of a vacancy in the next submarket, using the first order condition of the allocation problem with respect to productivity, and again calculate the job finding rate and the wage. 2. We continue at each subsequent submarket until we reach the boundary of at least one distribution and check if the labour market is cleared; If not, we change the cut-off point of firm entry. 3. We solve the consumer's dynamic non-linear programing problem exactly. 4. We check

[14] For computational reasons, we assume that there is no capital investment and that the interest rate is exogenous. Introducing both aspects does not affect the basic features of the directed search mechanism and its interaction with the consumption-savings decision, which is at the heart of our paper.

the convergence of the distribution of firm types and worker assets (both employed and unemployed) and update them. 5. We check if the total tax revenue and benefits paid are equal. 6. We check whether our guess on the dividend is correct and update.

Our objective is to study the role of policy on the equilibrium allocation and on welfare. As unemployment insurance changes, both the incentives to save and accumulate assets and the job search behavior change. This also affects the allocation of workers to jobs of different productivities. Different asset holders have different preferences for insurance and therefore for benefits. We decompose the channels through which unemployment insurance affects the workers welfare across the distribution. First, we calibrate the baseline model with suitably chosen parameters and report its basic properties.

4.1 Benchmark Calibration

We set one period to be six weeks. The output produced is y and the utility function is $u(c) = \log(c)$. We set the borrowing constraint $\underline{a}$ to be -1 and following Menzio and Shi (2011), we pick the CES contact rate functions, $m(\theta) = \theta(1+\theta^{\gamma})^{\frac{1}{\gamma}}$. The asset domain is $a \in \mathcal{A} = [-1, 3]$ and the measure of workers is equal to one. With the parameter configuration below, this implies that at steady state, 18% of workers have negative asset. This is comparable to 17.3% of workers in the US economy using PSID 2005 data.

We set the discount factor β to be 0.99, and the interest rate is 0.005 or an annual interest rate of 5%. Following Shimer (2005), we set the flow value of the unemployed household production (benefit, b) to be 0.8 which turns out to be 40% of average wages at steady state. We set the cost of posting a vacancy at 0.5, which means that at steady state the cost of a vacancy is 21% of the average productivity of active firms (see Shimer (2005)). The probability of separation λ is set to be 0.03 which implies an average job duration of 4 years, consistent with data from the Bureau of Labor Statistics. To calibrate the elasticity of the matching function γ we target the steady state unemployment rate to be 4.2%.

To discipline the domain of productivities and the measure of firms at each productivity level we use PSID data with industry and demographic characteristics to recover productivity, using the following regression (details are in the Appendix):

$$\log(\text{earnings}_{i,t}) = \rho_0 + f(\text{skill}_i, \text{age}_i) + \rho_1 \mathbb{1}\{\text{married}\} + \rho_2 \mathbb{1}\{\text{woman}\} + \rho_3 \mathbb{1}\{\text{high skill}\} + \rho_4\{\text{hours}\} + \sum \rho_{5,\kappa} \mathbb{1}\{\kappa = \text{family size}\} + \rho_6 \mathbb{1}\{\text{white}\} + \sum \nu_j \mathbb{1}\{j = \text{industry}_i\} + \delta_t + \epsilon_{i,t}.$$

In the above regression, ν is the measure of sector productivity. This basically reflects the average wage premium in the sector earned by a household, which is not explained by observables and therefore could be attributed to the sector productivity. This permits us to choose a realistic productivity domain: $y \in \mathcal{Y} = [1.5, 2.5]$. However, only those firms with productivity $y \geq \underline{y}$ enter the market, where $\underline{y} \in \mathcal{Y}$

is determined endogenously.[15] To parameterize the measure of firms at each productivity level, we use the share of employment distribution over industries in PSID, taking into account the probability of filling vacancies at steady states for different levels of productivities.

Parameter	Definition	Value
β	discount factor	0.99
r	interest rate	0.005
b	flow value unemployment	0.8
k	cost of vacancy	0.5
λ	probability of separation	0.03
γ	elasticity of matching function	1.2

Table 1: Exogenous parameters

The balanced budge tax rate on wage income $\tau = 0.02$ is set to finance the entire burden of unemployment benefits in the economy. The ergodic distribution of asset holdings of the employed $F_e(a)$ and the unemployed $F_u(a)$ is endogenous given optimal choices of a. Table 1 summarizes the value of the pre-calibrated parameters.

Table 2 details the summary statistics of the most important endogenous labor market outcomes for the benchmark steady state calibration.

u	τ	average θ	average w
4.2%	0.020	1.11	196.82

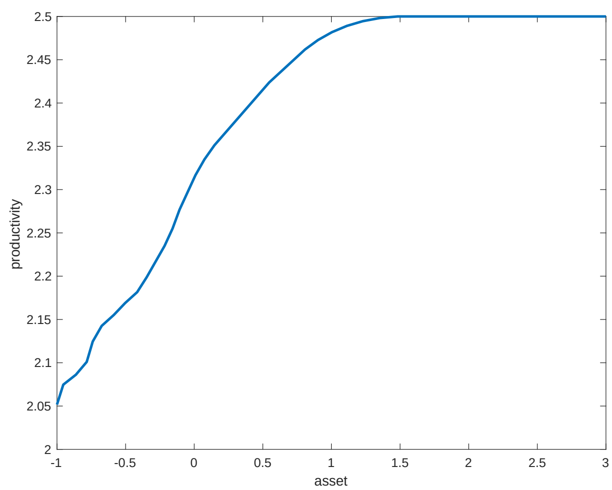
Table 2: Endogenous Outcomes

Characterization of the Steady State. We first display some key model features and then discuss our experiments.

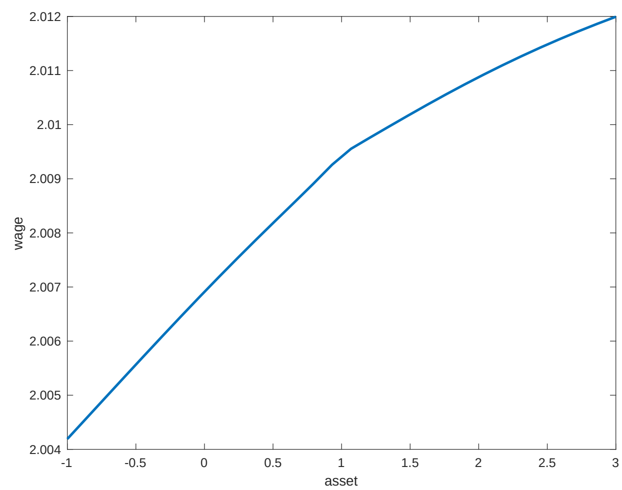
There is positive assortative matching between workers' asset holdings and firms' productivity: in equilibrium workers with a higher level of assets are matched with more productive firms.[16] Figure 1a shows the allocation of workers to firms in the labor market. The market clearing condition implies that all workers are allocated to submarkets while firms below a productivity threshold are staying out of the market. This threshold is obviously sensitive to different parameterizations of the model. In

[15] In the case with homogenous firms, the free entry condition implies that the value of posting a vacancy is zero at steady state. However, with two-sided heterogeneity, free entry implies that the value of posting a vacancy for firms at the threshold is zero and that firms above the threshold have positive values for posting vacancies. Moreover, a change in unemployment benefit when there is no heterogeneity among firms only affects the number of vacancies created by firms while with heterogenous firms it affects the number of vacancies as well as the quality of vacancies by shifting the productivity threshold.

[16] From Proposition 2 we know that under log preferences there is indeed positive sorting in the two period model. Because we cannot solve the general model analytically, we guess the allocation is positively assorted and verify ex-post that the match surplus along the equilibrium allocation is indeed supermodular, and the condition in Proposition 4 is satisfied.



(a) Allocation of firms and workers in the labor market



(b) Wages by asset level at hiring

Figure 1: Equilibrium allocation and wages

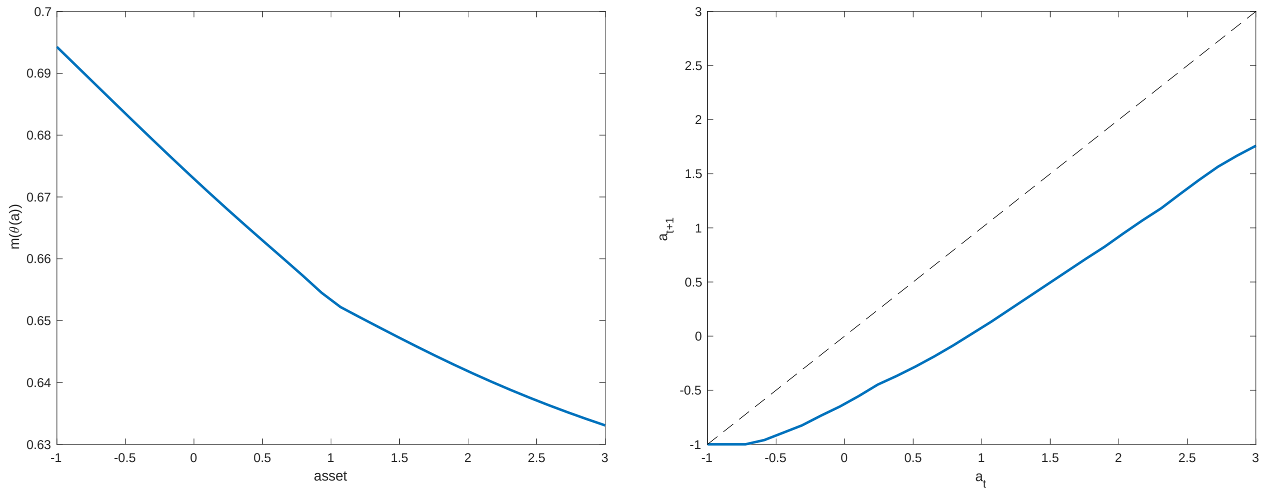
particular, below we will study the impact of a change in unemployment benefits on the threshold and therefore on job creation. A higher threshold means more firms stay out of the market and hence fewer jobs are created.

Figure 1b depicts equilibrium wages for different asset levels. Firms with more productive jobs post higher wages. This decreases the vacancy to unemployment ratio θ and allows them to fill the vacancy with higher probability. Workers with more assets apply for the high wage jobs because they are able to insure better against unemployment. Their assets allow them to maintain a higher level of consumption.

Quantitatively, the key aspect is the role that assets play in the productivity of *equally skilled* workers. Workers with higher assets apply for jobs with a substantially higher productivity than those with low assets (2.5 versus 2.05, Figure 1a). The reason why they are able to get those better jobs is that they are taking a substantial amount longer than those with low asset holdings. As shown in Figure 2a, the matching probability decreases from 69% for the low asset unemployed workers to just over 63% for those with high asset levels.[17] For the workers with low levels of assets, unemployment is a bad situation. If they do not find a job this period and deplete their asset stock further, next period they apply for lower productivity jobs which they can get with a higher probability. This dependence of the job search decision on assets is absent in the random search model: with random search, the probability of finding a job is the same for all workers regardless of their asset holding.

This channel is also absent in models with homogenous firms, which has important implications for the role of UI benefits. A change in unemployment benefit only affects the measure of vacancy creation

[17]The difference in job finding probabilities between low and high asset holders is higher for low level of UI compared to high level of benefit. For instance, at $b = 0$ the matching probability decreases from 85% to 72%.



(a) Job finding probability $m(\theta(a))$

(b) Savings decision a'

Figure 2: Policy functions of the unemployed

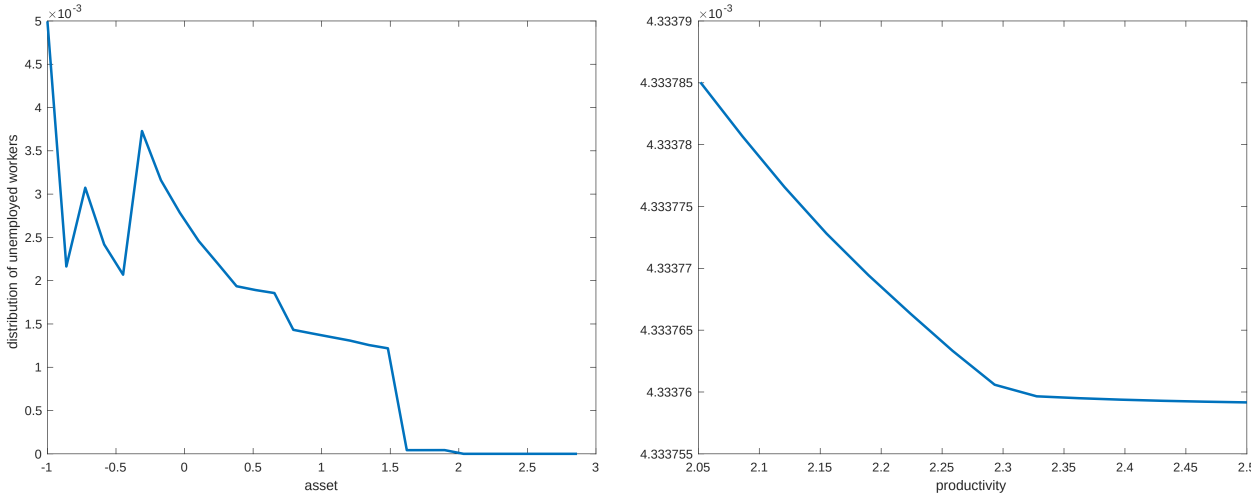
when all firms have identical productivity. In contrast, in our framework a change in UI not only affects the measure of job creation but also the productivity distribution of filled jobs and the productivity level in the economy even without a production function with decreasing return to scale.

The endogenous matching probability explains why the wage function is increasing whereas it is mostly flat in the random search model. At first sight, the slope of the wage function appears small. However, since the average duration of employment is around 33 periods (4 years), these small wage differences translate into big income differences over the duration of employment. In other words, workers choose submarkets with different probabilities of job finding, and different wages for the whole duration of employment. This is reflected in the fact that the value of jobs shows ample variation.

Interestingly, the dynamic nature of the problem now implies a time-varying job choice decision. A worker who fails to become employed sees their assets gradually deplete ($a_{t+1} < a_t$). But the optimal search decision dictates application to less productive, lower wage jobs when assets are lower. As a result, over the duration of unemployment, workers will gradually apply for less productive, lower wage jobs that they get with higher probability. Instead, while employed, they gradually increase their assets.

The probability of job finding is significantly lower for high asset holders. Those who are unlucky and do not find a job run down their assets in order to smooth consumption. In this process, they gradually apply to lower productivity jobs. As a result of this endogenous job finding probability, there are more unemployed workers with low assets than with high assets as depicted in Figure 3a.

Likewise, on the firm side (Figure 3b), we observe a fatter left tail for the stationary distribution of firms posting vacancies compared to the distribution of firms in the population, which is uniform. High productivity firms have a higher option value of filling a vacancy, so they increase the probability of



(a) Asset distribution unemployed workers $f_u(a)$ (b) Productivity distribution vacant firms $g(y)$

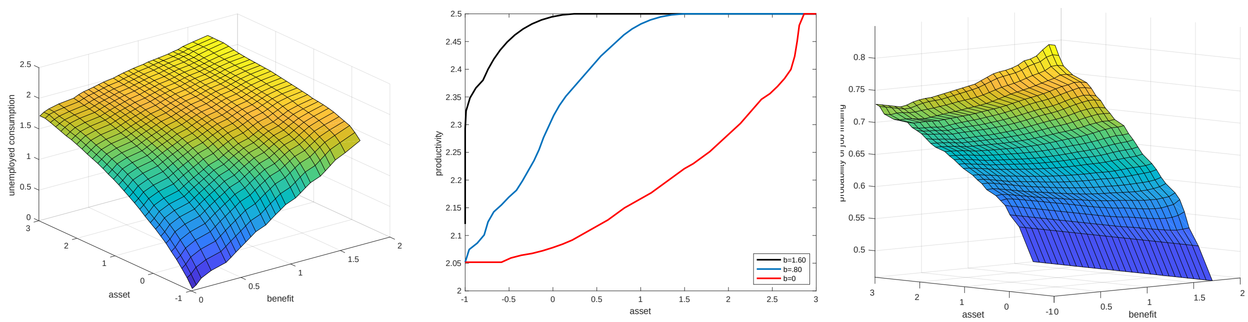
Figure 3: Ergodic distributions (Densities)

filling the vacancy by offering higher wages to the unemployed. Therefore more productive firms leave the pool of searching firms faster than less productive ones. As a result, in the steady state there are fewer high productive firms searching. In addition to the endogeneity of the vacancy distribution, also the marginal firm $\underline{y}$ is endogenous. This cutoff is thus a measure of job creation. Below, we investigate how the equilibrium allocation (including job creation) is affected by unemployment benefits.

4.2 Equilibrium effects of Unemployment Insurance

We now study the impact of different unemployment benefits on the equilibrium allocation. In the absence of complete markets to insure the employment risk, we ask how changes in the government mandated unemployment insurance policy that is financed with income taxes affects welfare. The direct impact of UI is that it allows workers to smooth consumption, as well as that it allows workers to apply for more productive jobs with lower job finding probabilities. However, UI will also affect welfare through various general equilibrium channels. In the first place, higher unemployment insurance reduces the firm's share of the match surplus. With a higher outside option, workers command a higher wage. This reduces job creation as only firms with higher productivity enter the market to post vacancies. This mechanism is similar to the one pointed out in Krusell et al. (2010) with random search, though with two-sided heterogeneity, a change in unemployment benefits also moves the productivity threshold above which firms enter to the market while with homogenous firms it only affects the measure of job opening but not the quality of jobs.

Specifically to our framework, direct insurance against unemployment affects the distribution of unemployed workers by influencing their saving decisions and therefore their allocation to jobs of different productivities. Guaranteed higher unemployment benefits, workers will save less, both while employed and while unemployed. In addition to the savings decision, benefits also affect the workers' job search behavior. Since workers with different asset levels direct their search to firms of different productivity, higher benefits will increase the unemployment rate (i.e., the matching probability) as well as the productivity of jobs that workers apply to. With less necessity to use their own assets for self insurance because of higher benefits, workers are more willing to take risk and will increasingly direct their search towards high productivity jobs that pay higher wages at the expense of lower matching probabilities.



(a) Consumption of unemployed (b) Allocation for different benefits (c) Job finding probability

Figure 4: Consumption, equilibrium allocation, and job finding probability for different levels of benefits.

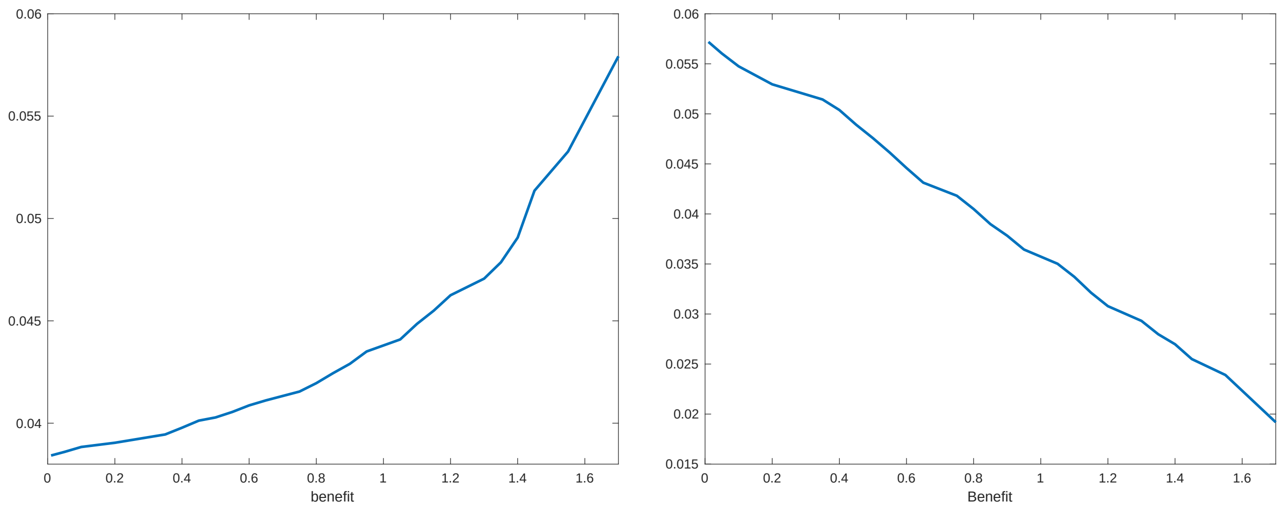
The general equilibrium effect of unemployment benefits are made explicit in the following series of figures where in the benchmark economy we vary the benefits b between 0 and 1.70.[18] First, consider the impact on consumption of the unemployed (Figure 4a). For all asset holders, equilibrium consumption of the unemployed increases in benefits. The effect however is much more pronounced for the low asset holders. In fact, those with assets close to the borrowing constraint nearly exclusively consume the entire benefits. For the high asset holders, benefits have a much more moderate impact on consumption.

Figure 4b illustrates the impact of benefits on the job search behavior and the resulting equilibrium allocation. When benefits are higher, all workers direct their search to more productive, high paying jobs. As a result, for all asset levels, the allocation of assets to productivities shifts upwards as benefits increase.

As benefits increase the productivity of the jobs, they also increases the competition for jobs and hence decrease the job finding probability (Figure 4c). This decrease is much more pronounced for the low asset holders. Benefits induce them to compete for higher productivity jobs. And while there is a big drop in the job finding probability as b increases, due to the general equilibrium effect there is only a minor increase in the allocation due to increased competition for high productivity jobs. This is most evident for the high asset holders. They apply for the high productivity jobs at any level of benefits, but they see their job finding probability drop from nearly 73% to 45% as benefits increase

[18] The average wage is endogenous, and in our simulated economies this range of benefits corresponds to the range of 0% and 90% of average wages.

from 0 to 1.70 while this drop is from 85% to 46% for low asset holders. Interestingly, at the highest level of benefits, the job search behavior in terms of matching probability is very similar (both low and high asset holders find a job with probability between 45 and 46%). Yet, there continues to be a big difference in the productivity of the jobs they obtain in equilibrium.



(a) Unemployment rate

(b) Total number of vacancies

Figure 5: Equilibrium unemployment and vacancy creation for different levels of benefits.

Not surprisingly then, the impact of increased benefits is an increase in aggregate unemployment (Figure 5a). The unemployment rate goes up by almost 2 percentage points as benefits increase from 0 to 1.70. At the same time, the number of firms entering the market to open vacancies decreases only modestly, the cutoff $\underline{y}$ going from of 2.08 to 2.15, with the average number of vacancies going from 5.8% to 1.8% (Figure 5b). Fewer firms enter, and in addition they leave the market faster, hence the total number of vacancies drops by nearly 70%. The effect on job creation is therefore big. Moreover, a rise in UI means that workers have higher outside options and that increases wages which in turn reduces worker's consumption from firms dividends.

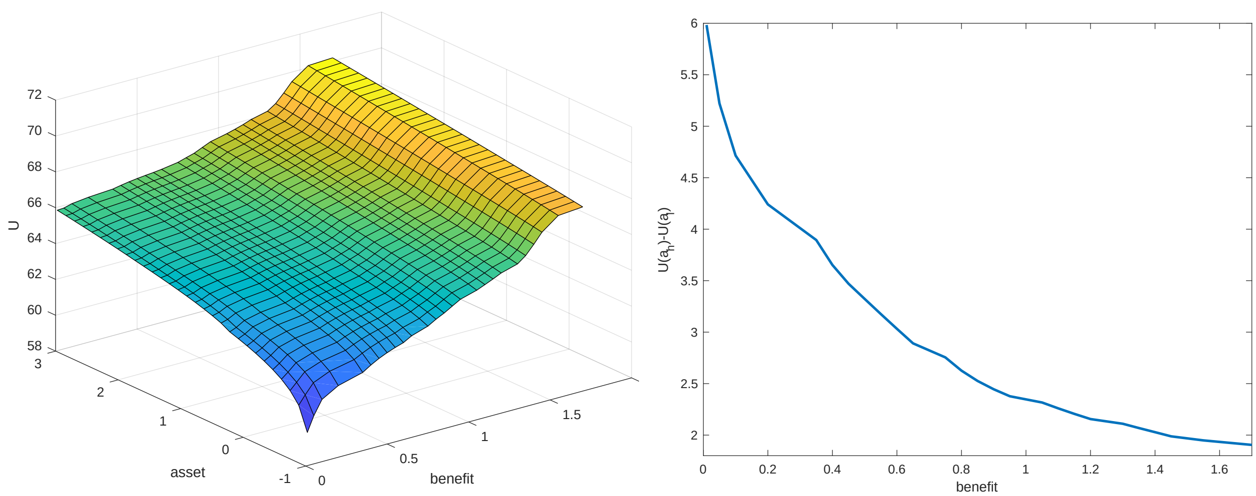
Higher benefit levels clearly pull the value of unemployment in opposing directions: search for better jobs and more consumption smoothing on the one hand, but lower vacancy creation, lower job finding rates and lower dividends on the other hand. To evaluate the overall impact, we look at the option value of unemployment as a function of assets and benefits. This is illustrated in Figure 6a.[19]

The value of unemployment increases in benefits for all unemployed workers. Nevertheless, it increases considerably more for low asset holders compered to high asset holders. A rise in unemployment benefit affects the value of unemployment in the following ways: i) increasing consumption; ii) decreasing job finding probability; iii) decreasing firm entry; iv) decreasing dividend; and v) increasing gross

[19]Given log preferences, the variation in utility is nominally small, even if assets and benefits drop to zero.

wages.[20] While the first effect increases the value of unemployment the next three are having the opposing impact. Higher benefits means more insurance and consumption smoothing during unemployment but on the other hand, when UI goes up it also means that workers tend to apply for better paying jobs with a lower job finding probability. Moreover, higher benefits imply less entry of firms which contributes further to the lower job findings rates in the aggregate, and they imply higher wages because workers have a higher outside option which in turn reduces the surplus of firms and therefore the dividend workers receive. For all unemployed workers the first positive effect of an increase in unemployment benefits dominates the other negative effects and therefore the value of unemployment is strictly increasing in benefit (see Figure 6a).

However, the effect of benefits is much more pronounced for the asset poor unemployed workers. Because they have a high marginal utility of consumption, the insurance effect of unemployment benefits is much stronger. Figure 6b depicts the difference in value of unemployment for a high asset holder unemployed worker compared to a low asset holder for all benefit levels. When UI increases, this difference shrinks because high asset holders need less insurance while they suffer more from lower probabilities of job findings.



(a) Value of unemployment as function of asset, benefits

(b) Difference in value of unemployment

Figure 6: Value of unemployment.

As with the value of unemployment, the value of employment is also increasing in benefits. Although higher benefits have a direct negative impact on the level of wages and dividends, higher insurance prospect in case of unemployment results in higher value of employment.

[20]this implies a reduction in net wages when the threshold of entry for firms in not changed.

4.3 Welfare

To study the welfare impact of a change in unemployment benefits, we compare steady states with different levels of UI. We measure welfare gains or losses by computing the percentage change in life time consumption required to give workers the steady state average lifetime utility. In our welfare analysis, we follow Krusell et al. (2010) and fix the distribution of workers' asset holdings at the benchmark economy. This implies that to compare the counterfactuals economies with the benchmark, we move all workers to a different economy which has a different level of UI but are otherwise is identical to our benchmark, and measure the consumption loses or gains of workers across the asset distribution. In this exercise, we hold fixed the distribution of assets in the welfare calculation so that welfare is always compared from the perspective of the same agents. We can thus isolate the welfare effect from a change in the distribution of asset holding.[21]

Denote our welfare measure in these comparisons by ψ. c_t is the consumption in the benchmark economy and $\hat{c}_t$ is the consumption in any of the counterfactual experiments.[22] Then, the welfare is calculated satisfying the following condition:

$$\mathbb{E}_0\Big[\sum_{t=0}^{\infty}\beta^t \log((1+\psi)c(a_t))\Big] = \mathbb{E}_0\Big[\sum_{t=0}^{\infty}\beta^t \log(\hat{c}(a_t))\Big] \tag{13}$$

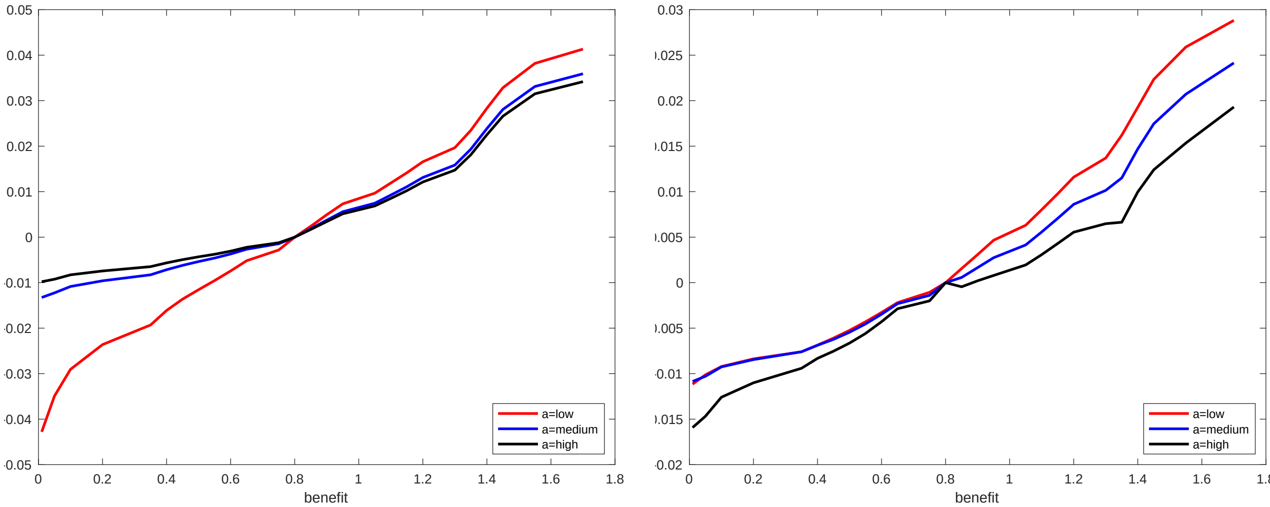
Figure 7 depicts the welfare measure for an unemployed and employed worker with low, medium and high asset holdings at different economies. The net gain or loss of changing UI is heterogenous across the distribution of asset. All unemployed workers gain from moving to an economy with a higher level of benefit though this gain is heterogenous across the distribution of asset holdings. Asset poor unemployed workers have a higher marginal utility of consumption and therefore value insurance more. The asset rich unemployed workers care more about their employment probability as they have enough assets to smooth consumption, while poor unemployed workers care more about the direct insurance effect of UI as they have high marginal utility of consumption.

The welfare gains or losses also differ across the distribution of asset holdings for employed workers. All employed workers experience a welfare gain when UI goes up. However, the welfare gain is higher for rich employed workers when benefits are low, and it is higher for poor employed workers when benefits are high. This is because they need less insurance if hit by a separation shock. The fact that increasing UI results in a welfare gain for all employed workers is the consequence of the direct insurance impact of UI. This effect dominates the negative impact of higher taxes on net wages.

To calculate the aggregate measure of the welfare change, we integrate the welfare measure ψ over

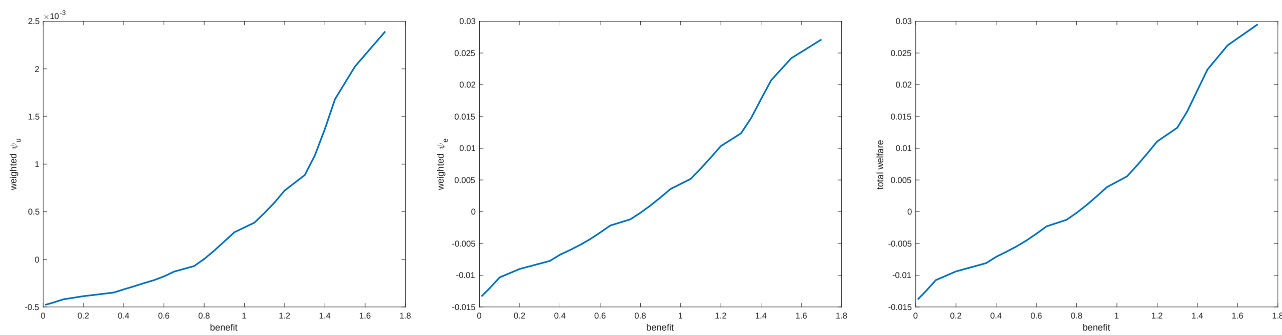
[21] We have repeated the entire welfare analysis with endogenous asset distributions, and find similar results. If anything, the qualitative findings are more pronounced. The results are available upon request.

[22] The value of consumption in the benchmark economy is $V_I = \mathbb{E}_0\Big[\sum_{t=0}^{\infty}\beta^t \log(c_i(a_t))\Big]$, and in the counterfactual economy is $\hat{V}_i = \mathbb{E}_0\Big[\sum_{t=0}^{\infty}\beta^t \log(\hat{c}_i(a_t))\Big]$ where $i \in \{u, e\}$, where the expectations operator is taken over the labor market uncertainty. The welfare gain or loss, ψ, can be calculated as $\psi_i = \exp[(1-\beta)(\hat{V} - V)] - 1$.



(a) unemployed workers (b) employed workers

Figure 7: Welfare measure: ψ



(a) Average value unemployed (b) Average value employed (c) Total welfare

Figure 8: Welfare

the distribution of asset holdings in the benchmark economy. This is depicted in Figure 8. The key point here is that after integrating over the distribution, the welfare of all workers is increasing in benefits.

In Table 3 we report the welfare implications of a change in benefits. Compared to the benchmark economy with $b = 0.8$, we calculate the welfare gain (total, for the unemployed and for the employed). We find that on average the unemployed and employed gain from an increase in the benefits. This implies that the total welfare effect from an increase in benefits is positive. But, this of course masks the fact that there is a clearly different gain for workers with different asset holdings. The last six columns aim to capture the heterogeneity due to different asset holdings within the pool of employed and unemployed. A low asset unemployed worker gains more than 4% by moving from a Laissez-faire

from $b = 0.8$ to $b =$	Total %	Unemp. %	Emp. %	$a_{u,l}$ %	$a_{u,m}$ %	$a_{u,h}$ %	$a_{e,l}$ %	$a_{e,m}$ %	$a_{e,h}$ %
0	-1.07	-0.09	-0.98	-4.28	-1.33	-0.98	-1.12	-1.09	-1.59
0.10	-0.91	-0.07	-0.84	-2.72	-1.08	-0.84	-0.91	-0.94	-1.26
0.35	-0.73	-0.05	-0.68	-1.85	-0.81	-0.64	-0.75	-0.73	-0.88
0.45	-0.63	-0.04	-0.59	-1.32	-0.60	-0.48	-0.63	-0.64	-0.76
0.65	-0.23	-0.02	-0.21	-0.56	-0.32	-0.28	-0.22	-0.23	-0.27
0.80	0	0	0	0	0	0	0	0	0
0.95	0.37	0.03	0.34	0.75	0.57	0.53	0.47	0.29	0.10
1.10	0.66	0.04	0.61	1.20	0.93	0.85	0.79	0.54	0.29
1.30	1.18	0.07	1.11	1.87	1.49	1.38	1.35	1.03	0.71
1.55	2.58	0.16	2.42	4.02	3.51	3.35	2.67	2.44	2.20

Table 3: Welfare change compared to benchmark economy (first three columns are welfare gains for all workers, the unemployed, and the employed. The last six columns are the welfare gain by an unemployed and employed worker with low(l), medium(m) and high(h) asset holding.

economy to the benchmark while this is less than 1% gain for an asset rich unemployed worker. A low asset employed worker gains 2.67% if he is thrown from the benchmark to a counterfactual economy with a level of benefit almost two times higher. A high asset employed workers gain is 2.20% in the same situation. On average moving workers from a Laissez-fair economy to an economy with double level of benchmark economy benefit increases the welfare by more than 3.5%.

4.4 Production and productivity

A novel feature of our model compared to the ones with identical firms is the impact of UI changes on workers productivity. In a model with homogenous firms and a linear production function, a rise in benefits only affects the measure of firms entering the market, and leaves the productivity of jobs unaffected. However, in this framework, a rise in UI affects labour productivity because the allocation of workers to jobs changes, which in turn affects the firms' entry decision. Figure 9 depicts the percentage change in average output per worker for different levels of benefits compared to the benchmark economy. By construction, at the benchmark $b = 0.8$, the change is zero.

We find that average output per worker is virtually unchanged for benefit levels between zero and the benchmark. Thereafter, labour productivity increases sharply to 1.4%. When UI goes up, there are three countervailing forces: 1. better insured workers tend to apply to more productive jobs; 2. asset holdings are lower, and asset poorer workers apply to less productive jobs; and 3. the threshold of firms' entry moves up so lower productive firms do not find it profitable to enter the market. What Figure 9 shows is that for low benefits, these forces neutralize each other. Instead, for high benefits, workers tend to apply to more productive jobs indicating that the incentive effect dominates the effect of lower asset holdings. This shows the impact of higher asset holding as well as higher entry threshold of firms.

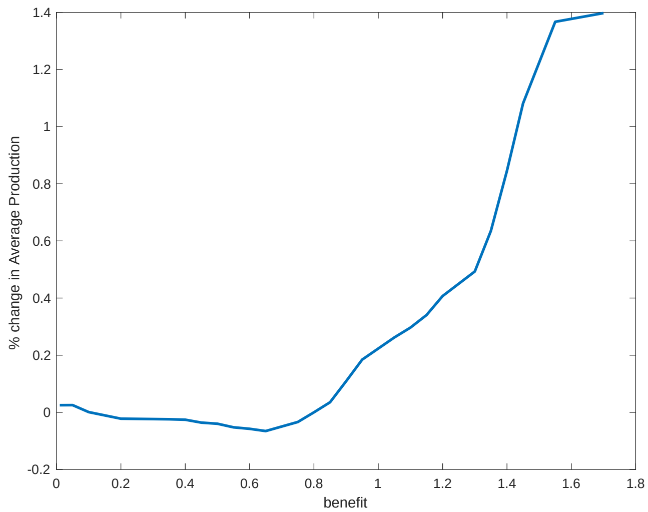


Figure 9: Productivity (average output)

This measure of output per worker is not equal to the measure of welfare because it does not take into account the employment probability, which is decreasing with benefits. We know from the welfare calculations that welfare is increasing in benefits, indicating that benefits increase worker productivity of the asset rich, while the decrease in overall employment (just over one percentage point) is limited. The incentive effect is therefore key in understanding the rise in welfare from an increase in benefits.

4.5 Severance Pay

An alternative UI regime to a per period payment while unemployed is to offer a lump sum payment upon separation, severance pay.[23] This has been heralded as a UI scheme with better incentives to search while unemployed, while at the same time offering insurance and a means to smooth consumption. Unlike per period benefits, the worker now has to manage her assets to smooth consumption. The trade off between the standard benefit system and severance pay is that severance pay does not offer any form of income if a worker continues to be unemployed and her assets are depleted.

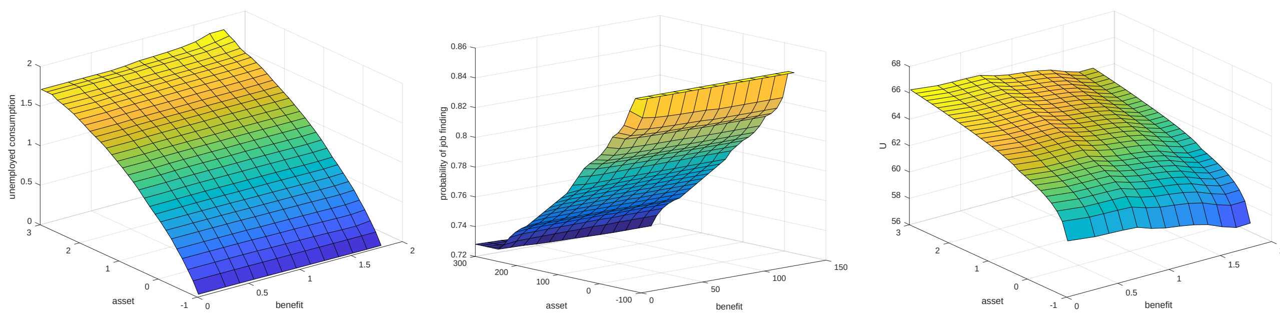
To that effect, in this section we assume workers get a lump sum transfer S upon separation and entry into unemployment and then receive nothing while being unemployed ($b = 0$) until they find a job again. We can rewrite the value function of the employed worker (the problem of other agents remains

[23] We are grateful to Melvyn Coles and Pierre Cahuc for suggesting to analyze severance pay and its comparison with per period benefits.

as before with $b = 0$):

$$E(a, y, w) = \max_{a'} \left\{ u(c_e) + \beta[\lambda U(a' + S) + (1 - \lambda)E(a', y, w)] \right\} \tag{14}$$
$$\text{s.t.} \quad c_e + a' = Ra + (1 - \tau)w + d \quad \text{and} \quad a' \geq \underline{a}.$$

As with the per period benefits b, the severance pay S is financed with a proportional wage tax and a balanced budget. To make the policies comparable, we express the severance pay S in terms of the equivalent expected benefit the average worker would receive under the per period benefit scheme, i.e., where $S = \frac{b}{m(\theta)}$. For example, if the average matching probability is $m(\theta) = 0.71$, and the per period benefit is $b = 0.8$, then the corresponding equivalent severance pay is $S = 1.1$. Therefore, in order to be able to compare per period benefits with severance pay, in the figures we express the severance pay in terms of the corresponding $b(S)$ (=0.8 in this example) and not in terms of S (=1.1 in the example).



(a) Consumption of unemployed

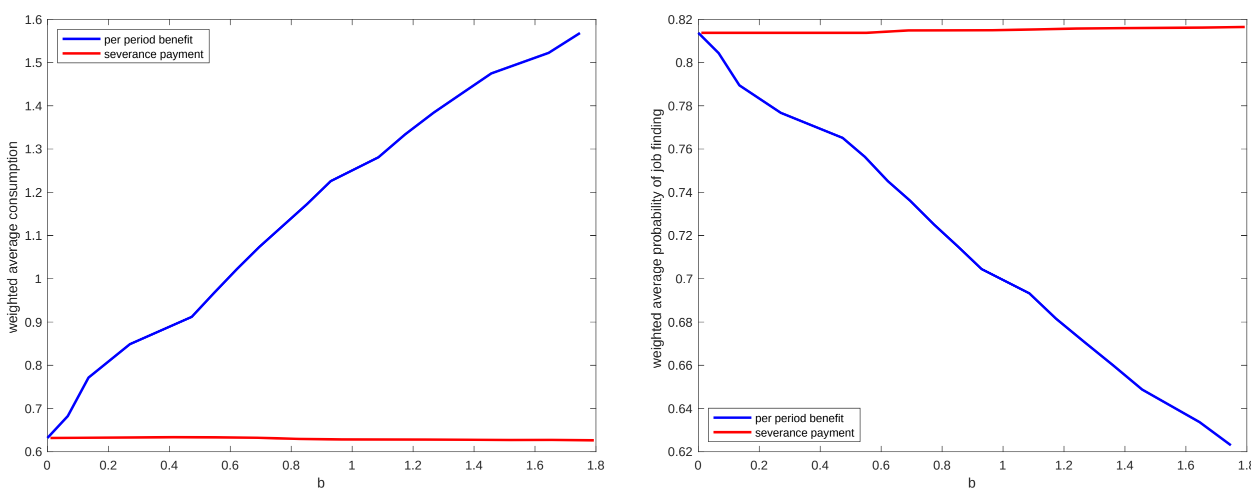
(b) Job finding probability

(c) Value of unemployment

Figure 10: Consumption, job finding probability, and value of unemployment for different levels of severance payments and assets

Figure 10 depicts consumption, the job finding probability and the value of unemployment for different asset levels and for different severance pay (expressed in b equivalence). A rise in severance pay does not significantly affect consumption and the probability of job finding: these two functions are almost flat in severance pay for each asset level. This is because severance pay is a one off payment upon becoming unemployed. Therefore its insurance power during joblessness is low and unemployed workers do not change by much their response in consumption or in job search behavior for different levels of severance pay. On the other hand, an increase in severance pay does increase taxes and reduces the net wage. Of course, workers dramatically change their behavior as assets deplete, much more so than with positive per period benefits. The effect that lower assets have on future consumption is reflected in the value of unemployment (Figure 10c): the value of unemployment as a function of severance payments is strictly decreasing in severance pay for each asset level.

To analyze the welfare implications of severance pay, we calculate the welfare gain or loss of moving from our benchmark economy with per period benefit ($b = 0.8$), to any of the counterfactual severance pay economies. As in the previous section, we compare utilities when we move the whole distribution of asset holdings from the benchmark economy to the counterfactual economies. We first compare the outcomes of per period benefits and severance payments, expressed in terms of the equivalent per period benefit b. Using the benchmark economy asset distribution, Figure 11 shows the change in average consumption and in the average job finding probability in two economies, one with per period benefits and one with severance pay.[24] Clearly, per period benefits offer better consumption outcomes as benefits increase, but lower job finding probabilities. With higher benefits, workers apply for better jobs and hence have lower job finding rates. By contrast, unemployed workers who have received higher severance pay increase their consumption as well but less so because they anticipate the possibility of not receiving benefits if unemployment lasts longer than average. This also translates in a more moderate change in the job finding probability (Figure 11b) due to precautionary job search.



(a) Average consumption of unemployed workers

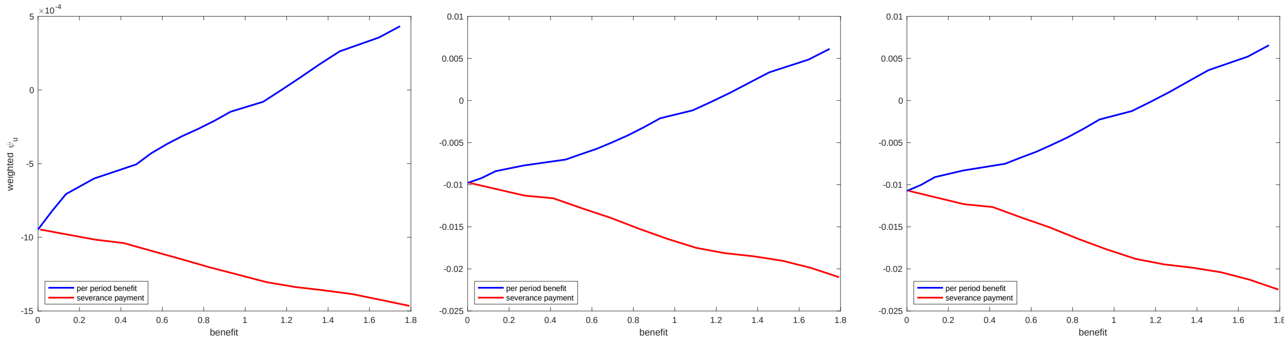
(b) Average job finding probability

Figure 11: Average weighted consumption and probability of job finding.

To make concrete the tradeoff between the insurance value of per period benefits and the incentive effects of severance pay, we evaluate the value of unemployment under both policies (compare Figure 10c to Figure 6a above). Evidently, with per period benefits, $U(a)$ increases more for low asset holders when total benefits increase, while a combination of higher taxes and low insurance offset the impact of higher job finding rates in severance payments economy and the value of $U(a)$ goes down.

Figure 12 shows the welfare gain and loss of workers across different regimes of UI (per period benefit and severance pay). In contrast to per period benefit, a rise in severance pay is welfare decreasing for unemployed workers (Figure 12a). An increase in severance payment means lower net wages while the consumption and probability of job findings do not change by much. In contrast, a rise in per period

[24]We calculate the severance pay to be equal to the average sum of all benefits received, given the expected duration of unemployment.



(a) Average value unemployed (b) Average value employed (c) Total welfare

Figure 12: Comparing welfare effect of per period benefit and severance payment

benefit offers considerable increase in insurance to unemployed workers. Employed workers under severance pay regimes prefer lower UI. An increase in severance pay is associated with a high welfare loss compared with a rise in per period benefit. The direct impact of a change in UI on employed workers is through net wages. But agents also internalize the indirect effect of a change in the UI regime by taking into account the change in probability of job finding and consumption if they become unemployed. This implies that the insurance welfare effect of a higher per period benefit dominates the incentive effect of a higher severance pay.

To conclude, the welfare gain of per period benefits dominates that of the severance pay regime. Moreover, moving from the benchmark economy to almost any per period benefit regimes gains higher net welfare compared to moving to a severance pay economy with the equivalent per period benefit.

5 Conclusion

We have analyzed the effect of asset holdings on worker productivity in the presence of frictional job search. In the absence of complete insurance markets, workers have precautionary search motive: the job search decision is an important source of self insurance for those with low assets levels. To analyze this, we solve a model with directed search and consumption smoothing where workers with high asset holdings sort into more productive jobs. Because their asset holdings allow them to smooth consumption, they can afford to face a substantially lower job finding probability. The difference in the job finding probability depends on the level of benefits. For our benchmark calibration, we find that the job finding rate for those with high assets is 7.1% lower, and it is 18% lower if benefits are zero. This is consistent with independent findings that the poor without liquid assets find jobs faster.

In this framework, we analyze the welfare effect of an income tax financed UI policy. Not only is there the usual conflict of interest between the unemployed who receive the benefits and those with a job who pay for it, there is also a conflict between the unemployed with assets and those without. Both receive benefits, but the rich can rely more on their savings for insurance.

A novel feature of our model is the impact of UI changes on workers productivity. UI affects the average productivity of workers through 1. the allocation of workers to jobs of different productivities; and 2. through entry decision of firms. We show that for low UI benefit levels, an increase in benefits has no effect on average productivity per worker. However, for high enough benefit levels, a rise in benefits has a cleansing effect, meaning that firms with lower productivities do not enter the market anymore which leads to a rise in average worker productivity.

We also compare per period benefits to a one off severance payment. Severance pay provides better job search incentives, but comes at the cost of poorer consumption smoothing. We find that per period benefits dominate severance pay as workers value more the insurance effect than the search incentive effect.

Given our setup, the potential for some obvious extensions immediately come to mind. For example, it is clear that the optimal benefit scheme should be contingent on assets. However, as Koehne and Kuhn (2015) show, the effect of asset tested benefits on welfare is rather minor. Moreover, when asset holdings are private information, the dynamic mechanism design problem in infinite horizon, with a continuum of agents and with time varying strategies is virtually impossible to solve.

Appendix A Proofs and Derivations

A.1 Partial Derivatives of U and a'

$$\begin{aligned}
U_y &= \beta m u'(c_e') + U_{a'} a_y' + U_\theta \theta_y = \beta m u'(c_e') \\
U_a &= u'(a - a') + U_{a'} a_a' + U_\theta \theta_a = u'(a - a') \\
U_V &= \beta m u'(c_e') \frac{-1}{\beta q} + U_{a'} a_V' + U_\theta \theta_V = \beta u'(c_e') \frac{-\theta}{\beta} \\
U_{ay} &= -u''(a - a') a_y' \\
U_{aV} &= -u''(a - a') a_V'
\end{aligned}$$

where $U_{a'} = 0$ and $U_\theta = 0$ from the envelope theorem.

Denote the maximand of U by $\phi(a', \theta) = u(a - a') + \beta \left[m u\left(c_e'\right) + (1 - m)\, u(Ra') \right]$, i.e., the objective function that is maximized with respect to a', θ. We calculate the derivative of a' using the implicit function theorem. For the problem to have a maximum, we require that the Hessian of the maximand is positive $|\mathbf{H}| > 0$ (recall that $\phi_{\theta\theta}$ is assumed negative), where:

$$|\mathbf{H}| = \begin{vmatrix} \phi_{a'a'} & \phi_{a'\theta} \\ \phi_{\theta a'} & \phi_{\theta\theta} \end{vmatrix}$$

Applying the implicit function theorem,

$$a_y' = \frac{\partial a'}{\partial y} = -\frac{\begin{vmatrix} \phi_{a'y} & \phi_{a'\theta} \\ \phi_{\theta y} & \phi_{\theta\theta} \end{vmatrix}}{|\mathbf{H}|} = \frac{\phi_{a'y}\phi_{\theta\theta} - \phi_{\theta y}\phi_{a'\theta}}{|\mathbf{H}|} \quad \text{and} \quad a_V' = \frac{\partial a'}{\partial V} = -\frac{\begin{vmatrix} \phi_{a'V} & \phi_{a'\theta} \\ \phi_{\theta V} & \phi_{\theta\theta} \end{vmatrix}}{|\mathbf{H}|} = \frac{\phi_{a'V}\phi_{\theta\theta} - \phi_{\theta V}\phi_{a'\theta}}{|\mathbf{H}|}$$

A.2 Proof of Proposition 1

Proof. $U_{ay} > \frac{U_y}{U_V} U_{aV}$ provided (where the partial derivatives of U are derived in the Appendix):

$$\begin{aligned}
-u''(a - a') a_y' &> \frac{\beta m u'(c_e')}{\beta u'(c_e') \frac{-\theta}{\beta}} (-u''(a - a') a_V') \\
a_y' &> -\beta q a_V'
\end{aligned}$$

We obtain the expressions for a_y' and a_V' from the first order conditions (above). Then the condition for positive sorting of a on y becomes:

$$\left(\phi_{a'y} + \beta q \phi_{a'V}\right) \phi_{\theta\theta} \quad < \quad \left(\phi_{\theta y} + \beta q \phi_{\theta V}\right) \phi_{a'\theta}$$

Observe that from the first order conditions to the maximization problem, we obtain the cross partials on ϕ. First, note that $\phi_{a'y} = -\beta q \phi_{a'V} = \beta R m u''(c'_e)$ so that the LHS is zero. This follows from the envelope theorem since ϕ is maximized with respect to a' and y. Then we derive the following:

$$\begin{aligned}
\phi_{\theta y} &= \beta m' u'(c'_e) + \beta u''(c'_e)\frac{\theta q' V}{\beta q} \\
\phi_{\theta V} &= \beta m' u'(c'_e)\frac{-1}{\beta q} + \beta u'(c'_e)\frac{\theta q'}{\beta q} + \beta u''(c'_e)\frac{-1}{\beta q}\frac{\theta q' V}{\beta q} \\
&= \frac{-1}{\beta q}\phi_{\theta y} + \beta u'(c'_e)\frac{\theta q'}{\beta q}
\end{aligned}$$

Therefore, the inequality can be written as:

$$0 \quad < \quad \beta u'(c'_e)\theta q' \phi_{a'\theta}$$

The term $\beta u'(c'_e)\theta > 0$ but $q' < 0$, so the condition for positive sorting of a on y is $\phi_{a'\theta} < 0$. Equivalently:

$$\beta R\left(m'[u'(c'_e) - u'(Ra')] + u''(c'_e)\frac{\theta q' V}{\beta q}\right) < 0.$$

From the first order condition $\phi_\theta = 0$ we obtain:

$$\frac{\theta q' V}{\beta q} = -m' \frac{u(c'_e) - u(Ra')}{u'(c'_e)}.$$

Substituting in the condition $\phi_{a'\theta} < 0$:

$$m'[u'(c'_e) - u'(Ra')] - u''(c'_e) m' \frac{u(c'_e) - u(Ra')}{u'(c'_e)} < 0,$$

or, noting that $m' > 0$,

$$u'(c'_e)[u'(c'_e) - u'(Ra')] < u''(c'_e)\left[u(c'_e) - u(Ra')\right].$$

or alternatively

$$\frac{u'(c'_e) - u'(Ra')}{u(c'_e) - u(Ra')} < \frac{u''(c'_e)}{u'(c'_e)}.$$

■

A.3 Proof of Proposition 2

Proof. We calculate the derivatives:

$$\begin{aligned} u'(c) &= \alpha \left(\frac{\alpha c}{1-\gamma} + \beta \right)^{\gamma - 1} \\ u''(c) &= -\alpha^2 \left(\frac{\alpha c}{1-\gamma} + \beta \right)^{\gamma - 2} \end{aligned}$$

and condition (**U**) becomes (where $c = Ra'$):

$$\alpha \left(\frac{\alpha c_e}{1-\gamma} + \beta \right)^{\gamma-1} \left[\alpha \left(\frac{\alpha c_e}{1-\gamma} + \beta \right)^{\gamma-1} - \alpha \left(\frac{\alpha c}{1-\gamma} + \beta \right)^{\gamma-1} \right] <$$
$$-\alpha^2 \left(\frac{\alpha c_e}{1-\gamma} + \beta \right)^{\gamma-2} \left[\frac{1-\gamma}{\gamma} \left(\frac{\alpha c_e}{1-\gamma} + \beta \right)^{\gamma} - \frac{1-\gamma}{\gamma} \left(\frac{\alpha c}{1-\gamma} + \beta \right)^{\gamma} \right]$$

and after dividing by α^2 and by $\left(\frac{\alpha c_e}{1-\gamma} + \beta \right)^{2\gamma-2}$, which under our assumptions are both positive, this implies:

$$1 - \left(\frac{\frac{\alpha c}{1-\gamma} + \beta}{\frac{\alpha c_e}{1-\gamma} + \beta} \right)^{\gamma-1} < -\frac{1-\gamma}{\gamma} \left[1 - \left(\frac{\frac{\alpha c}{1-\gamma} + \beta}{\frac{\alpha c_e}{1-\gamma} + \beta} \right)^{\gamma} \right],$$

or

$$1 - x^{\gamma-1} < -\frac{1-\gamma}{\gamma} \left[1 - x^{\gamma} \right] \quad \text{where} \quad x = \frac{\frac{\alpha c}{1-\gamma} + \beta}{\frac{\alpha c_e}{1-\gamma} + \beta} \in (0,1).$$

First consider $\gamma > 0$. After rearranging and multiplying by $\gamma x^{1-\gamma}$, which is positive for $\gamma > 0$:

$$\begin{aligned} x^{1-\gamma} - (\gamma + (1-\gamma)x) &< 0 \\ G(\gamma) - H(\gamma) &< 0. \end{aligned}$$

At $\gamma = 0$ and $\gamma = 1$ the expression is exactly zero, i.e., G and H cross at 0 and 1. Now, $G'(\gamma) = -x^{1-\gamma} \log x, H'(\gamma) = 1 - x$, and $G''(\gamma) = x^{1-\gamma}(\log x)^2 > 0, H''(\gamma) = 0$. Observe that $G(\gamma)$ is convex, $G''(\gamma) = x^{1-\gamma}(\log x)^2 > 0$, while $H(\gamma)$ is linear. As a result, for $\gamma \in (0,1)$ condition (**U**) holds with strict inequality. For $\gamma = 1$, (**U**) holds with equality and for $\gamma > 1$ it holds with opposite inequality.

Now consider $\gamma < 0$. Since we multiplied by $\gamma < 0$, condition (**U**) now implies that $G(\gamma) - H(\gamma) > 0$. Using the same logic, we establish that condition (**U**) holds for $\gamma < 0$.

This establishes that for a risk averse worker with HARA utility function, condition (**U**) holds strictly if and only if $\gamma < 1$, i.e., there is DARA. The condition holds with opposite inequality when there is IARA and $\gamma > 1$. ■

A.4 Proof of Corollary 1

Proof. All the cases can immediately be verified from Proposition 2, except for the case of CARA. There, $u'(c) = \alpha e^{-\alpha c}, u''(c) = -\alpha^2 e^{-\alpha c}$, so that condition (**U**) becomes:

$$\begin{aligned} \alpha e^{-\alpha c_e} \left(\alpha e^{-\alpha c_e} - \alpha e^{-\alpha c}\right) &\leq -\alpha^2 e^{-\alpha c_e} \left(1 - e^{-\alpha c_e} - 1 + e^{-\alpha c}\right) \\ e^{-\alpha c_e} - e^{-\alpha c} &\leq -\left(-e^{-\alpha c_e} + e^{-\alpha c}\right) \end{aligned}$$

which holds with equality. ∎

A.5 Proof of Proposition 3

Proof. It is immediate that this condition is not satisfied when $u''' = 0$. To see this, observe that then $u'(c_e') - u'(Ra') = wu''(c_e')$ and the condition (**U**) can be written as $u'(c_e')wu''(c_e') < u''(c_e')\left[u(c_e') - u(Ra')\right]$, or $u'(c_e')w > u(c_e') - u(Ra')$. This condition only holds under convexity of u, and therefore is never satisfied for risk averse agents.

When $u''' < 0$, we have instead that $u'(c_e') - u'(Ra') > wu''(c_e')$, so the left hand side is even smaller, and again, condition (**U**) implies $u'(c_e')w > u(c_e') - u(Ra')$, which is not satisfied for risk averse agents.

Now consider $u''' > 0$. Then we can write the utility function and its derivative as

$$\begin{aligned} u(c) &= u(c_e') + u'(c_e')(c - c_e') + \frac{u''(c_e')}{2}(c - c_e')^2 + \cdots \\ u'(c) &= u'(c_e') + u''(c_e')(c - c_e') + \frac{u'''(c_e')}{2}(c - c_e')^2 + \cdots \end{aligned}$$

and therefore condition (**U**) becomes:

$$\begin{aligned} u'(c_e') \left[u''(c_e')(c_e' - c) - \frac{u'''(c_e')}{2}(c_e' - c)^2 + \frac{u''''(c_e')}{6}(c_e' - c)^3 - \cdots\right] &< \\ u''(c_e') \left[u'(c_e')(c_e' - c) - \frac{u''(c_e')}{2}(c_e' - c)^2 + \frac{u'''(c_e')}{6}(c_e' - c)^3 - \cdots\right]&. \end{aligned}$$

Canceling terms and dividing by $(c_e' - c)^2$, this condition implies that at least for small $c_e' - c = w$ implies

$$u'''(c_e') > \frac{u''(c_e')^2}{u'(c_e')}.$$

This is equivalent to requiring that the coefficient of risk aversion $A(c) = -\frac{u''}{u'}$ is decreasing, i.e., $A' = -\frac{u'''u' - (u'')^2}{u'^2}$ or $u''' > \frac{(u'')^2}{u'} = -u''A(c)$. ∎

A.6 A Counter Example

Example 1 *Let w be large enough and find a u-function with u'''' suitably chosen such that the condition is not satisfied. Let $u(c)$ be defined as:*

$$u(c) = u(c_e') + u'(c_e')(c - c_e') + \frac{u''(c_e')}{2}(c - c_e')^2 + \frac{u'''(c_e')}{6}(c - c_e')^3 + \frac{u''''(c_e')}{24}(c - c_e')^4.$$

Evaluating u at $c = Ra'$ and observing that $c_e' - Ra' = w$ we can then write

$$\begin{aligned} u(c_e') - u(Ra') &= u'(c_e')w - \frac{1}{2}u''(c_e')w^2 + \frac{1}{6}u'''(c_e')w^3 - \frac{1}{24}u''''(c_e')w^4 \\ u'(c_e') - u'(Ra') &= u''(c_e')w - \frac{1}{2}u'''(c_e')w^2 + \frac{1}{6}u''''(c_e')w^3. \end{aligned}$$

*Now we can write condition (**U**) as (where u denotes $u(c_e')$):*

$$\begin{aligned} u'\left[u''w - \frac{1}{2}u'''w^2 + \frac{1}{6}u''''w^3\right] &< u''\left[u'w - \frac{1}{2}u''w^2 + \frac{1}{6}u'''w^3 - \frac{1}{24}u''''w^4\right] \\ u'u''' &> u''^2 - \frac{1}{3}u''u'''w + \frac{1}{3}u''''w\left[u' + \frac{1}{4}u''w\right] \end{aligned}$$

Observe that $u'u''' > u''^2$ is the standard condition for Decreasing Absolute Risk Aversion. But for any $u''' > 0$, however large, we can find a utility function with $\frac{1}{3}u''''w\left[u' + \frac{1}{4}u''w\right]$ sufficiently large such that the inequality is not satisfied. For example, if $u' + \frac{1}{4}u''w > 0$ we can choose u'''' positive and large. Conversely, if $u' + \frac{1}{4}u''w < 0$ we can choose u'''' sufficiently negative such that the inequality does not hold.

A.7 Proof of Proposition 4

Proof. The solution to the (interior) maximization problem is $a'(a, y, V), \theta(a, y, V)$ and satisfies:

$$\begin{aligned} -u'(c_u) + \beta[mE_{a'}(a', y, w') + (1 - m)U_{a'}(a')] &= 0 \\ m'[E(a', w') - U(a')] + mE_{w'}(a', y, w')\frac{\partial w'}{\partial \theta} &= 0 \end{aligned}$$

Now we have monotone matching of a in y provided: $U_{ay} > \frac{U_y}{U_V} U_{aV}$.

$$\begin{aligned}
U_y &= m\beta E_{w'}(a', y, w').\frac{\partial w'}{\partial y} + U_{a'} a'_y + U_\theta \theta_y = m\beta E_{w'}(a', y, w') \\
U_a &= u'(c) + U_{a'} a'_a + U_\theta \theta_a = u'(c_u) \\
U_V &= m\beta E_{w'}(a', y, w').\frac{\partial w'}{\partial V} + U_{a'} a'_V + U_\theta \theta_V = -m\beta E_w(a', y, w') \left(-\beta\lambda + \frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{\beta q} \right) \\
U_{ay} &= -u''(a - a') a'_y \\
U_{aV} &= -u''(a - a') a'_V
\end{aligned}$$

where $U_{a'} = 0$ and $U_\theta = 0$ from the envelope theorem. Then:

$$\begin{aligned}
U_{ay} &> \frac{U_y}{U_V} U_{aV} \\
-u''(a - a') a'_y &> \frac{m\beta E_w(a', y, w')}{-m\beta E_w(a', y, w') \left(-\beta\lambda + \frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{\beta q} \right)} (-u''(a - a') a'_V) \\
a'_y &> -\frac{1}{-\beta\lambda + \frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{\beta q}} a'_V
\end{aligned}$$

Writing the Hessian $|\mathbf{H}| > 0$ as:

$$|\mathbf{H}| = \begin{vmatrix} \phi_{a'a'} & \phi_{a'\theta} \\ \phi_{\theta a'} & \phi_{\theta\theta} \end{vmatrix}$$

then

$$a'_y = \frac{\partial a'}{\partial y} = -\frac{\begin{vmatrix} \phi_{a'y} & \phi_{a'\theta} \\ \phi_{\theta y} & \phi_{\theta\theta} \end{vmatrix}}{|\mathbf{H}|} \quad \text{and} \quad a'_V = \frac{\partial a'}{\partial V} = -\frac{\begin{vmatrix} \phi_{a'V} & \phi_{a'\theta} \\ \phi_{\theta V} & \phi_{\theta\theta} \end{vmatrix}}{|\mathbf{H}|}$$

$$\begin{aligned}
a'_y &> -\frac{1}{-\beta\lambda + \frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{\beta q}} a'_V \\
\phi_{a'y}\phi_{\theta\theta} - \phi_{\theta y}\phi_{a'\theta} &< -\frac{1}{-\beta\lambda + \frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{\beta q}} (\phi_{a'V}\phi_{\theta\theta} - \phi_{\theta V}\phi_{a'\theta}) \\
\left(\phi_{a'y} + \frac{1}{-\beta\lambda + \frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{\beta q}} \phi_{a'V} \right) \phi_{\theta\theta} &< \left(\phi_{\theta y} + \frac{1}{-\beta\lambda + \frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{\beta q}} \phi_{\theta V} \right) \phi_{a'\theta} \quad \text{(A.1)}
\end{aligned}$$

Observe that from the first order conditions to the (interior) maximization problem, we obtain the cross partials on ϕ. First, note that:

$$\phi_{a'y} = -\frac{1}{-\beta\lambda + \frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{\beta q}}\phi_{a'V}$$

so that the LHS is zero. Then deriving the expression for $\phi_{\theta y}$ and $\phi_{\theta V}$: Note that: $m'[E(a',w') - U(a')] + qE_{w'}\frac{\delta w'}{\delta\theta}] = 0$ and this implies:

$$\begin{aligned}
\phi_{\theta y} &= \beta m' E_{w'}(a',y,w') + \beta E_{w'w'}(a',y,w')\frac{\partial w'}{\partial y}\frac{(1-\beta)(1-\beta(1-\lambda))\theta q'}{\beta q}V \\
\phi_{\theta V} &= \beta\frac{\partial w'}{\partial V}\left(m' E_{w'} + E_{w'w'}\frac{(1-\beta)(1-\beta(1-\lambda))\theta q'}{\beta q}V\right) + \beta E_{w'}\frac{(1-\beta)(1-\beta(1-\lambda))\theta q'}{\beta q} \\
&= (\beta\lambda - \frac{[1-\beta(1-q)][1-\beta(1-\lambda)]}{\beta q})\phi_{\theta y} + \beta E_{w'}\frac{(1-\beta)(1-\beta(1-\lambda))\theta q'}{\beta q}
\end{aligned}$$

the RHS reduces to:

$$\frac{(1-\beta)(1-\beta(1-\lambda))\theta q'}{q}E_{w'}(a',y,w')\phi_{a'\theta}$$

Therefore, the inequality (A.1) is satisfied provided $\phi_{a'\theta} < 0$, since $q' < 0$:

$$\begin{aligned}
\phi_{a'\theta} &= \beta m'[E_{a'}(a',y,w') - U_{a'}(a')] + \beta m E_{a',w}(a',y,w')\frac{\partial w'}{\partial\theta} \\
&= \beta m'[E_{a'}(a',y,w') - U_{a'}(a')] + \beta m E_{a',w'}(a',y,w')\frac{-1}{mE_{w'}(a',y,w')}m'[E(a',y,w') - U(a')]
\end{aligned}$$

from the FOC for θ

$$\frac{\partial w'}{\partial\theta} = \frac{-m'}{mE_{w'}(a',y,w')}[E(a',y,w') - U(a')]$$

Therefore $\phi_{a'\theta} < 0$ provided

$$\frac{E_{a'}(a',y,w') - U_{a'}(a')}{E(a',y,w') - U(a')} < \frac{E_{a',w'}(a',y,w')}{E_{w'}(a',y,w')}$$

■

Appendix B Data

We use data from the Panel Study of Income Dynamics (PSID). The PSID is a longitudinal study collecting biennial data on households in the US. The PSID started in 1968 and follows individuals from the 1968 sample as well as additional households added over time. Relative to other widely used datasets such as the SCF or CPS the PSID has a long panel element with rich data on household characteristics, wealth, employment and demographics. Using the pooled PSID panel we restrict the attentions to the post recession 2009 through 2013 waves. In our sample we include all households where the household head is employed and we have non-missing information for their industry, race, marital status, hours worked, earnings and completed education. These restrictions leave 8,560 observations in the core analysis sample. We look at the following sectors

- (1) Agriculture, (2) Construction, (3)Manufacturing, (4) Wholesale Trade, Retail trade, (5) Finance and Insurance, Real estate, rental and housing, (6) Professional, Scientific and technical services, (7) Management, Administrative and Support, and Waste, (9) Educational Services, (10) Health Care and social assistance, (11) Accommodations and Food Services, (12) Public Administration and (13) Active Duty Military

When measuring sector productivity we use the Agriculture sector as the base sector. At this point in the data we also assign NAICS codes to some households. The BLS time series trend has data on a subset of industries defined by NAICS code (approximately 25% of the sample). We use a crosswalk to determine the correct Census code (Table B.1).

Industry	ν	Implied multiplier	Industry Share	CDF
1	0	1	0.025	0.025
2	0.341	1.406	0.075	0.101
3	0.581	1.788	0.186	0.287
4	0.556	1.744	0.041	0.329
5	0.059	1.061	0.106	0.436
6	0.778	2.338	0.049	0.485
7	0.361	1.436	0.017	0.503
8	0.708	2.031	0.054	0.557
9	0.224	1.251	0.044	0.601
10	0.421	1.524	0.087	0.689
11	0.495	1.641	0.151	0.840
12	-0.010	0.989	0.062	0.902
13	0.702	2.018	0.097	1

Table B.1: Industry Classficication

References

Acemoglu, D. and Shimer, R. (1999). Efficient unemployment insurance. *Journal of Political Economy*, 107(5):893–928.

Aiyagari, S. R. (1994). Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics*, Vol. 109, No. 3.:659–684.

Andolfatto, D. (1996). Business cycles and labor-market search. *The American Economic Review*, 86:112–132.

Bewley, T. F. (1980). Interest bearing money and the equilibrium stock of capital. mimeo.

Browning, M. and Crossley, T. F. (2001). Unemployment insurance benefit levels and consumption changes. *Journal of public Economics*, 80(1):1–23.

Card, D., Chetty, R., and Weber, A. (2007). Cash-on-hand and competing models of intertemporal behavior: New evidence from the labor market. *The Quarterly Journal of Economics*, 122(4):1511–1560.

Chade, H., Eeckhout, J., and Smith, L. (2017). Sorting through search and matching models in economics. *Journal of Economic Literature*, 55(2):493–544.

Chaumont, G. and Shi, S. (2017). Wealth accumulation, on the job search and inequality. Penn State.

Chetty, R. (2008). Moral hazard versus liquidity and optimal unemployment insurance. *Journal of Political Economy*, 116(2):173–234.

Danforth, J. (1979). On the role of consumption and decreasing absolute risk aversion in the theory of job search. In S., L. and McCall, J., editors, *Studies in the Economics of Search*, pages 109–131. North-Holland, NewYork.

den Haan, W. J., Ramey, G., and Watson, J. (2000). Job destruction and propagation of shocks. *American Economic Review*, 90:482–498.

Eeckhout, J. and Kircher, P. (2010). Sorting and decentralized price competition. *Econometrica*, 78:539–574.

Golosov, M., Maziero, P., and Menzio, G. (2012). Taxation and redistribution of residual income inequality. University of Pennsylvania mimeo.

Herkenhoff, K., Phillips, G., and Cohen-Cole, E. (2015). How credit cosntraints impact job finding rates, sorting, & aggregate output.

Herkenhoff, K. F. (2013). The impact of consumer credit access on unemployment. University of Califomia Los Angetes mimeo.

Hopenhayn, H. A. and Nicolini, J. P. (1997). Optimal unemployment insurance. *Journal of Political Economy*, 105(2):412–438.

Huggett, M. (1993). The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of Economic Dynamics and Control*, 17:953–969.

Jerez, B. (2012). Competitive equilibrium with search frictions: a general equilibrium approach. working paper.

Kaneko, M. (1982). The central assignment game and the assignment markets. *Journal of Mathematical Economics*, 10:205–232.

Koehne, S. and Kuhn, M. (2015). Should unemployment insurance be asset tested? *Review of Economic Dynamics*, 18(3):575–592.

Krusell, P., Luo, J., and Ríos-Rull, J.-V. (2018). Wage dispersion in the aiyagari model. UPenn.

Krusell, P., Mukoyama, T., and Sahin, A. (2010). Labour-market matching with precautionary savings and aggregate fluctuations. *Review of Economic Studies*, 77.

Legros, P. and Newman, A. (2007). Beauty is a beast, frog is a prince: Assortative matching with nontransferabilities. *Econometrica*, 75:1073–1102.

Lentz, R. (2009). Optimal unemployment insurance in an estimated job search model with savings. *Review of Economic Dynamics*, 12(1):37–57.

McAfee, R. P. (1993). Mechanism design by competing sellers. *Econometrica*, 61(6):1281–1312.

Menzio, G. and Shi, S. (2011). Efficient search on the job and the business cycle. *Journal of Political Economy*, 119:468–510.

Merz, M. (1995). Search in the labor market and the real business cycle. *Journal of Monetary Economics*, 36.

Michelacci, C. and Ruffo, H. M. (2014). Optimal life cycle unemployment insurance. *American Economic Review*. forthcoming.

Peters, M. (1997). A competitive distribution of auctions. *Review of Economic Studies*, 64(1):97–123.

Peters, M. (2000). Limits of exact equilibria for capacity constrained sellers with costly search. *Journal of Economic Theory*, 95(2):139–168.

Rendon, S. (2006). Job search and asset accumulation under borrowing constraints. *International Economic Review*, 47(1):233–263.

Shimer, R. (2005). The assignment of workers to jobs in an economy with coordination frictions. *Journal of Political Economy*, 113:996–1025.

Shimer, R. and Werning, I. (2007). Reservation wages and unemployment insurance. *The Quarterly Journal of Economics*, 122(3):1145–1185.

Shimer, R. and Werning, I. (2008). Liquidity and insurance for the unemployed. *American Economic Review*, 98(5):1922–1942.