

Competing Norms of Cooperation*

JAN EECKHOUT[†]

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Abstract

We model a firm in the context of a repeated partnership game with random matching. This captures the notion that a firm's organizational design (or its norm) is shaped by market forces. The main result is that in equilibrium, firms' norms differ: in the level of effort of its members, in turnover, and in the compensation schedule. Norms can be Pareto ranked, and the bad norms necessarily persist because they make the good norms incentive compatible. The welfare implications are surprising: economies where firms have flat compensation schedules are more efficient than when they are steep.

Keywords. Non-market Interaction. Inequality. Welfare. Optimal Organizational Design. Repeated Matching Games.

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[†]Department of Economics, University of Pennsylvania, 3718 Locust Walk, Philadelphia, PA 19104; eeckhout@ssc.upenn.edu; <http://www.ssc.upenn.edu/~eeckhout/>

1 INTRODUCTION

Why are organizations designed the way they are, and why do some firms in similar circumstances choose a different design than others? What are the welfare implications of different designs? There is substantial evidence of heterogeneity in organizational design.¹ In addition to heterogeneity, the same evidence shows a significant relation between productivity and a measure for the quality of the organizational design. This implies that some designs can be (Pareto) ranked as “better” than others. The purpose of this paper is to provide a theoretical foundation for the persistence and continuity of *differences* in designs. The design here is interpreted as the prevailing norm within the firm. Then we ask what the welfare implications are for economies in which firms with different norms coexist.

The central premise in this paper is that outside employment opportunities shape the organizational design of a firm. Employee relationships are increasingly governed by the appearance of market forces.² The traditional firm-employee relation of life-long commitment, with a fair day’s work that was uniquely rewarded through internal promotions, has long gone. But a fine balance remains between a long-term relationship based on trust, flexibility, performance, and a spot market. Relationships are long-run agreements that are not enforceable, but market forces and outside options determine the terms of the agreement: employees get rewarded for high degrees of cooperation, and other job opportunities fix the negotiated agreement.

To capture the interplay between market forces and non-market interactions within the firm, we model this economy as a repeated partnership problem with an endogenous outside option. The outside option consists in the ability of any partner to leave the current partnership and join a new partnership, selected at random and anonymously. In our model, all partners are ex ante identical. Even though myopically, the best response is not to cooperate in a partnership, cooperation can be sustained in equilibrium, when interaction is repeated. However, because there is an outside option in leaving the firm to form another partnership, punishment is determined endogenously. This will establish the main result: differences in firm norms (i.e. the existence of firms with bad norms) will provide sufficient incentives to sustain cooperation in good norms. This leads to the observation that ex ante identical firms adopt different organizational designs.³ This is surprising because intuitively, one would expect that competitive pressures

¹See for example Kotter and Heskett (1992) and Cappelli and Neumark (1999). They quantify measures of commitment, rules, compensation packages, cooperation,... which are commonly labeled as corporate culture or norms.

²Our approach is complementary to Hermalin (1994). He considers the effect of product (rather than labor) market competition on the organizational form of the firm.

³A familiar example is the difference in design between UPS and FedEx. Both firms are comparable in terms of the services provided, the output generated,... Moreover, both firms employ full time employees that are very similar in terms of ability. Still, there is a substantial difference in wages (30 to 50% higher for a driver in UPS), turnover (the majority of

would decrease differences between firms, rather than generate differences. Why should bad firms persist in the long run – and they do in reality –? The rationale proposed here is based on the incentives that are needed to sustain the good firms.

The implication of this theory is that a firm’s norm persists, even though its members are changing over time. Because members enter the partnerships at different times, the new entrant adopts the prevailing norm.⁴ But even if all partners have turned over, the norm has not changed. Recent evidence by Jovanovic and Rousseau (2000) establishes that organizational capital is persistent over time. Looking at historical data, they find that firms maintain productivity differences over very long stretches of time.

Our theory draws heavily on existing work on repeated games and random matching (Datta (1993), Greif (1993), Kranton (1996), Ghosh and Ray (1996)). This work shows that in the context of random matching, cooperation can be sustained through symmetric strategies that exhibit increasing degrees of cooperation. This makes cooperation incentive compatible, because after a deviation, in a new partnership there is a period of costly low cooperation before full cooperation is established. In equilibrium, partnerships exhibit increasing compensation schedules: because effort (or cooperation) initially is low, compensation is low, and eventually, increasing cooperation leads to an increase in compensation.

However, our paper differs in essence from this literature by solving for equilibrium strategies that are more efficient. First, we show that asymmetric strategies (in some partnerships there is, and in others there is no cooperation) can obtain cooperation in equilibrium (as opposed to symmetric strategies with increasing levels of cooperation). Second, and more importantly, we establish that asymmetric strategies are more efficient than symmetric strategies (i.e. with increasing compensation schedules). This is surprising because increasing compensation schedules have long been recognized as effective incentive devices. While this is true in the context of an exogenous outside option, this is no longer the case when the outside option is endogenous as in our model.

The type of employment contracts that are observed in equilibrium is self-enforcing. They are of the termination type, i.e. deviation is punished by termination of the employment relation (the partner quits or is laid off).⁵ What is novel with respect to the existing literature on self-enforcing contracts is the endogeneity of the outside option.⁶ Observe also that the outside option here does not imply UPS employees are employed for life, and absenteeism, which indicates the degree of cooperation within the firm. There is no doubt that the norm (or corporate culture) differs substantially between these two firms.

⁴This notion is reminiscent of the one proposed by Cremer (1986), where cooperation is derived in repeated games with overlapping generations of players.

⁵MacLeod and Malcomson (1989) show that these contracts are contained in the set of self-enforcing contracts.

⁶A notable exception in this literature is the paper by Felli and Roberts (1999) who introduce the effect of competition

unemployment. All workers are always employed. This is where the current model (and its conclusions) starkly differs from the standard efficiency wage model (see for example Macleod and Malcomson (1989) and Shapiro and Stiglitz (1984)). Though unemployment certainly has an incentive effect, it cannot be ignored that a substantial (and increasing) share of job movements is job-to-job rather than job-to-unemployment-to-job. It is precisely the option of moving between jobs that determines the equilibrium compensation schedules. Consider an economy where firms offer a flat compensations schedule, i.e. high compensation for a new partner. Then a firm that offers a steep compensation schedule will induce free riding by the new partner. Though free riding implies the partner will have to leave the firm, she will be better off since her new firm will offer a high compensation upon arrival. Free riding and quitting will yield a higher payoff than climbing the ladder within the firm and waiting for higher future compensation.

The remainder of the paper is organized as follows. In the following section, the competing norms model is presented. Given exogenous sharing rules, in section 3 the model is solved and it is shown how firms differ in equilibrium. This is illustrated with an example and further discussed with some comparative statics results. In section 4, compensation schedules are allowed to increase and the efficiency result is derived. The robustness of the model to the introduction of capital, renegotiation and deviations by coalition is discussed in section 5. In section 6, the implications for the model from extensions to include heterogeneous agents, complementary inputs in production and unemployment are considered. Finally, some concluding remarks are made.

2 THE COMPETING NORMS MODEL

In this section, the basic model is presented. We describe the incentives employees face when joining a firm with a certain social norm, and define equilibrium.

Workers, Firms and the Stage Game. The economy is populated with an infinite number of *identical* agents. The set of agents \mathcal{A} has measure 1 and each agent is interpreted as an infinitesimally small subset of \mathcal{A} . Production occurs in organizations of a fixed and finite number of $m > 2$ agents. Index agents within an organization by $i = 1, \dots, m$. The set of all organizations is given by \mathcal{N} and has measure $\frac{1}{m}$. A generic organization is referred to as $n \in \mathcal{N}$. For the purpose of the characterization below, consider the partition $\{\mathcal{C}, \mathcal{D}\}$ of \mathcal{N} , where $c \in \mathcal{C}$ is an organization with a norm of cooperation and $d \in \mathcal{D}$ is one with a norm of non-cooperation.

on the (static) hold up problem. They show conditions under which Bertrand competition between *heterogeneous* agents can solve hold up. This paper differs from theirs in two aspects: the repeated game and the fact that all agents here are identical.

We want to capture the notion of joint production. The stage game is therefore as Holmström's (1982) moral hazard in teams model. Total output y produced in an organization is a function of all individuals' effort. Let e_i be agent i 's level of effort and let $\mathbf{e} = (e_1, \dots, e_m)$ be the vector of all effort levels in n . The organization's total output produced $y = Q(\mathbf{e})$ is deterministic and symmetric in e_i . Agents receive a share $s_i(Q), \forall i$ of total output. The utility cost of effort to each individual is $C(e_i)$, with C convex. The utility of agent i is given by

$$u_i = s_i(Q(\mathbf{e})) - C(e_i) \quad (1)$$

Given the sharing rule, agents choose their level of effort e_i , they produce, and in function of the vector \mathbf{e} , output Q is realized. Effort is not contractible, which gives rise to the moral hazard problem. Ex ante sharing rules are binding because they are contracted. Ex post output is perfectly observed.

In a competitive environment, firms' profits are zero. Given a technology without physical capital, it follows that the total wage bill is equal to total production. We have chosen this simple production function to economize on notation. In section 5, the model is shown to be robust to the introduction of a production function with physical capital in addition to effort. Throughout the paper, the following assumption is maintained: the sharing rule $\{s_i(Q)\}$ satisfies Balanced Budget: $\sum_{i=1}^m s_i(Q) = Q$.

Holmström (1982) shows that the solution to the static game with budget balancing sharing rules is inefficient. Given the vector of effort choices by all other agents $e_{-i}, \forall -i (\neq i) \in n$, the best response correspondence of agent i satisfies $\arg \max_{e_i} \{s_i(Q(e_i, e_{-i})) - C(e_i)\}$. The Nash equilibrium effort e_i^* , with corresponding utility u^* satisfying (1), solves for the fixed point $e_i^* = \arg \max_{e_i} \{s_i(Q(e_i, e_{-i}^*)) - C(e_i)\}, \forall i$. Pareto optimal effort e_i^o yields utility u^o , and satisfies $e_i^o = \arg \max_{e_i} \{Q(e_i, e_{-i}^o) - C(e_i)\}$.

Theorem 1 (Holmström) *There do not exist sharing rules $\{s_i(Q)\}$ which satisfy $\sum_i s_i(Q) = Q$, and which yield e_i^o as a Nash equilibrium in the non cooperative game with payoffs u_i^o .*

Would all agents cooperate and provide optimal effort levels e^o , then an individual best response is to deviate and provide effort $e^d \neq e^o$ such that $e^d = \arg \max_{e_i} \{s_i(Q(e_i, e_{-i}^o)) - C(e_i)\}$, which yields u^d .⁷ As a corollary to the theorem it follows that for a given sharing rule, equilibrium effort $e^* < e^o$ is suboptimal and that $u^d > u^o > u^*$. The theorem holds for a general production function and for general sharing rules.

The inefficiency result crucially hinges on the assumption of budget balancing sharing rules. A large part of the literature has given attention to studying incentives in environments where this assumption can

⁷Formally, $u_i^d = s_i(Q(e_i^d, e_{-i}^o)) - C(e_i^d)$.

be relaxed, for example involving an independent principal (see Holmström (1982)). Perhaps of equal importance is the interaction between joint production and mobility across firms. Our analysis is an attempt to complement the incentives approach.⁸ The objective here is to find solutions for the moral hazard problem even in environments where the budget is balanced. This is the case for example where it is not possible to involve a completely independent principal (for example a residual claimant principal who cannot be stopped from colluding with one of the agents). Any dependent principal needs to be considered as one of the employees, which brings us back to the inefficiency. In the case of partners in a law firm for example, partners are both the owners and employees.

Matching and Monitoring. Consider now the repeated game, where utility that is delayed for one unit of time is discounted at the common rate $1 + r$. Time is discrete, and organizations of m agents are formed through random and anonymous matching. This implies that newly matched agents cannot observe the past history of actions of their new partners. Within any partnership, all workers choose their input of production e_i , and after production, output $Q(\mathbf{e})$ is realized and shared according to the sharing rule $\{s_i(Q)\}$, contracted upon ex ante. Output Q is perfectly observable and verifiable. This also implies that players can condition their strategies on the realization of Q . Note that given $m > 2$, even within partnerships there is incomplete information in the case of deviations. An agent knows that a partner has deviated, but she does not know the identity of the deviator.

At the end of play, an agent consumes and decides whether to continue the current partnership or to terminate it. Irrespective of whether an agent continues or terminates the partnership, there is some exogenous attrition: with probability $m\alpha$, an agent in the firm will be separated from the match. Because out of all partners each agent has equal probability of being separated, an individual's exogenous rate of attrition is α . Note that while attrition is crucial for anonymity (deviators cannot be distinguished from agents who have been separated because of attrition), α can be infinitely small. Some partnerships, entirely or a fraction of it, may remain matched. All unmatched players go in the pool of unmatched and get randomly assigned to a new partnership⁹, so that again a mass of $\frac{1}{m}$ firms of size m are formed.

When matched to a partnership and before effort is chosen, a new entrant can observe that partnership's last period output level. This assumption will allow the norm to play the role of a public randomization device. Upon arrival, an agent can infer from the past output whether she is matched to a good or a bad norm firm.

⁸A similarly complementary approach has been taken by Meyer (1994) in studying learning in task assignment of team members.

⁹There is no friction and no agents is ever unmatched. Remaining unmatched with zero utility is an option, but never individually rational.

Social Norms and Equilibrium. Loosely speaking, a social norm is a totality of common characteristics, behavior patterns, beliefs,... that applies to each organization individually. More precisely, the social norm consists of the strategy or the behavior rule that agents follow and which will in general differ across organizations. It is a full contingent plan of action: for a given history, in each period workers choose effort and, after realization of Q , they decide whether or not to terminate the partnership. Of course, we will not be looking for just *any* set of strategies that constitute a firm's norm, but those that are an equilibrium, both within the firm and in an economy as a whole.¹⁰ We return to equilibrium in more detail below.

The interest here is in equilibria where a norm of cooperation within some firms can be maintained, despite the non-cooperative outcome in the static game. A norm is an implicit dynamic agreement between the agents in an organization. Because agents have the option to terminate the partnership, matching is endogenous and the standard folk theorem for infinitely repeated games between a given set of agents does not apply. In deriving equilibrium, we will be looking for those strategies that can support social norms of cooperation in the presence of endogenous matching.

Two remarks. First, in concentrating on equilibria that are supported by strategies specific to each firm's norm, the focus is on pure strategy equilibria. Nothing prevents agents from playing a mixed strategy, and such equilibria may exist. Second, the main objective of this paper is to derive those competing norms that exhibit the highest degree of cooperation. As is the case with the standard folk theorem, the set of individually rational payoffs that can be sustained in equilibrium will typically not be the singleton.

Whenever an agent is matched to a new organization, she forms a belief about the norm in that firm. Given the norm, i.e. the belief about the strategy of all other $m - 1$ agents, an optimal strategy must be a best response. An equilibrium is then described by a rule, such that given the best response of all other agents in the economy, each player chooses effort to maximize expected discounted utility. Suppose that all other agents cooperate, cooperation is a best response only if the payoff is higher from cooperating, and remaining matched to the firm with a norm of cooperation. A norm of cooperation is not merely the choice of effort, but also the decision whether or not to terminate the partnership. An agent's best response will depend on her belief whether her colleagues will cooperate and decide not to separate. The outside option (i.e. the distribution of norms) will ultimately tie down the economy's equilibrium. This is precisely the role of different types of norms. A sequential equilibrium is then determined by all individuals' best replies within a firm's norm, given the distribution of norms. The focus of attention will be on stationary equilibria.

¹⁰As a result, norms are self-enforcing. This is by now standard in the economics literature on norms: see for example Cole, Mailath and Postlewaite (1998), and Rob and Zemsky (1999).

Two more remarks are worth noting. First, all matches must be individually rational. For symmetric exogenous sharing rules, this is always satisfied as matches are formed instantaneously and being matched has a higher value than being unmatched. The issue does have immediate relevance in the case of asymmetric sharing rules. This issue will be taken up in section 4. Second, the assumption of having more than two agents in a firm ($m > 2$) is not without consequences. We want to capture the notion of ongoing organizations, which would be impossible for a two worker firm as it dissolves if either of the agents terminates partnership.

3 HETEROGENEITY OF NORMS

In this section, the model is solved for exogenously given symmetric sharing rules (see for example Farrell and Scotchmer (1988)). This assumption implies $s_i(Q) = s_j(Q) = s, \forall i, j \in n$. The problem individuals face at the beginning of each period is to choose effort that maximizes the continuation payoff. We will now construct an equilibrium that can sustain cooperation. Consider therefore the following strategy in any period t after observing Q_{t-1} :

1. at the beginning of period t
 - if $Q_{t-1}(\mathbf{e}) = Q(\mathbf{e}^o)$, then choose $e = e^o$
 - otherwise, choose $e = e^*$
2. at the end of period t
 - if $Q_t = Q(\mathbf{e}^o)$, then continue the match
 - otherwise, terminate the match

Suppose some firms of type $c \in \mathcal{C} \subset \mathcal{N}$ exist and are characterized by the fact that all workers always cooperate, and never choose to terminate the partnership unless other workers deviate. If all agents in the economy use the strategy above, then the continuation payoff v^o of cooperation (i.e. choosing $e = e^o$) in a firm of type c is given by¹¹

$$rv^o = u^o + \alpha [V - v^o] \tag{2}$$

¹¹At the beginning of period t , the expected continuation payoff v_t^o satisfies

$$v_t^o = \frac{1}{1+r} [u^o + (1-\alpha)v_{t+1}^o + \alpha V_{t+1}]$$

which, under stationarity, implies equation (2).

An agent gets u^o at the end of each period, and given the strategy above she only gets separated from the firm due to exogenous attrition at rate α . In the case of termination, the expected continuation payoff of entering a new match is denoted by V . Below, we will derive V explicitly. Whether or not an agent will be willing to follow the strategy described above depends on the continuation payoff of deviating from it. Any agent who deviates from this strategy by playing $e = e^{d12}$ gets a continuation payoff v^d satisfying

$$rv^d = u^d + [V - v^d] \quad (3)$$

It yields a higher utility $u^d > u^o$, but given the strategy by all other players, it implies that at the end of the period, the match will get terminated: the output observed will be below the optimal level $Q < Q(\mathbf{e}^o)$, in which case all other players' strategy prescribes termination.

When all players follow the strategy above, this can equally well give rise to a firm of type $d \in \mathcal{D} \subset \mathcal{N}$. When newly matched to a firm of which $Q_{t-1}(\mathbf{e}) \neq Q(\mathbf{e}^o)$ (for example because all agents are newly matched and there was no past output). The strategy then prescribes to choose $e = e^*$ and to terminate the partnership at the end of the period. The continuation payoff is then determined by the utility from playing Nash and the expected continuation payoff of a future match:

$$rv^* = u^* + [V - v^*] \quad (4)$$

The crucial variable here is the expected continuation payoff of a future match V . It is determined by the belief any agent has about the whole population of agents' behavior. A first preliminary result is that a strategy where none of the agents cooperates is an equilibrium.

Proposition 2 (No Cooperation) *Non Cooperative behavior, $e = e^*$ in all firms in \mathcal{N} is an equilibrium*

Proof. In Appendix. ■

We now show that equilibria do exist with cooperation, given that all players follow the strategy above. Suppose at time t , a newly matched agent observes $Q_{t-1}(\mathbf{e}) = Q(\mathbf{e}^o)$. She will follow the strategy, provided the continuation payoff satisfies the “no deviation” constraint (ND)

$$v^o \geq v^d \quad (5)$$

This is a necessary condition for a worker to be induced to cooperate in a firm c , rather than free ride on the other members and rematch in the next period. From equations (2) and (3), this condition can be written as

$$u^o \geq \frac{\alpha + r}{1 + r} u^d + \frac{r(1 - \alpha)}{1 + r} V \quad (6)$$

¹²Below we show that no player ever wants to deviate by terminating a match after choosing e^o .

the current payoff from cooperating must be large enough to make cooperating incentive compatible. This condition is therefore a function of u^d , the utility of deviating, and of V , the expected continuation payoff of rematching. V is determined by the distribution of norms in the economy, and it is easy to see that, in order to induce the agent to cooperate rather than free ride, the payoff from cooperating must be larger the larger the expected outside option V .

The outside option will pin down the equilibrium distribution of firm norms in the economy. Let $F(n)$ be the cumulative density function of all norms in the economy, where $\sum_n F(n) = 1$. We are constructing equilibria where the norm is either one of two types: the norm $c \in \mathcal{C}$ with the optimal level of effort and no endogenous separation; or the norm $d \in \mathcal{D}$, with the static Nash equilibrium level of effort followed by immediate termination. Denote f the upper bound on $F(c)$ in equilibrium. In each period of time, the total mass of agents in the pool of newly matched is proportional to $1 - F(c) + \alpha F(c)$: all the bad norm agents rematch each period and only the exogenously separated good norm agents do so. As a result, the fraction of newly matched workers that will be matched to a firm with a norm of cooperation is

$$p = \frac{\alpha F(c)}{1 - F(c) + \alpha F(c)} \quad (7)$$

This now determines the expected continuation payoff: $V = pv^o + (1 - p)v^*$. It is the weighted sum of the continuation payoffs of each type of firm. We can now state the main result.

Theorem 3 *There exists a pair $(\bar{r}, \bar{\alpha})$ such that for any $r \in (0, \bar{r}]$ and for any $\alpha \in (0, \bar{\alpha}]$, an equilibrium exists where a fraction f of firms $c \in \mathcal{C} \subset \mathcal{N}$ have a norm for cooperation, with*

$$f = 1 - \frac{(u^d - u^o)(r + 1)}{u^o(r + 1) - ru^d - u^*} \frac{\alpha}{1 - \alpha} < 1 \quad (8)$$

Proof. In Appendix. ■

The fraction of firms with a norm of cooperation f as derived in the theorem is the upper bound. It now follows immediately that an economy where all firms have a norm for cooperation (i.e. $f = 1$) cannot be an equilibrium. The outside option after termination is no worse, which makes cooperation not credible. This is confirmed by mere observation of equation (29). When $f = 1$, then $p = \gamma = 1$. Since u^d is strictly larger than u^o , the ND constraint is always violated. The way the upper bound (8) is determined is precisely by solving for highest possible f such that the ND is binding. Note that though agents are identical, and even with mobility, wages (and for that matter continuation payoffs) differ between firms. There is a gap between the utility derived from being in the high norm firm compared to the utility in the low firm.¹³ This gap is necessary to stop agents from deviating.

¹³This result relates to Eeckhout and Jovanovic (1998), who show in a production economy that inequality *necessarily* arises in a dynamic framework, when an economy-wide production externality involves higher moments of the distribution

An Example and Some Comparative Statics Results

We illustrate the result with a simple example. Let $m = 3$, $Q = \sum_i e_i$ and $C(e) = \frac{e^2}{2}$. Output is shared equally $s(Q) = \frac{1}{3}Q$. We can calculate the Nash equilibrium effort and utility $e^* = \frac{1}{3}$, $u^* = \frac{5}{18}$ and the optimal effort and utility $e^o = 1$, $u^o = \frac{1}{2}$. Deviating when both other partners supply optimal effort implies $e^d = \frac{1}{3}$, $u^d = \frac{13}{18}$. Suppose that the rate of discounting is $r = 0.1$ and that the exogenous termination rate $\alpha = 0.1$. Then Theorem 3 allows us to calculate f , and from equation (8) it follows that $f \approx 0.86$. Eighty six percent of the firms have a norm of cooperation, with the remaining fourteen percent having a norm of non cooperation.

Attrition Rate. The exogenous rate of attrition has two different effects. It determines the fraction of high cooperation jobs that are opened each period of time, and as a result, the expected value of a new match V . Second, it also determines the probability with which cooperative behavior will be "unjustly" punished. Both effects go the same way:

$$\frac{\partial f}{\partial \alpha} = -\frac{(u^d - u^o)(r + 1)}{(r + 1)u^o - ru^d - u^*} \frac{1}{(1 - \alpha)^2} < 0$$

The higher the exogenous attrition rate, the more attractive free riding becomes and as a result, the higher the fraction of firms with bad norms needs to be in order for cooperation to remain incentive compatible.

Discounting. An increase in the interest rate implies that the future is discounted more which makes agents more myopic. The more myopic agents are, the less they care about future low utility matches in their trade off between current effort and future utility. It follows that a larger fraction of non cooperating firms is needed to enforce cooperation, i.e. to satisfy the ND constraint.

$$\frac{\partial f}{\partial r} = -\frac{(u^d - u^o)(u^d - u^*)}{[(r + 1)u^o - ru^d - u^*]^2} \frac{\alpha}{1 - \alpha} < 0$$

In the limit of complete myopia, the future is not valued at all, so that all firms are non-cooperative. As was shown in Theorem 3, there is an upper upper bound \bar{r} in order to assure existence of c organizations.

Firm Size. The effect of larger organizations implies that free riding becomes more attractive. Ceteris paribus, an increase in m results in a higher value u^d , while keeping u^o and u^* constant. This in turn brings about a larger fraction of norms of non cooperation. Free riding is more lucrative, hence punishment is required to be stronger (i.e. a larger probability of a bad match). Formally, we show of types. This is the case in our model: low norm firms induce a positive externality (allowing credible punishment in the high norm organizations).

this for a linear additively separable production function, for which $e^d = e^*$. Let $Q = \sum_i e_i$, then $u^d = \frac{1}{m} [(m-1)e^o + e^d] - C(e^d)$, and as a result, $\frac{\partial u^d}{\partial m} = \frac{e^o + e^d}{m^2} > 0$. Now, it immediately follows that

$$\frac{\partial f}{\partial m} = -\frac{\alpha}{1-\alpha}(r+1)\frac{\partial u^d}{\partial m} \frac{u^o - u^*}{(r+1)u^o - ru^d - u^*} < 0$$

The larger the firms, the lower the fraction of cooperating firms. Note that this is true also for non linear sharing rules as the budget balancing requirement implies that the budget must be balanced at any level of effort, thereby restricting the set of admissible sharing rules.

4 HETEROGENEITY VERSUS INCREASING COMPENSATION SCHEDULES

In this section, we compare firm heterogeneity as an incentive device with wage-tenure schedules. It has long been recognized that steep wage-tenure schedules can provide incentives within the firm (see for example MacLeod and Malcomson (1989)). Moreover, in a context of random matching of new partnerships, that is precisely what the symmetric strategies in Datta (1993), Ghosh and Ray (1996) and Kranton (1996) amount to. The longer the tenure of the partnership, the higher the payoff. Therefore, we allow compensation schedules to increase with tenure. The most important result in this section is that conditions are identified under which heterogeneity is more efficient than wage-tenure schedules. In addition, we show that there is a coordination problem in the setting of an increasing compensations schedule: if all firms choose a steep schedule, a firm will set a steep schedule whereas if the schedule is chosen flat if all firms choose a flat schedule.¹⁴

We distinguish between *junior* and *senior* workers, indexed by the subscript j and s respectively. Junior workers are new entrants to the firm. Seniors are all other incumbent workers. Being junior lasts until a senior gets separated (or until the junior gets separated herself). In every firm, there is one junior and $m-1$ identical seniors.¹⁵ As before, output shares are contracted upon ex ante: $s_j(Q)$ for juniors and $s_s(Q)$ for seniors. As a result, the flow utility to any agent is $u_i = s_i(Q(\mathbf{e})) - C(e), \forall i \in \{j, s\}$.

In what follows, we make the following assumption:

¹⁴Note also that because we have stage games of more than two players and entry into the partnership is not simultaneous, the incentive scheme here can involve *full* cooperation in each period, i.e. there need not be inefficiency in production.

¹⁵All senior incumbents are treated equally. There is no a priori reason to do so. What is captured here is that all $m-1$ senior incumbents enter jointly as one party - the organization - in the market for junior applicants. While this simplification already conveys the main message of introducing a market, it reduces the complexity of the problem considerably. Of course, the model could be extended to the case of a complete seniority schedule. In that case, there would be “a market” between each level in the organization.

Assumption A. $Q(\mathbf{e}) = \sum_i e_i$ and $s_i(Q(\mathbf{e})) = s_i Q(\mathbf{e})$

Assumption A does not affect the results qualitatively and is made mainly because of analytical convenience. Some of the results have been extended for more general sharing rules, but at the cost of tractability. Note that a different assumption made earlier – balanced budget – *does* affect the results qualitatively (see Holmström (1982)).

4.1 EXOGENOUS SHARING RULES

For a given, exogenous sharing rule $\{s_j, s_s\}$, we can now derive the equivalents to equations (2),(3),(4). Let v_j^o denote the continuation payoff in a type c firm when junior and v_s^o when senior. In the firms of type d , all workers are newly matched and the surplus is split equally. A junior worker now has the prospect of becoming senior¹⁶:

$$rv_j^o = u_j^o + \alpha [V + (m-1)v_s^o - mv_j^o] \quad (9)$$

$$rv_s^o = u_s^o + \alpha [V - v_s^o] \quad (10)$$

The continuation payoff in a firm of type d is as before: $rv^* = u^* + [V - v^*]$. The fundamental difference in a firm of type c is that when joining the firm as a junior, there is the prospect of becoming a senior. Once a senior has been separated exogenously, the junior gets promoted to senior and a new junior is hired. Because a senior in general receives a share of the output different from that of a junior, there is a gap between the continuation payoff of a senior and that of a junior. Let Δ be defined as $\Delta = v_s^o - v_j^o$, then equation (9) can be written as $rv_j^o = u_j^o + \alpha [V - v_j^o + (m-1)\Delta]$. Using equations (9) and (10), for any given sharing rule $\{s_j, s_s\}$, Δ is given by

$$\Delta = \frac{u_s^o - u_j^o}{r + m\alpha} \quad (11)$$

It is now shown that it is decreasing in s_j .

Lemma 4 For any given sharing rule $\{s_j, s_s\}$: $\frac{\partial \Delta}{\partial s_j} < 0$.

Proof. Since $e_j^o = e_s^o = e^o$, and $u_i^o = s_i(Q^o) - c(e^o)$, the utility difference is equal to $u_s^o - u_j^o = s_s(Q^o) - s_j(Q^o)$. Under budget balancing, $s_j(Q) + (m-1)s_s(Q) = Q$ which implies $u_s^o - u_j^o = \frac{Q^o - ms_j(Q^o)}{m-1}$. Taking the derivative of (11) with respect to s_j :

$$\frac{\partial \Delta}{\partial s_j(Q)} = \frac{-m}{(r + m\alpha)(m-1)} < 0$$

¹⁶At the beginning of period t , the continuation payoff (given stationarity) satisfies

$$v_j^o = \frac{1}{1+r} [u_j^o + \alpha V + (1 - \alpha m)v_j^o + (m-1)\alpha v_s^o]$$

This completes the proof. ■

Not surprisingly, for $s_j = s_s$, there is no difference in the continuation payoff of juniors and seniors: $\Delta = 0$. Then from Lemma 4, for any $s_s > s_j$, Δ is strictly positive. As in the former section, we calculate the continuation payoff of a deviator (junior and senior) when in a type c firm:

$$rv_i^d = u_i^d + [V - v_i^d], \forall i \in \{j, s\} \quad (12)$$

No deviation by any agent in a firm of type c requires the condition ND to be satisfied for both juniors and seniors, i.e.

$$v_i^o \geq v_i^d, \forall i \in \{j, s\} \quad (13)$$

With authority, the firm in addition has to ensure that the sharing rules are individually rational (IR) for the junior. Because the outside option is endogenous, any agent will reject offers which give a continuation payoff that is lower than in a firm of type d :

$$v_j^o \geq v^* \quad (14)$$

Note that this allows for utilities in a c firm that are lower than those in a d firm: $\exists s_j : u_j^o < u^*$. In fact, when the IR constraint is binding, utility u_j^o may even be negative. The following lemma derives a lower bound on s_j .

Lemma 5 *There is a lower bound \underline{s}_j on the sharing rule, satisfying*

$$\underline{s}_j(Q^d) - c(e_j^d) = u^* \quad (15)$$

Proof. In appendix ■

At $s_j = \underline{s}_j$, $v_j^o = v^*$ and any agent is indifferent between joining an organization with a norm c or one with a norm d . Given this sharing rule, there is no longer any involuntary continuation payoff difference, in the sense that workers are indifferent and hence equally well off in both types of firms. That does not rule out the existence of the two types of different norms.

For any exogenously given s_j, s_s , proposition 6 now establishes the existence of equilibrium and derives the distribution of firms in the presence of authority. This Proposition is the equivalent of Theorem 3, where $s_j = s_s$.

Proposition 6 (Exogenous Asymmetric Sharing Rules) *Under assumption A, there exists a pair $(\widehat{r}, \widehat{\alpha})$ such that for any $r \in (0, \widehat{r}]$ and for any $\alpha \in (0, \widehat{\alpha}]$, and for a sharing rule $\{s_j, s_s\}_c, \forall c \in \mathcal{C}$, where $s_j \in [\underline{s}_j, s_s]$, an equilibrium exists where a fraction f of firms $c \in \mathcal{C} \subset \mathcal{N}$ have a norm of cooperation, with*

$$f = 1 - \frac{(u_j^d - u_j^o)(r + 1) + \alpha(m - 1)\Delta}{u_j^o(r + 1) - ru_j^d - u_j^* + \alpha(m - 1)(1 + r)\Delta} \frac{\alpha}{1 - \alpha} \quad (16)$$

Proof. In appendix ■

The proposition states that equilibrium exhibiting authority relations within firms with a norm of cooperation, exists. In fact, any type of authority is an equilibrium (i.e. the proposition holds for any feasible s_j) as long as all firms in \mathcal{C} use the same exogenous sharing rule. We have derived equilibrium when authority is “assumed”. We now turn to the case where the sharing rule (i.e. the price for entry) is determined in equilibrium. Authority is endogenous.

4.2 ENDOGENOUS SHARING RULES: LIMITED BY THE MARKET

Consider an organization with a norm of cooperation. Juniors are better off in the high norm firm than in a low norm firm, from the *IR* constraint (14). Lemma 7 shows that whatever the symmetric equilibrium sharing rule $\{s_j, s_s\}$ in the economy, seniors increase their continuation payoff by decreasing s_j .

Lemma 7 *The continuation payoff of a senior worker is increasing with decreasing s_j*

$$\frac{\partial v_s^o}{\partial s_j} < 0$$

Proof. From equation (10) it follows that

$$v_s^o = \frac{1}{r + \alpha} \{u_s^o + \alpha V\}$$

Derivation with respect to s_j ,

$$\frac{\partial v_s^o}{\partial s_j} = \frac{1}{r + \alpha} \frac{\partial u_s^o}{\partial s_s} \frac{\partial s_s}{\partial s_j}$$

which is negatives since budget balance implies that $\frac{\partial s_s}{\partial s_j} < 0$. ■

Now within each firm, we allow for the seniors to determine s_j . An equilibrium with endogenous sharing rules is now as before, with the additional requirement that the budget balancing sharing rule $\{s_j, s_s\}_c, \forall c \in \mathcal{C}$ for each firm is optimally chosen to maximize v_s^o , given the choice of an optimal sharing rules by all other firms $\{s_j, s_s\}_{-c}, \forall -c (\neq c) \in \mathcal{C}$, subject to the *ND* and *IR* constraints. From Lemma 7, it seems as if seniors will want to choose s_j as low as possible ($s_j = \underline{s}_j$). Proposition 8 shows that in

general this is not true and establishes that there is a limit to the rents the senior incumbents can extract from the entrants. The entrants willingness to provide effort depends on the sharing rule that is set in all other firms (i.e. the continuation payoff from deviation). If all other firms set a high s_j , then deviating is more attractive, so firm i will choose a high s_j to stop the entrant from deviating.

Proposition 8 (Endogenous Sharing Rules are Limited) *Under assumption A, there exists a pair (r^*, α^*) and an \hat{r} such that for any $r \in [r^*, \hat{r}]$ and for any $\alpha \in (0, \alpha^*)$, an equilibrium exists where seniors in a firm $c \in \mathcal{C}$ with a norm of cooperation, choose $\{s_j, s_s\}_c = \{s_j, s_s\}_{-c}$, satisfying $s_j \in [\underline{s}_j, s_s]$ and where the fraction f of firms in \mathcal{C} is*

$$f = 1 - \frac{(u_j^d - u_j^o)(r+1) + \alpha(m-1)\Delta}{u_j^o(r+1) - ru_j^d - u_j^* + \alpha(m-1)(1+r)\Delta} \frac{\alpha}{1-\alpha} \quad (17)$$

Proof. We proceed to prove the proposition in two steps. First, in Lemma 9 we show that, for a given sharing rule of all other firms $\{s_j, s_s\}_{-c}$, firm c 's best response is $\{s_j, s_s\}_c = \{s_j, s_s\}_{-c}$. Then we apply Proposition 6 to show existence and derive f as in equation (17).

Lemma 9 (Best Response) *Under assumption A, and provided ND_j is binding, there exists a pair (r^*, α^*) , such that for any $r \in (r^*, 1]$ and for any $\alpha \in (0, \alpha^*)$, a firm i 's best response $\{s_j, s_s\}_c, \forall c \in \mathcal{C}$ satisfies $\{s_j, s_s\}_c = \{s_j, s_s\}_{-c}$.*

Proof. In Appendix. ■

The proof of Proposition 8 is now nearly complete. We only need to show that there is an $r^* < \hat{r}$, so that Proposition 6 applies. For any \hat{r} , there exists an α low enough such that this is satisfied. It follows from the proof of Lemma 9 (in appendix, equation (37)) that r^* is decreasing in α

$$\frac{dr^*}{d\alpha} = \frac{Q^o r^*}{Q^o(1-r^*) + Q^d \frac{(r^*+\alpha)(r^*+m\alpha)}{(1+r^*)^2}} > 0$$

and with α going to zero, r^* becomes negative since $Q^o > Q^d$

$$\lim_{\alpha \rightarrow 0} r^* = \frac{-Q^o}{Q^o - Q^d} < 0$$

As a result, there is always an $r^* < \hat{r}$. This completes the proof of Proposition 8. ■

The intuition is that even though the seniors' continuation payoff is increasing for a decreasing s_j , the incentive constraint ND of the juniors is affected by the change in s_j . What the proposition shows is

the conditions under which a decrease in s_j violates the ND_j constraint. For sufficiently high r and sufficiently low α , a decrease in s_j decreases v_j^o marginally more than a decrease in v_j^d , which violates the ND constraint. Consider $v_j^o = v_j^d$ binding, then a decrease in s_j decreases both v_j^o and v_j^d . Since v_j^o depends on both r and α , and v_j^d only on r , both continuation payoffs have a different marginal effect for different pairs (r, α) .

The behavior by other firms in the market clearly limits a firm to extract authority rents from newly entering juniors. The best one individual firm can do is extract as much as the other firms. Of course, there is a continuum of equilibria in this economy: if all other firms extract more from the juniors (i.e. have a low s_j) then an individual firm can extract that much as well. It is important to note that the equilibrium level of s_j , associated with each of these equilibria, affects the equilibrium distribution, and hence efficiency. Next, we illustrate this result with an example.

4.3 AN EXAMPLE WITH INCREASING COMPENSATION SCHEDULES

Consider the same example as in section 3, where each time, two senior incumbents hire one junior. Note that assumption A is satisfied, and that the sharing rule satisfies budget balancing: $s_j + 2s_s = 1$. Utility is given by $u_i = s_i Q - \frac{e_i^2}{2}, \forall i \in \{j, s\}$. Optimal effort is unchanged $e^o = 1$ and adjusting for the shares, optimal utility $u_i^o = s_i 3 - \frac{1}{2}$. Effort for deviating is determined by the first order condition, where $C'(e) = e$ implies $s_i = e_i$. It follows that

$$u_i^d = s_i(2 + s_i) - \frac{s_i^2}{2} = 2s_i + \frac{s_i^2}{2}, \forall i \in \{j, s\}$$

Making use of budget balancing $s_j + 2s_s = 1$, we get

$$\begin{aligned} u_j^d &= 2s_j + \frac{s_j^2}{2} \\ u_s^d &= 1 - s_j + \frac{(1 - s_j)^2}{8} \end{aligned}$$

As before, in firms with a norm of non-cooperation, output is shared equally: $e^* = \frac{1}{3}$ and $u^* = \frac{5}{18}$. From equation (11) it follows that $\Delta = \frac{3}{2} \frac{1-3s_j}{r+3\alpha}$. Note that for $s_j = s_s = \frac{1}{3}$, we have the case of symmetric exogenous sharing rules, and $\Delta = 0$. From the individual rationality condition (14), $u_j^d = u^*$ it follows that $2s_j + \frac{s_j^2}{2} = \frac{5}{18}$, which is satisfied for $\underline{s}_j = 0.13$. Note that $u^o = s_j 3 - \frac{1}{2}$ is negative for any $s_j < \frac{1}{6} \approx 0.17$ (at $s_j = \underline{s}_j$, $u_j^o = -0.097$).

We first verify the conditions of Proposition 6:

1. The junior's ND is binding

$$\Delta \geq \frac{u_s^d - u_j^d}{r + 1}$$

implies

$$\frac{3}{2} \frac{1 - 3s_j}{r + 3\alpha} \geq \frac{1}{8} \frac{9 - 26s_j - 3s_j^2}{r + 1}$$

which is satisfied for all the examples we give below. Hence f is derived from (16)

$$f = 1 - \frac{\left(2s_j + \frac{s_j^2}{2} - 3s_j + \frac{1}{2}\right) (1 + r) + \alpha \frac{3(1-3s_j)}{r+3\alpha}}{\left(3s_j - \frac{1}{2}\right) (1 + r) - r\left(2s_j + \frac{s_j^2}{2}\right) - \frac{5}{18} + \alpha \frac{3(1-3s_j)}{r+3\alpha} (1 + r)} \frac{\alpha}{1 - \alpha} \quad (18)$$

2. Limited by the market. From the proof of lemma 9 (in appendix, equation (39)) it follows that the firm's schedule is limited by the other firms' schedules (i.e. the junior share cannot be lower than in other firms $\{s_j\}_c \geq \{s_j\}_{-c}$)

$$3 \left(\frac{r}{(r + \alpha)(r + 3\alpha)} \right) > \frac{2 + s_j}{1 + r}$$

It is easy to verify that this condition holds for $r = \alpha = 0.1$. And though it does not hold for $r = \alpha = 0.2$ over the whole range of s_j (in particular near $s_j = \frac{1}{3}$), it does hold over the whole range for $r = 0.3$ and $\alpha = 0.1$. This implies that when it holds, authority is limited to what the market offers. Firms cannot offer an s_j that is lower than the rest of the firms. If they would, that would violate the juniors' ND constraint. When this condition is not satisfied, firms can exercise unlimited authority by offering the lowest share possible.

Figure 1 plots the distribution f in function of s_j from equation (18) for different combinations of r and α . The junior's share is bounded above by $\frac{1}{3}$ and below by $\underline{s}_j = 0.13$. The solid line gives equation (18). Note that in this case, a firm's schedule is limited by the market, there are multiple equilibria: all shares $s_j \in [.15, .33]$, chosen equally by all firms, are equilibria.

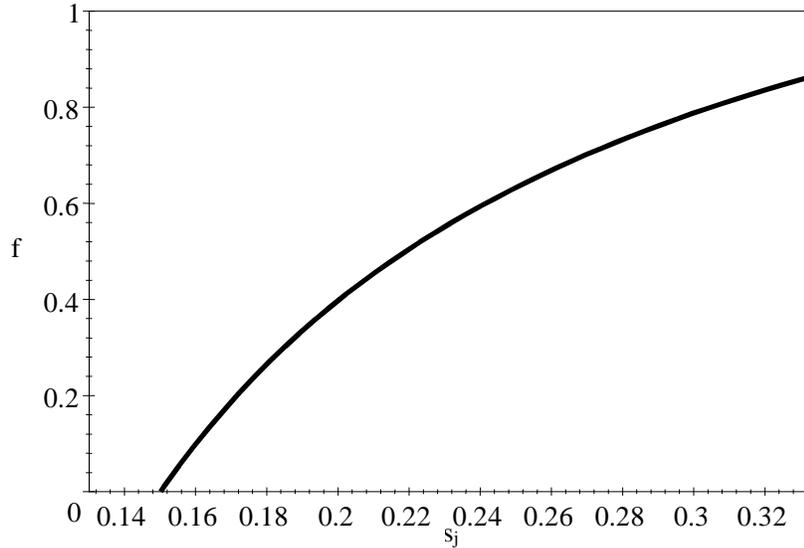
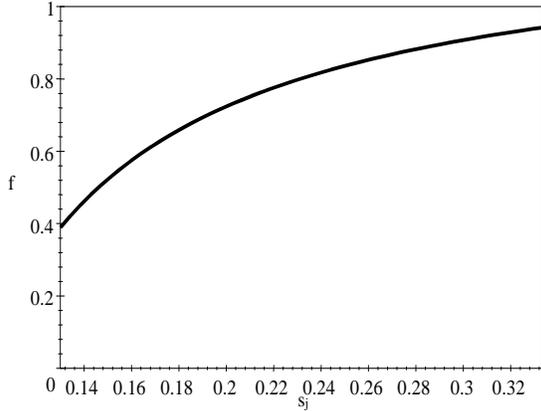


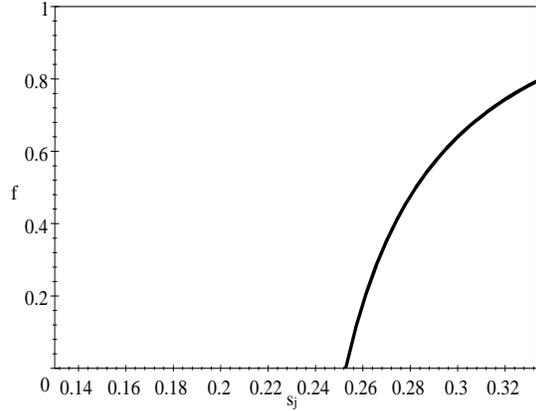
Figure 1: The Share of f in function of s_j for $r = \alpha = .1$.

Below, the different types of equilibrium distributions are illustrated. In figure 2, (as is the case in figure 1), s_j is limited. If all firms choose pay a share s_j , then the best response for a firm that employs a new entrant is to offer an identical share s_j . Then, no firm will offer a share different than any other firm. If it would do so, that would violate the junior's incentive not to deviate. As a result, all outcomes of f within the feasible range are possible ($f \in [.42, .94]$ in part (a) and $f \in [0, .79]$ in part (b)). Note also that in the figure 2(a), f is bounded from below for any feasible s_j . The minimum level is at $s_j = \underline{s}_j$, where $f = .42$. For r and α even lower (for example equal to .01), f increases to $f = 0.89$. As α and r go to zero, all firms in the limit have a norm of cooperation. On the other hand, as r and α increase, the equilibrium with heterogeneity in norms eventually does not exist, as in the example for $r = \alpha = .4$, and all firms have a norm of non-cooperation.

Figure 3 depicts the opposite case when s_j is unlimited by the market. Whatever share s_j other firms offer, senior incumbents increase their continuation payoff v_s^o by offering the lowest junior share possible without violating the ND_j constraint. In figure 3(a), it is illustrated that norms of cooperation simply do not exist (f hits zero before the IR constraint is binding). In figure 3(b), the seniors are constrained by the IR condition to offer shares above \underline{s}_j . Hence the only equilibrium is one with unlimited authority but where a fraction of roughly half of the firms has a norm for cooperation.

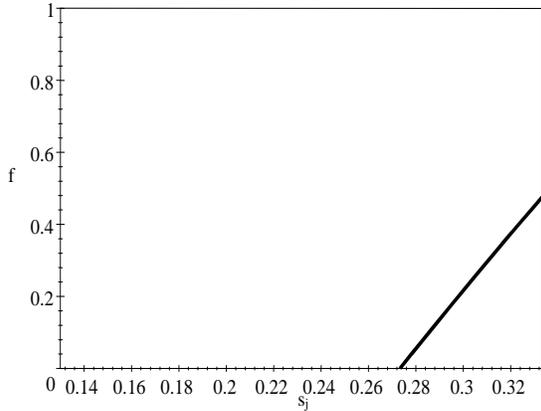


(a) $f \in [.42, .94]$ for $r = \alpha = .05$

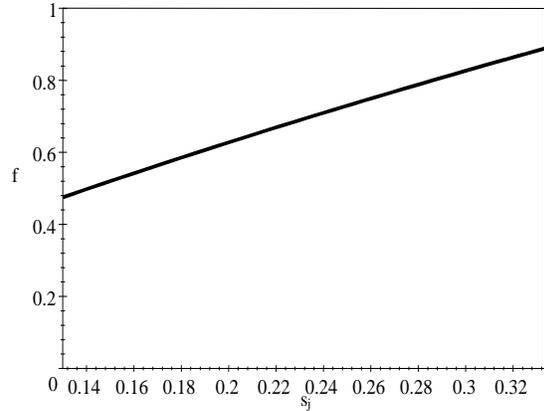


(b) $f \in [0, .79]$ for $r = .3, \alpha = .1$

Figure 2: s_j limited in equilibrium



(a) $f = 0$ for $r = .1, \alpha = .3$



(b) $f = .49$ for $r = .01, \alpha = .1$

Figure 3: s_j unlimited in equilibrium

4.4 EFFICIENCY

One major question remains: which organizational designs are more efficient? Here, we compare economies where organizations have a different compensation schedule. The result is that efficiency is higher the flatter the compensation schedule. That is, the most efficient design is the one with equal sharing rules $s_j = s_s$. This is surprising because without competition (i.e. in the presence of an exogenous outside option), increasing compensation schedules improves the incentives within the firm. However, in the presence of competing firms, increasing compensation schedules increase the incentives for deviation. As a result, more bad norms need to exist in equilibrium in order to sustain cooperation by the juniors in the

good norms. Obviously, a necessary condition for flat schedules to be more efficient is that junior shares are limited by the market. If not, there is no equilibrium with flat shares.

The next proposition shows that the conditions that limit s_j constitute a sufficient condition for this to be the case.

Lemma 10 *Under assumption A, and provided s_j is limited, the fraction f of firms of type $c \in \mathcal{C}$ is increasing in s_j .*

Proof. From equations (9) and (12), it follows that

$$\begin{aligned} v_j^o &= \frac{1}{r + \alpha} \{u_j^o + \alpha [V + (m - 1) \Delta]\} \\ v_j^d &= \frac{1}{r + 1} (u_j^d + V) \end{aligned}$$

As in Lemma 9 we calculate the effect of changes of s_j on the ND constraint. Now, we do not verify for a unilateral deviation but compare between different equilibria. As a result, we take the effect of s_j on V into account. Then denote the total derivative to take into account the effect on V , then

$$\frac{dv_j^o}{ds_j} \geq \frac{dv_j^d}{ds_j} \tag{19}$$

implies

$$\frac{\partial v_j^o}{\partial s_j} + \frac{\alpha}{r + \alpha} \frac{\partial V}{\partial s_j} \geq \frac{\partial v_j^d}{\partial s_j} + \frac{1}{r + 1} \frac{\partial V}{\partial s_j} \tag{20}$$

From Lemma 9 it follows that there exists a pair (r^*, α^*) such that $\frac{\partial v_j^o}{\partial s_j} = \frac{\partial v_j^d}{\partial s_j}$, then it follows that for all $\alpha \in [0, \alpha^*]$

$$\frac{\partial v_j^o}{\partial s_j} + \frac{r(\alpha - 1)}{(r + \alpha)(r + 1)} \frac{\partial V}{\partial s_j} > \frac{\partial v_j^d}{\partial s_j} \tag{21}$$

since $\frac{\partial V}{\partial s_j} > 0$. It now follows that for the ND constraint to hold with equality for a decrease in s_j , p and hence f have to decrease. This completes the proof. ■

Proposition 11 *Consider the set of equilibria with f good norm firms, and where s_j is limited. Then efficiency is increasing in s_j .*

Proof. This follows immediately from Lemma 10. ■

To see this, note that V is a convex combination in f of v_j^o and v_j^d , and V is increasing in s_j .

5 ROBUSTNESS

In this section, we verify whether the results derived are robust to changes in the assumptions. We consider the introduction of capital in production, renegotiation, and deviations by coalitions.

5.1 PRODUCTION WITH CAPITAL

Consider the model from section 3, with capital, competing for labor. The output production function is Cobb-Douglas with capital in addition to additively separable effort

$$y = f(\mathbf{e}, k) = \left(\sum e_i \right) k^a$$

This represents a situation as before: the firm can announce wages depending on the whole bundle \mathbf{e} of effort choices. Agents and capitalists simultaneously choose effort and capital, respectively. Given an effort bundle \mathbf{e} , a firm hires capital k at a capital rental rate R in order to maximize profits $\pi = y - mw(\mathbf{e}) - kR$, where $mw(\mathbf{e})$ is the total wage bill that is paid by the firm, which is shared according to the sharing rule $\{s_i\}$. This implies the first order condition

$$ak^{a-1} \left(\sum e_i \right) = R$$

The equilibrium level of capital k

$$k = \left(\frac{aQ}{R} \right)^{\frac{1}{1-a}}$$

The first order condition for labor is

$$\frac{dw(\mathbf{e})}{de_i} = \frac{dy}{de_i} = \frac{d \sum e_i}{de_i} k^a$$

The increase in the wage for extra effort is equal to the increase in the additional production of output. As before, we look for equilibria with equal effort supply by all agents within one firm. Then the first order condition for $e_i = e, \forall i$ is

$$\frac{dw(\mathbf{e})}{de} = k^a$$

Using (since $Q = me$)

$$k = \left(\frac{ame}{R} \right)^{\frac{1}{1-a}}$$

If there is a total amount of capital \bar{k} in the economy, then the rental rate of capital R is determined endogenously by equating supply of capital with demand

$$\int \left(\frac{ame}{R} \right)^{\frac{1}{1-a}} d \frac{F(e)}{m} = \bar{k}$$

which gives $R = \left(\int \frac{1}{km} (ame)^{\frac{1}{1-a}} dF(e) \right)^{1-a}$ where $F(e)$ is the cumulative distribution of all workers, with m of them in each firm. We can now substitute

$$\frac{dw(\mathbf{e})}{de} = \left(\frac{ame}{R} \right)^{\frac{a}{1-a}}$$

which after integration gives the wage schedule (with K the constant of integration)

$$w(\mathbf{e}) = (1-a) \left(\frac{am}{R} \right)^{\frac{a}{1-a}} e^{\frac{1}{1-a}} + K$$

And substituting for the equilibrium amount of capital k

$$w(\mathbf{e}) = (1 - a) ek^a + C$$

The total wage bill is $mw(\mathbf{e}) = m(1 - a) ek^a + mK$ and payment to capital is equal to ay , so that the zero profit condition implies that $K = 0$. The wage bill is a fixed proportion of y : $mw(\mathbf{e}) = (1 - a)y$. This competitive equilibrium implies that capital is efficiently hired, given the firm's belief about the effort supply \mathbf{e} of the workers. The total wage bill $mw(\mathbf{e}) = (1 - a)mek^a$ corresponds to total output available to the workers as modeled before

$$Q(\mathbf{e}) = (1 - a) \left(\sum e_i \right) k^a$$

In this setting, capital is proportional to the effort level in the firm and as a result capital yields the same return in all firms, irrespective of the norm. Capital does not earn a higher return in the firms with a norm of cooperation. Recent empirical work by Cappelli and Neumark (1999) supports this result. They report evidence that "high performance" work practices increase labor productivity. At the same time, these work practices raise labor cost and employee compensation, while keeping the return on capital constant. "High performance" work practices are good for employees and harm nor hurt employers. They conclude that "high road" human resources practices do raise employee compensation without affecting the firm's (i.e. the capital's) competitiveness.

5.2 RENEGOTIATION

The equilibrium derived in the former sections is not renegotiation proof. The strategy that supports equilibrium requires a punishment that involves termination of the entire c firm in the case of a deviation. Because $v^o \geq V$, this is not renegotiation proof.

In the former sections, no individual monitoring technology was available. In this section, it is illustrated how a renegotiation proof equilibrium arises if there is a positive probability that a deviator is detected. Suppose now that after production, in addition to Q , a worker's individual effort e_i is observed with probability $\beta \in (0, 1]$.

Consider the same strategy as above, except for the fact that there is no punishment unless the true effort is observed to be different from e^o . The punishment then implies that the match is terminated for the deviator only. All remaining $m - 1$ workers continue in the c firm with a newly hired worker replacing the deviator. The continuation payoff of a deviator in a firm of type c , in the case of no authority (i.e. $s_j = s_s$) then is

$$rv^d = u^d + (\alpha + \beta(1 - \alpha)) [V - v^d] \tag{22}$$

As in Theorem 3 we can derive the condition $v^o \geq v^d$ for no deviation, which is a modified version of (29)

$$u^o \geq \gamma' u^d + (1 - \gamma') u^* \quad (23)$$

where

$$\gamma' = \frac{r + p + \alpha(1 - p)}{r + \alpha + \beta(1 - \alpha) + p[1 - \alpha - \beta(1 - \alpha)]} \quad (24)$$

The proportion of cooperating firms now is

$$f(\beta) = 1 - \frac{(u^d - u^*)(r + 1)}{u^o(r + \beta) - ru^d - \beta u^*} \frac{\alpha}{1 - \alpha} \quad (25)$$

It can be shown that an equilibrium exists with a fraction $f(\beta)$ of firms of type c . Note that for $f(1)$, the outcome is identical to the one in the case of no monitoring. It is easy to show that $f(\beta)$ is increasing in β .

The main difference is however that this equilibrium is renegotiation proof. In case of a unilateral deviation, the deviator is detected (with probability β) while the remaining workers remain in the firm and continue to cooperate. For them the continuation payoff of punishment is not dominated, and hence satisfies Farrell and Maskin's (1989) criterion for renegotiation proofness. This holds for any $m > 2$.

5.3 DEVIATIONS BY COALITIONS

Equilibrium derived here is non-cooperative, in the sense that only deviations by one individual at the time are considered. Allowing for deviations by coalitions of m workers certainly does change the equilibrium. In particular, a firm of type d would always gain if all its workers were to coordinate their actions and start to cooperate (an individual firm has zero mass). However, equilibrium now does not exist. When all firms are cooperating, an individual will deviate. And we just pointed out that a coalition that does not cooperate will deviate otherwise. It follows that equilibrium does not exist. Note also that a mixed strategy by coalitions would be problematic. Given a mixed strategy by all other firms, one firm's best response is to cooperate with probability one. Being of zero mass, this does not change the no deviation constraint of one individual worker. This is a dominant strategy as the payoff from cooperating is higher than not cooperating.

6 EXTENSIONS

We consider three extensions to the model of section 3.

6.1 HETEROGENEOUS AGENTS

Consider two types θ of agents, h and l and such that, in addition to effort, the agent types are inputs in production. Types θ are observable. Let firms consist of $m = 2$ agents. For sorting to matter, let agent types be complementary inputs: $Q = \Pi_{\theta} \sum_i e_i$. There is now a productivity gain from matches that are positively assorted, as for a given level of effort, $Q(h, h) + Q(l, l) > 2Q(h, l)$. In the earlier sections, rematching is assumed to be frictionless. That implies that any high type can always reject a low type, and redraw a partner until she gets another high type. We now modify the model slightly and let agents make one draw from the pool of unmatched per period. Because the value of not being matched to anyone is normalized to zero, it always pays to remain matched, even if that includes negatively sorted matches (h, l) .

Now, effort choice by high types includes the consideration of "bad" (h, l) matches, in addition to the possibility of being in a (h, h) match with a norm for non-cooperation. All high types will immediately want to separate from a match with a low type implying there is no cooperation in mixed matches. The difference between h and l , and hence the marginal productivity of effort in different matches is now crucial in determining equilibrium. With a high difference, all high types will be induced to cooperate as there is sufficient punishment in the threat of being matched to a type l . So the heterogeneity has an efficiency gain by inducing all the high types to coordinate. Consider now the low types. They still need sufficient matches with a norm for non-cooperation in order to credibly sustain cooperation in others. However, there is also a possibility of being rematched to an exogenously separated high type worker. The larger the difference between low and high types, the higher the benefit to a low type and the higher here incentive to try a rematch each time. This will induce her not to cooperate even if she is matched to another low type, as she wants to try her luck by possibly rematching a high type. While increasing dispersion in the types provides incentives for the high types to cooperate, it provides incentives for the low types not to cooperate. The result is that the initial dispersion is exacerbated in the payoffs through effort choice.

6.2 COMPLEMENTARY INPUTS

When inputs are complementary, multiple static Nash equilibria can exist. The marginal productivity of a worker's effort increases as effort by other workers in the firm increases. As a result, multiple fixed points to the static game can exist.¹⁷ Suppose there are two pure strategy Nash equilibria with utilities

¹⁷Consider an example with $m = 3$, but where the production function is now multiplicative (i.e. effort is a complementary input) $Q = 3\Pi_i e_i$ and cost of effort is $c(e_i) = \frac{e_i^4}{4}$, which implies $c' = e_i^3$. When output is equally shared, there are two pure strategy Nash equilibria: $\bar{e}^* = 1$ and $\underline{e}^* = 0$. Then either $\bar{u}^* = \frac{1}{2}$ or $\underline{u}^* = 0$. The Pareto optimal level of effort is $e^o = 3$,

associated $\underline{u}^* < \bar{u}^*$ such that $\underline{u}^* < \bar{u}^* < u^o < u^d$. Let the corresponding continuation payoffs \underline{v}^* and \bar{v}^* be defined as above. To derive the equilibrium distribution of firm norms in this economy, consider the following expected continuation payoff of being rematched: $V = p_1 v^o + p_2 \underline{v}^* + p_3 \bar{v}^*$ where p_1 is the probability of matching to firm with a norm for cooperation and $p_1 + p_2 + p_3 = 1$.

An equilibrium distribution will now depend on what the level of effort is in the firms without a norm for cooperation. The condition (5) will now write $u^o \geq \gamma_1 u^d + \gamma_2 \underline{u}^* + \gamma_3 \bar{u}^*$. If $p_2 = 0$ (and hence $\gamma_2 = 0$), the fraction of cooperating firms f where the ND constraint is binding will be smaller than if $p_2 = 1 - p_1$. In fact, as p_2 is increasing, f is decreasing. The value of being in a firm with a norm for non-cooperation \underline{e}^* is the lowest possible, which implies that punishment is sufficiently severe that a large number of firms with a norm for cooperation can be sustained. In principle, any distribution of between p_2 and p_3 can be envisaged, as long as it satisfies the constraint.

Now consider the following case: let $\bar{v}^* > V$. Then a worker in a firm with a norm for non-cooperation (the higher one of the two), will not want to separate as the current value is higher than the expected value of rematching. However, even if these non-cooperating stay together, it will not be an equilibrium to start cooperating if the ND constraint is binding with equality. Hence there is an equilibrium with three types of norms: high turnover, low non-cooperative effort; low turnover, high non-cooperative effort; cooperation. We now derive distribution, always under the assumption that $\bar{v}^* > V$.

6.3 UNEMPLOYMENT

The result of the model is surprising because in all periods, agents are productive and can choose their effort levels. In this section, costly search is introduced: matching is not instantaneously so that a worker, whose match has terminated, necessarily spends some time without producing output. Now, in addition to the firm norm, there is unemployment. This is reminiscent of the efficiency wage model as in Shapiro and Stiglitz (1984). It will now become immediately apparent that this model differs from the efficiency wage model in three substantial aspects: 1. production involves cooperation between workers in the Holmström framework; 2. matching is costless¹⁸; 3. in the repeated game, histories of outcomes are

implying that $u^o = \frac{27}{4} \approx 6.75$. The utility from deviation is given by $u^d = 9e_i - \frac{e_i^4}{4}$, which solves $e^d = \sqrt[3]{9} \approx 2.08$ and yields $u^d = \frac{3}{4} 9^{\frac{4}{3}} \approx 14.04$.

¹⁸In the efficiency wage model without costly search, incentive compatibility becomes infinitely costly. To see this, consider the steady state condition in that model: $b(N - L) = aL$ with the original notation: L is the level of unemployment, N is the total labor force, a is the job separation rate and b is the arrival rate of jobs. Without costly search, the arrival rate of a job $a = \infty$ which implies there is no punishment device. From the steady state condition it follows that there is full employment (as in our model): $L = N$. The only wage that can sustain positive effort is $w = \infty$. Since that is not feasible (firms make losses: $\pi = f(L) - \infty < 0$), the equilibrium without frictions is $w = 0$ and $e = 0$.

observed within the existing matches, and the firm norm serves as a public randomization device.

We now extend our model to include costly search in the matching process. Matches arrive at a rate $\lambda \in [0, \infty)$. The implication is that a worker whose match has terminated, now has to spend some time without a positive flow of utility (unemployment benefits are normalized to zero). Let z be the continuation payoff of unemployment:

$$ru = \lambda(V - u) \quad (26)$$

where V is the expected continuation payoff of a future match. Note that for $\lambda \rightarrow \infty$ frictions disappear and $z = V$. As before, we consider firms of type c where a worker has a continuation payoff v^o and firms of type d with v^* . The continuation payoffs satisfy:

$$\begin{aligned} rv^* &= u^* + \alpha z + \lambda V - (\alpha + \lambda)z^* \\ rv^o &= u^o + \alpha(z - v^o) \\ rv^d &= u^d + (z - v^d) \end{aligned}$$

We can now rewrite the no deviation constraint ND as

$$\begin{aligned} u^o &\geq \frac{\alpha + r}{1 + r}u^d + \frac{r(1 - \alpha)}{1 + r}z \\ &\geq \frac{\alpha + r}{1 + r}u^d + \frac{r(1 - \alpha)}{1 + r} \frac{\lambda V}{r + \lambda} \end{aligned} \quad (27)$$

The difference between the ND constraint here with the model without frictions is in the term $\frac{\lambda}{\lambda + r}$ (see equation (6) above), which is equal to 1 for $\lambda \rightarrow \infty$, i.e. immediate arrival of jobs (or no frictions).

Steady state implies that the total flow out of unemployment is equal to the total flow into unemployment $\lambda \#z = \alpha(1 - \#z)$ (where $\#z$ is the measure of unemployed) and that the flow into good jobs is equal to the flow out of good jobs: $\lambda(1 - f) = \alpha f$. Note that this may give rise to an equilibrium where $f = 1 - \#z$: since the bad firm types immediately change to a good firm when they get the opportunity, equilibrium implies there are only good firms. However, this is only true if the (ND) constraint is satisfied. Note that the proportion of good firms in general is

$$p = \frac{\alpha f}{1 - f - \#z + \alpha f}$$

Consider the extreme case where only firms with a norm of cooperation exist. Then $f = 1 - \#z$, $p = 1$ and $V = v^o$. The ND constraint now reduces to

$$u^o \geq \frac{\alpha + r}{1 + r}u^d + \frac{(1 - \alpha)\lambda}{(1 + r)(r + \alpha + \lambda)}u^o \quad (28)$$

For a rate of attrition α , there exists a critical λ for which this equation is satisfied with equality. For a given pair (r, α) , let $\underline{\lambda}$ satisfy $u^o = \frac{\alpha+r}{1+r}u^d + \frac{(1-\alpha)\lambda}{(1+r)(r+\alpha+\lambda)}u^o$. Then since $\frac{d}{d\lambda} \frac{(1-\alpha)\lambda}{(1+r)(r+\alpha+\lambda)} = \frac{(1-\alpha)(r+\alpha)}{(1+r)(r+\alpha+\lambda)^2} > 0$, the ND constraint is violated $\forall \lambda > \underline{\lambda}$. The implication is that only equilibria can exist where $V < v^o$, which is only satisfied for $p < 1$ since $V = pv^o + (1-p)v^*$. It follows that $f < 1 - \#z$ and that the fraction of firms with a norm for non cooperation is $1 - f - \#z > 0$. Moreover, the fraction of high cooperation firms is constant, while the fraction of low cooperation firms is strictly increasing in λ . When $\lambda \rightarrow \infty$, $u = 0$ and the fraction of low norm firms is $1 - f$, as in the competing norms model with frictionless matching. As an example, the consider the production technology as above with $m = 10$, then $u^d = 0.93$. Let $r = 0.1$ and $\alpha = 0.3$. The lower bound $\underline{\lambda}$ then solves the equation (28) which implies $\underline{\lambda} = 0.41$. For any $\lambda > 0.41$, the equilibrium necessarily involves some degree of inequality of firm norms.

7 CONCLUDING REMARKS

The theory of competing norms provides an explanation for the persisting differences in organizational design: norms that are bad for its members do not disappear because they affect incentives within good norm firms. Moreover, compensation schedules are shaped by the market and the schedules adopted in other firms. The fundamental implication for efficiency is that flat compensation schedules within firms, combined with heterogeneity between firms generates highest aggregate welfare.

In a different economic environment, the competing norms model may provide the parallel to Tiebout's theory of local public goods, even for identical agents. The social capital associated with the norm can be interpreted as a local public good. In Tiebout's model, heterogeneous citizens move between different neighborhoods (by "voting with their feet") and sort themselves into homogeneous communities in order to provide the local public good (e.g. education). As a result, heterogeneity between neighborhoods increases. What the theory of competing norms shows, is that even with identical citizens and with sufficient mobility, neighborhoods will have different degrees of contribution to the public good, as long as the contribution cannot be contracted upon ex ante. The low contribution neighborhoods will exhibit a high rate of turnover.

8 APPENDIX

Proof of Proposition 2

Proof. If there is no cooperation in none of the firms, then the continuation payoff in all firms is v^* . Since all firms are identical, the expected continuation payoff of a future match is $V = V^*$. As a result, the agent chooses e_i to maximize $rv^* = \max_{e_i} \{s_i(Q(e_i, e_{-i}^*)) - c(e_i)\}$, the solution of which by definition of the static Nash equilibrium is $e_i = e_i^*$. Because all agents in all firms are indifferent between rematching and remain matched to the current partner ($V = v^*$), an equilibrium may involve any termination strategy, i.e. with any probability $\in [0, 1]$. ■

Proof of Theorem 3

Proof. Consider the strategy described above. The continuation values are given by equations (2), (3), (4). Substituting for $V = pv^o + (1-p)v^*$ implies

$$\begin{aligned} rv^o &= u^o + \alpha(1-p)[v^* - v^o] \\ rv^d &= u^d + (1-p)[v^* - v^d] \\ rv^* &= u^* + p[v^o - v^*] \end{aligned}$$

We can now rewrite the no deviation constraint (5) which implies

$$u^o \geq \gamma u^d + (1-\gamma)u^* \tag{29}$$

where

$$\gamma = \frac{r+p+\alpha(1-p)}{r+1} \tag{30}$$

It is easy to verify that $v^o \geq v^*$ so that no agent who has cooperated wants to terminate the match when $Q(\mathbf{e}) = Q(\mathbf{e}^o)$. So no agent in a firm c wants to deviate if condition (29) is satisfied.

We now verify deviations by agents in firms of type d . Suppose she chooses a level of effort $e \neq e^*$, then by definition of Nash equilibrium, her utility $u(e_i, e_{-i}^*) < u^*$. Given the termination strategy of her partners, she will be separated with probability 1, thus giving her the expected continuation payoff V . As a result, her continuation payoff from choosing $e \neq e^*$ is lower than v^* . Given the termination strategy of all other agents in a type d firm, her termination strategy does not affect her payoff. Note however that a strategy where all players in a d firm choose not to terminate cannot be an equilibrium. Suppose it were, then deviating by termination yields a continuation payoff $V \geq v^*$ (from $v^o \geq v^*$ and given that V is a convex combination of v^o and v^*).

It now suffices to demonstrate the existence of a non negative pair $(\bar{r}, \bar{\alpha})$ such that condition (29) is satisfied. To establish (29) we can choose an \bar{r} and $\bar{\alpha}$ to satisfy (29) with equality. To see this is possible, note that $\lim_{r \rightarrow 0} (\lim_{\alpha \rightarrow 0} \gamma) = 0$ and $\lim_{r \rightarrow 1} (\lim_{\alpha \rightarrow 1} \gamma) = 1$, and that $\frac{d\gamma}{d\alpha} > 0$ and $\frac{d\gamma}{dr} > 0$, making use of equation (7). Since by definition, u^o, u^* , and u^d satisfy $u^d \geq u^o \geq u^*$, we choose $(\bar{r}, \bar{\alpha})$ so that $u^o = \gamma u^d + (1 - \gamma) u^*$. This is satisfied with equality for $F(c) = f$. Now, for a given $(r, \alpha) < (\bar{r}, \bar{\alpha})$, the ND constraint is satisfied. Using (7) and (30) to substitute at the ND constraint (29), yields equation (8). This completes the proof. ■

Proof of Lemma 5

Given that the incentive compatibility constraint is binding, IR requires that $v_j^o = v_j^d \geq V^*$. From equations (12) and (4), IR then implies

$$\frac{u_j^d + V}{1 + r} \geq \frac{u^* + V}{1 + r}$$

and hence $u_j^d \geq u^*$. Where the IR constraint is binding, $u_j^d = u^*$ can be rewritten as $\underline{s}_j (Q^d) - C(e_j^d) = u^*$, where \underline{s}_j is the minimal s_j . This is a lower bound because u_j^d is increasing in s_j . ■

Proof of Proposition 6

To prove this Proposition, we proceed by showing two Lemmata. In Lemma 12, for a given sharing rule $\{s_j, s_s\}$, common to all firms, we derive the equivalent distribution function as in Theorem 3. As in Theorem 3 we can verify that we only have to make sure no deviations are made by workers (both junior and senior) in c firms (as before, no one in a d firm wants to deviate from $e = e^*$ nor the the separation strategy, and no worker in the c firm wants to deviate by early termination. In Lemma 13, assumption A allows us to determine that ND_j is binding, and we show existence.

Lemma 12 *For any given sharing rule $\{s_j, s_s\}_c, \forall c \in \mathcal{C}$ the fraction f_1 of firms with a norm for cooperation, is given by*

$$f_1 = 1 - \frac{(u_j^d - u_j^o)(r + 1) + \alpha(m - 1)\Delta}{u_j^o(r + 1) - ru_j^d - u_j^* + \alpha(m - 1)(1 + r)\Delta} \frac{\alpha}{1 - \alpha} \quad (31)$$

provided $\frac{u_s^o - u_j^o}{r + m\alpha} \geq \frac{u_s^d - u_j^d}{r + 1}$ and provided equilibrium exists.

Proof. Consider the same strategies as in Theorem 3. Then the proportion p of c firms is given by equation (7). The expected continuation payoff of rematching is now $V = pv_j^o + (1 - p)v^*$. Substituting

V in equations (9), (12) and (4), using (11) implies

$$\begin{aligned} rv_j^o &= u_j^o + \alpha [(1-p)(v^* - v_j^o) + (m-1)\Delta] \\ rv_j^d &= u_j^d + (1-p)[v^* - v_j^d] \\ rv^* &= u^* + p[v_j^o - v^*] \end{aligned}$$

No deviation by the junior requires $v_j^o \geq v_j^d$ (ND_j), implies:

$$u_j^o + \alpha(m-1)\Delta \geq u_j^d\gamma + u^*(1-\gamma) \quad (32)$$

where γ is as before and given by equation (30).

The continuation payoffs for the senior workers can be rewritten in a similar way: $rv_s^o = \alpha [(1-p)(v^* - v_s^o) - p\Delta]$.

We then get a parallel condition ND_s for the senior workers derived from $v_s^o \geq v_s^d$

$$u_s^o + p(1-\alpha)\Delta \geq u_s^d\gamma + u^*(1-\gamma) \quad (33)$$

Both ND_j and ND_s need be satisfied. To determine which one of the two is binding, consider

$$\begin{aligned} v_j^o &\geq v_j^d \\ v_s^o &\geq v_s^d \end{aligned}$$

Now given the definition of $\Delta = v_s^o - v_j^o$, we can write ND_s as

$$v_j^o + \Delta \geq v_j^d + \frac{u_s^d - u_j^d}{r+1}$$

since

$$v_s^d - v_j^d = \frac{u_s^d - u_j^d}{r+1} > 0$$

This implies that ND_j is binding iff $\Delta \geq \frac{u_s^d - u_j^d}{r+1}$ and ND_s if $\Delta \leq \frac{u_s^d - u_j^d}{r+1}$ (note that both are binding at $s_j = s_s$: then $\Delta = 0$ and $u_s^d = u_j^d$). From the definition of Δ

$$ND_j \text{ binding} \Leftrightarrow \frac{u_s^o - u_j^o}{r+m\alpha} \geq \frac{u_s^d - u_j^d}{r+1} \quad (34)$$

Assuming existence of a non degenerate distribution, we now proceed as in the proof of Theorem 3 by calculating the distribution. If (34) holds, from (32) (holding with equality), we can calculate f_1 which gives (16). This completes the proof. ■

In the following Lemma, we make use of assumption A in order to determine when ND_j is binding.

Lemma 13 Under assumption A , and for any sharing rule $\{s_j, s_s\}$, with $s_j \in [\underline{s}_j, s_s]$, there exists a pair (r_1, α_1) such that for any $r \in (0, \tilde{r}]$, and for any $\alpha \in (0, \tilde{\alpha}]$, ND_j is binding.

Proof. We show that $u_s^o - u_j^o \geq u_s^d - u_j^d$. The left hand side can be written as $s_s(Q^o) - s_j(Q^o)$. The right hand side is $s_s(Q_s^d) - s_j(Q_j^d) - [C(e_s^d) - C(e_j^d)]$. For any $s_j \leq s_s$, and given A , it follows that $e_j^d \leq e_s^d$ (from $\frac{\partial u}{\partial e_i} = s_i Q_e - c'(e_i) = 0$, and c convex the envelope theorem implies that $\frac{\partial e_i}{\partial s_i} < 0$) and as a result, $Q_s^d \geq Q_j^d$. Since $Q^o > Q^d$, it immediately follows that $u_s^o - u_j^o \geq u_s^d - u_j^d$.

For a finite m , there always exists a pair (r, α) small enough such that equation (34) is satisfied. To see this, for any r , let $\alpha \leq \frac{1}{m}$, which is sufficient. Then let (r_1, α_1) be chosen such that (34) holds with equality. From Lemma 12, it follows that the binding constraint is ND_j . ■

We can now finalize the proof of Proposition 6 and derive the distribution f . As in theorem 3, there exists a pair (r, α) such that (32) holds with equality. To see this, note that $\lim_{r \rightarrow 0} \lim_{\alpha \rightarrow 0} \alpha(m-1)\Delta = \frac{m-1}{m} (u_s^o - u_j^o)$ so that in the limit, the left hand side of ND_j in equation (32) is equal to $u_j^o + \frac{m-1}{m} (u_s^o - u_j^o) = \frac{1}{m} Q^o - c(e^o) > u^*$. Choose (r_2, α_2) to satisfy (32) with equality. Let $(\hat{r}, \hat{\alpha}) = \min\{(r_1, \alpha_1), (r_2, \alpha_2)\}$. Then, under assumption A , Lemma 13 holds, so that from Lemma 12, it follows that $f = f_1$

$$f = 1 - \frac{(u_j^d - u_j^o)(r+1) + \alpha(m-1)\Delta}{u_j^o(r+1) - ru_j^d - u_j^* + \alpha(m-1)(1+r)\Delta} \frac{\alpha}{1-\alpha} \quad (35)$$

This completes the proof of Proposition 6. ■

Proof of Lemma 9

Proof. The constraint ND_j binding implies, from equation (13) that $v_j^o = v_j^d$. From equations (9) and (12) it follows that

$$\begin{aligned} v_j^o &= \frac{1}{r+\alpha} \{u_j^o + \alpha[V + (m-1)\Delta]\} \\ v_j^d &= \frac{1}{r+1} (u_j^d + V) \end{aligned}$$

The problem of the senior is to choose s_j (and as a result s_s , from budget balancing) in order to maximize v_s^o subject to ND_j

$$\begin{aligned} &\max_{s_j} v_s^o \\ &\text{s.t. } v_j^o \geq v_j^d \end{aligned}$$

Since v_s^o is always increasing for decreasing s_j (from Lemma 7) it suffices to verify whether for a lower s_j the ND_j constraint is still binding, i.e. whether

$$\frac{\partial v_j^o}{\partial s_j} \leq \frac{\partial v_j^d}{\partial s_j} \quad (36)$$

A unilateral deviation requires the effect on V is ignored, this then implies, using assumption A:

$$\frac{Q^o}{r + \alpha} \left(1 - \frac{m\alpha}{(r + m\alpha)} \right) \leq \frac{Q^d}{1 + r} \quad (37)$$

We now show that there exists a pair (r^*, α^*) for which equation (36) holds with equality. To see this, we consider two extreme points. At $r = 0$, equation (36) holds with strict inequality for any $\alpha > 0$, since

$$\lim_{r \rightarrow 0} \frac{Q^o}{r + \alpha} \left(1 - \frac{m\alpha}{(r + m\alpha)} \right) = 0$$

$$\lim_{r \rightarrow 0} \frac{Q^d}{1 + r} = Q^d > 0$$

At $r = 1$, the inequality is violated if

$$Q^o \frac{1}{(1 + \alpha)(1 + m\alpha)} > \frac{Q^d}{2} \quad (38)$$

which is the case for all $\alpha \in (0, \alpha^*)$, where α^* solves equation (38) with equality (note that the left hand side is monotonically decreasing in α and goes to zero as α goes to infinity). It now follows that, provided $\alpha < \alpha^*$ there exists an r^* such that equation (37) holds with equality, since $\forall r \in (0, 1)$, $\frac{d}{dr} \frac{\partial v_j^o}{\partial s_j} = Q^o \frac{1-r}{(1+\alpha)(1+m\alpha)} > 0$ and $\frac{d}{dr} \frac{\partial v_j^d}{\partial s_j} < 0$.

For any pair (r, α) such that $r \in (r^*, 1]$ and $\alpha \in (0, \alpha^*)$, the ND_j constraint satisfies

$$\frac{\partial v_j^o}{\partial s_j} > \frac{\partial v_j^d}{\partial s_j} \quad (39)$$

A decrease in s_j implies a higher marginal effect on v_j^o than on v_j^d . Given that ND_j is binding for the strategy $\{s_j, s_s\}_{-c}$ by all other norms $-c \in \mathcal{C}$, it follows that $v_j^o = v_j^d$, for $\{s_j, s_s\}_c = \{s_j, s_s\}_{-c}$. Equation (39) implies that $v_j^o < v_j^d$ for $\{s_j\}_c < \{s_j\}_{-c}$ implying that the best response is $\{s_j, s_s\}_c = \{s_j, s_s\}_{-c}$. This completes the proof of the Lemma. ■

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