Unemployment Cycles

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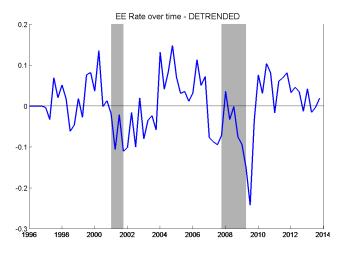
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This Paper

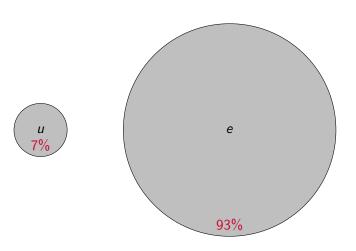
- Theory of cycles, solely driven by the labor market
- Labor market by itself can generate cyclical outcomes
 - 1. Mechanism: search behavior of the employed
 - 2. We illustrate theory with a Quantitative Exercise

SEARCH BEHAVIOR OF THE EMPLOYED



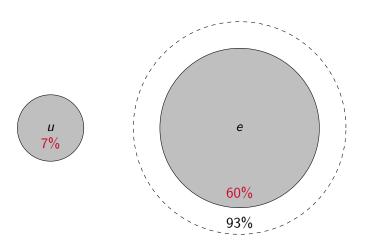
Composition Externality

LABOR FORCE (ON AVERAGE)



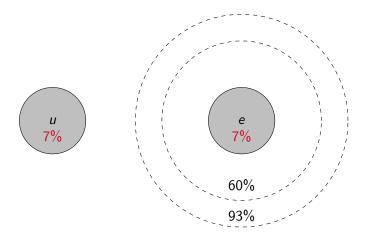
COMPOSITION EXTERNALITY

Searchers



COMPOSITION EXTERNALITY

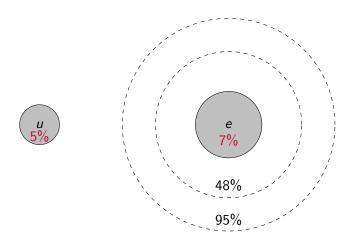
EFFECTIVE SEARCHERS



 \rightarrow on average 50% $\simeq \frac{7}{7+7}$ of jobs are filled by employed

Composition Externality

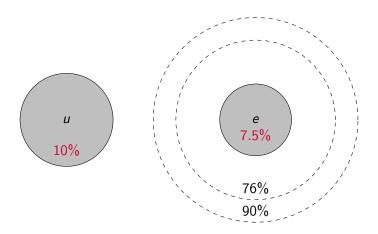
Воом



 \rightarrow Boom: 62% $\simeq \frac{7}{7+5}$ of jobs are filled by employed

COMPOSITION EXTERNALITY

RECESSION



 \rightarrow Recession: 42% $\simeq \frac{7.5}{7.5+10}$ of jobs are filled by employed

THE MECHANISM

- Pro-cyclical on-the-job search (OJS) intensity of employed
 - ⇒ Multiple equilibria
- Strategic complementarity betw. search effort and vac. posting due to:
 - 1. Composition externality + job quality: newly created jobs by employed are more productive and more prevalent in Boom: 42% (R) \rightarrow 62% (B)
 - 2. Duration: average job duration shorter in Boom

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Boom: OJS intensity \uparrow \Rightarrow composition \succ duration \Rightarrow profits \uparrow \Rightarrow v \uparrow \Rightarrow matching prob \succ search cost \Rightarrow OJS intensity \uparrow
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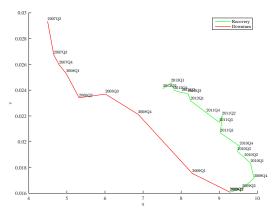
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Boom: OJS intensity \uparrow \Rightarrow composition \succ duration \Rightarrow profits \uparrow \Rightarrow v \uparrow \Rightarrow matching prob \succ search cost \Rightarrow OJS intensity \uparrow
Recession: OJS intensity \downarrow \Rightarrow composition \prec duration \Rightarrow profits \downarrow \Rightarrow v \downarrow \Rightarrow matching prob \prec search cost \Rightarrow OJS intensity \downarrow
```

IMPLICATIONS

- 1. Large fluctuations in u, v, EE without shifts in fundamentals
- Jobless recovery: OJS crowds out unemployed searchers during recovery
- 3. Outward shift Beveridge curve in recovery (no change match efficiency)

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THE LITERATURE

Multiple Equilibria in Search Markets:

Increasing Returns: Diamond (1982)

Selection: Burdett-Coles (1998)

Demand External.: McAfee (1992), Kaplan-Menzio (2014), Schaal-Taschereau

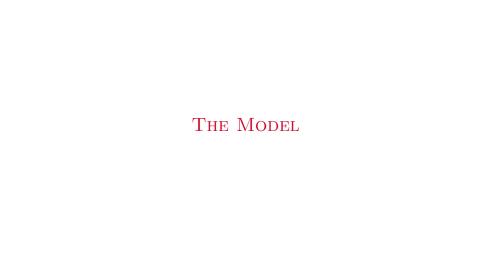
(2014)

Decreasing Returns: Golosov-Menzio (2015) Marriage Market: Burdett-Imai-Wright (2004)

Housing Market: Moen-Nenov (2014)

• Business Cycles and Search:

Shimer (2005), Hall (2005), Hagedorn-Manovskii (2008)



THE MODEL: KEY INGREDIENTS

- 1. On-the-job search
- 2. Job ladder (sorting)
- 3. Endogenous vacancy creation

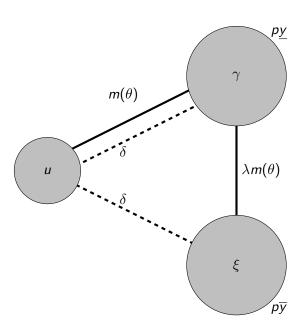
THE MODEL: KEY INGREDIENTS

- 1. On-the-job search
- 2. Job ladder (sorting)
- 3. Endogenous vacancy creation
 - Natural setup: random arrival diff. jobs \Rightarrow selection + duration issue
 - All action comes from OJS of those in low productivity job who transit to high productivity job
- ⇒ Focus on simple model: out of U, low prod. job; out of E high prod.

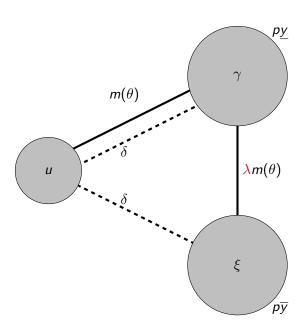
AGENTS, ACTIONS, PAYOFFS + WAGE SETTING

- Workers: measure one; risk-neutral and homogenous
 - Employed (get w) or unemployed (get b)
 - Decision: Once on the job, active OJS at cost k?
 - Cost of search during unemployment (or passive OJS) normalized to zero
 - Objective: maximize discounted value of employment
- Firms: large number; ex-ante homogenous and risk-neutral
 - Decision: post a vacancy at cost c; free entry
 - Ex-post heterogeneity in their job productivity $y \in \{\underline{y}, \overline{y}\}$:
 - \underline{y} for UE match, \overline{y} for EE match \rightarrow Job ladder
 - Objective: maximize discounted sum of profits
- Wage setting: sequential auction; firms match outside offers

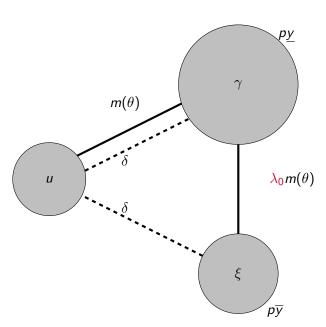
LABOR MARKET



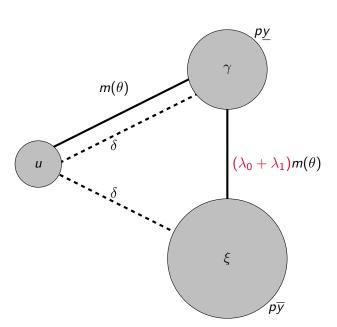
LABOR MARKET



No Active OJS



ACTIVE OJS



FIRMS

Bellman Equations

$$rV = -c + q(\theta(\Omega)) \left[\frac{u}{s(\Omega)} \underline{J} + \frac{\lambda(\Omega)\gamma}{s(\Omega)} \overline{J} - V \right] + \dot{V}$$

$$r\underline{J} = p\underline{y} - \underline{w}(\Omega) - [\lambda(\Omega)m(\theta(\Omega)) + \delta](\underline{J} - V) + \dot{\underline{J}}$$

$$r\overline{J} = p\overline{y} - \overline{w}(\Omega) - \delta(\overline{J} - V) + \dot{\overline{J}}$$

where

- $\Omega \in [0,1]$ all workers' search decision
- we suppress time indices

•
$$\theta(\Omega) = \frac{v}{s(\Omega)} = \frac{v}{u + \lambda(\Omega)\gamma}$$

• $\underline{w}(\Omega), \overline{w}(\Omega)$ set by PVR bargaining



FIRMS

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Workers

Bellman Equations

$$rU = pb + m(\theta(\Omega))(\underline{E} - U) + \dot{U}$$

$$r\underline{E} = \underline{w}(\Omega) - \omega pk + \lambda(\omega)m(\theta(\Omega))(\overline{E} - \underline{E}) - \delta(\underline{E} - U) + \dot{\underline{E}}$$

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where

• $\omega \in [0,1]$ individual worker's search decision

LABOR MARKET DYNAMICS

$$\dot{\gamma} = um(\theta(\Omega)) - \gamma[\delta + \lambda(\Omega)m(\theta(\Omega))]
\dot{\xi} = \gamma\lambda(\Omega)m(\theta(\Omega)) - \xi\delta
1 = u + \gamma + \xi$$

EQUILIBRIUM

DEFINITION

An equilibrium is a path $\{U_t, \underline{E}_t, \overline{E}_t, V_t, \underline{J}_t, \overline{J}_t, \theta_t, \underline{w}_t, \overline{w}_t, u_t, \gamma_t, \xi_t, \omega_t, \Omega_t\}$ s.t. for all $t \geq 0$

- 1. $U_t, \underline{E}_t, \overline{E}_t, V_t, \underline{J}_t, \overline{J}_t$ satisfy the Bellman equations above;
- 2. Given Ω_t , $\omega_t = \Omega_t$ maximizes \underline{E}_t ;
- 3. There is free entry: $V_t = 0$;
- 4. Wages: \underline{w}_t such that $\underline{E}_t = U_t$ and \overline{w}_t such that $\underline{J}_t = V_t$;
- 5. u_t, γ_t, ξ_t satisfy the laws of motion;
- 6. $\lim_{t\to\infty} \underline{J}_t$ is finite for initial conditions u_0, γ_0, ξ_0 .

MULTIPLE STEADY STATE EQUILIBRIA: EXISTENCE

- Check one-shot deviations of workers in y-jobs in interval dt
- Denote $\underline{E}(\omega|\Omega)$: value of y job when worker action is ω given Ω
 - 1. $\Omega=1$: all workers active OJS \Rightarrow profitable to stop active OJS $\omega=0$?

$$\underline{\underline{F}}(1|1) > \underline{\underline{F}}(0|1) \quad \Longleftrightarrow \quad m^{-1}\left(\frac{k(r+\delta)}{\lambda_1(y-b)}\right) < \theta(1).$$

2. $\Omega = 0$: all workers no active OJS \Rightarrow profitable active OJS $\omega = 1$?

$$\underline{\underline{E}}(0|0) > \underline{\underline{E}}(1|0) \iff \theta(0) < m^{-1} \left(\frac{k(r+\delta)}{\lambda_1(y-b)} \right).$$

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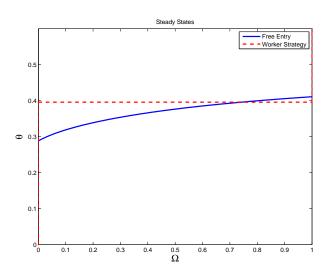
LEMMA

There are multiple steady states if and only if

$$\theta(0) < m^{-1}\left(\frac{k(r+\delta)}{\lambda_1(y-b))}\right) < \theta(1).$$



STEADY STATE EQUILIBRIA

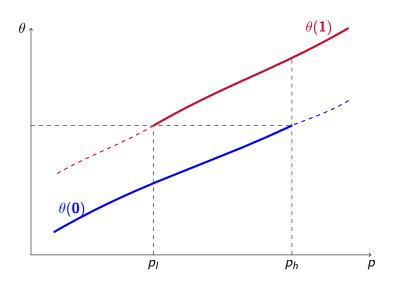


MULTIPLE STEADY STATES: EXISTENCE

PROPOSITION

Let $m(\theta) = \phi \frac{\alpha \theta}{\alpha \theta + 1}$. Then there are multiple steady state equilibria if and only if $p \in [p^l, p^u]$. The set $[p^l, p^u]$ is non-empty for an open set of parameters.

Multiplicity Bounds: p



MULTIPLE STEADY STATE EQUILIBRIA: EXISTENCE

SUFFICIENT SORTING NEEDED FOR ACTIVE OJS

Proposition

Let
$$m(\theta) = \phi \frac{\alpha \theta}{\alpha \theta + 1}$$
.

- 1. If $(\overline{y} \underline{y} < \epsilon)$ then there is a unique steady state with no active OJS;
- 2. If \overline{y} is arbitrarily high (given \underline{y}), there is a unique steady state with active OJS;
- 3. For $\overline{y} \in [\overline{y}^I, \overline{y}^u]$ (given \underline{y}), there are multiple steady states.



STEADY STATE EQUILIBRIA: PROPERTIES

PROPOSITION

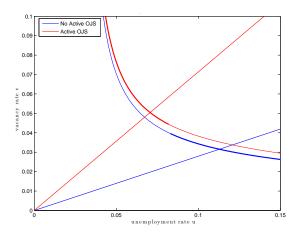
Assume there are multiple steady states. Then:

- 1. unemployment is lower with active OJS: u(1) < u(0);
- 2. EE flows are higher with active OJS: EE(1) > EE(0);

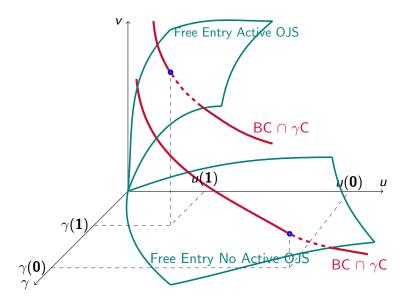
and under $m(\theta) = \phi \alpha \theta / (\alpha \theta + 1)$

- 3. vacancies are higher with active OJS: v(1) > v(0);
- **4**. conventional market tightness is higher with active OJS: $\Theta(\mathbf{1}) > \Theta(\mathbf{0})$;
- 5. BC(1) is shifted outward relative to BC(0)
- 6. $BC^{s}(1)$ is shifted outward relative to $BC^{s}(0)$
- 7. Share of OJSearchers is higher with active OJS: $\frac{\lambda(1)\gamma(1)}{s(1)} > \frac{\lambda(0)\gamma(0)}{s(0)}$.

STEADY STATE EQUILIBRIA: PROPERTIES



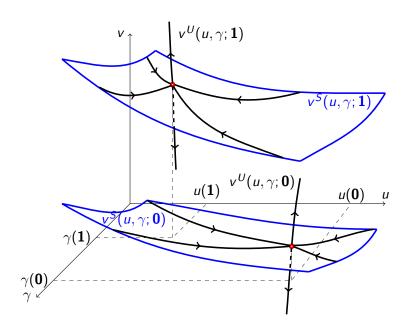
STEADY STATE EQUILIBRIA: PROPERTIES



DYNAMICS

- Our model can be reduced to a dynamic system in \mathbb{R}^3 : $\dot{u}(\Omega), \dot{\gamma}(\Omega), \dot{\theta}(\Omega)$ System
- \bullet Multiple SS equilibrium \to multiple equil. paths in dynamic economy

SADDLE-PATH STABILITY



Validation and Quantitative Exercise

VALIDATION AND QUANTITATIVE EXERCISE

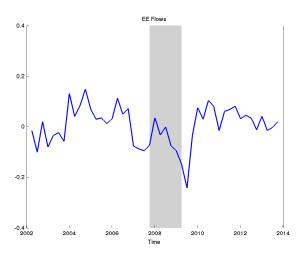
- 1. Direct evidence for mechanism: pro-cyclical search intensity
- 2. Quantitative exercise
 - Calibrate the model to US economy
 - Quantitative assessment:
 - Steady States: Labor Market Fluctuations and counterfactuals
 - Dynamics: Jobless recovery

THE DATA

- US quarterly data
- Main data source: Current Population Survey (CPS)
- Data on vacancies, unemployment, labor market transitions
- Vacancies: JOLTS (BLS) + online help-wanted ads
- Data spans 1996-2013 but main focus on Great Recession



EE FLOWS (DETRENDED)

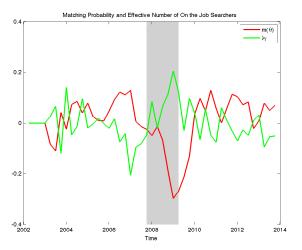


Decomposition of EE Flows: $EE = \lambda \gamma m(\theta)$

$$m(\theta) = \frac{UE}{u}$$
 and $\lambda \gamma = \frac{EE \cdot u}{UE}$

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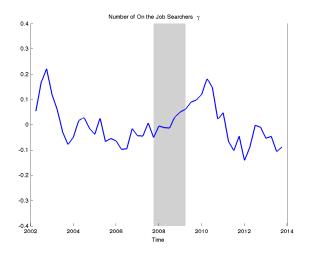
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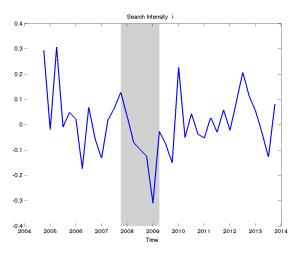
Decomposing $\lambda \gamma$

- ullet Problem: No direct measure of search intensity λ
- Use CPS micro-data panel structure
- Check whether individuals was unemployed before current job or transited from another job
- Construct γ (employed after UE transition) and ξ (after EE transition)
- Then, search intensity is computed as: $\lambda = \frac{EE}{m(\theta)\gamma}$

Decomposition of EE Flows: γ



Decomposition of EE Flows: $\lambda = \frac{EE}{m(\theta)\gamma}$



⇒ Pro-cyclical search intensity!





CALIBRATION

- Set parameters (r, b, δ, p, y) outside the model
- Calibrate $(\lambda_0, \lambda_1, \alpha, \phi, c, k, \overline{y})$ using GMM
- Target business cycle moments from the Great Recession
 - EE fluctuations (peak and trough)
 - $m(\theta)$ -fluctuations (peak and trough)
 - wage differentials $\overline{w}/\underline{w}$ in boom (peak)
 - v, u-levels in boom (peak)
- Focus on 2 data points from last cycle with largest differences in EE
- \Rightarrow 2006Q3 boom ($\Omega = 1$) and 2009Q3 recession ($\Omega = 0$)

CALIBRATION

- We do not target unemployment and vacancy levels in the recession
- We do not restrict the estimates to fall into range of multiple SS (we get it)

EXOGENOUSLY SET PARAMETERS

Variable	Value		Notes
r	0.0113	discount factor	standard
У	1	productivity first job	normalization
\overline{b}	0.919	unemployment value	92% of y ; 58% of \overline{y} (see below)
δ	0.05	job separation rate	average separation rate
p	1	productivity	normalization

ESTIMATED PARAMETERS

	Estimate	Parameter Description
λ_0	0.092	passive OJS intensity
λ_1	0.073	active OJS intensity
α	0.863	curvature matching function
ϕ	3.258	overall matching efficiency
С	9.404	vacancy posting cost
\overline{y}	1.577	high productivity
k	0.080	search cost

ESTIMATED PARAMETERS

λ_0 0.092 passive OJS intensity	
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ϕ 3.258 overall matching efficience	Су
c 9.404 vacancy posting cost	
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 \Rightarrow Multiple Steady States Exist: $p \in [p^l, p^u] = [0.994, 1.026]$

Moments

TARGETED

ullet Model 1: Benchmark model, multiple steady st., fixed productivity p

	Data	Model
<i>EE</i> (1)	0.066	0.035
<i>EE</i> (0)	0.036	0.022
u(1)	0.047	0.055
v(1)	0.029	0.039
$m(\theta(1))$	0.852	0.853
$m(\theta(0))$	0.511	0.513
$\frac{\overline{w}(1)}{\underline{w}(1)}$	1.230	1.230

MOMENTS

TARGETED

• Model 1: Benchmark model, multiple steady st., fixed productivity p

	Data	Model
<i>EE</i> (1)	0.066	0.035
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$\frac{\overline{w}(1)}{\underline{w}(1)}$	1.230	1.230

Discrepancy between model and data: constant separation rate

Moments

Non-Targeted

	Data	Model	
<i>u</i> (0)	0.096	0.089	
v(0)	0.016	0.029	
$\frac{\lambda(0)\gamma}{s(0)}$	0.423	0.327	
$\frac{\lambda(1)\gamma}{s(1)}$	0.625	0.425	

LABOR MARKET FLUCTUATIONS

• Fluctuations between peak and trough of Great Recession

•
$$\Delta x = \frac{x(0)-x(1)}{x(1)}$$

	Data	Model 1	Model 2
ΔEE	-0.46	-0.37	
$\Delta m(\theta)$	-0.40	-0.40	
Δv	-0.47	-0.28	
Δu	1.06	0.60	
$\Delta \theta$	-0.61	-0.47	
$\Delta\Theta$	-0.74	-0.55	
$\Delta \lambda \gamma / s$	-0.32	-0.23	

LABOR MARKET FLUCTUATIONS

Fluctuations between peak and trough of Great Recession

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$$\Delta x = \frac{x(\mathbf{0}) - x(\mathbf{1})}{x(\mathbf{1})}$$

	Data	Model 1	Model 2
ΔEE	-0.46	-0.37	-0.05
$\Delta m(\theta)$	-0.40	-0.40	-0.15
Δv	-0.47	-0.28	-0.08
Δu	1.06	0.60	0.17
$\Delta \theta$	-0.61	-0.47	-0.20
$\Delta\Theta$	-0.74	-0.55	-0.22
$\Delta \lambda \gamma / s$	-0.32	-0.23	-0.02

Model 1: Multiple equilibria, fixed productivity $\Delta p = 0$.

Model 2: Active OJS equil., Δp : +2% deviation from trend in boom, -3% in recession.

I. A SIMPLE EXERCISE

- Myopic agents: in recession $(\Omega=0)$ change beliefs to boom $(\Omega=1)$
- Searchers: $s(\mathbf{0}) = u(\mathbf{0}) + \lambda_0 \gamma(\mathbf{0}) \rightarrow s^R = u(\mathbf{0}) + (\lambda_0 + \lambda_1) \gamma(\mathbf{0})$
- Fraction κ of *u*-hires:

$$\kappa(\mathbf{0}) = \frac{u(\mathbf{0})}{u(\mathbf{0}) + \lambda_0 \gamma(\mathbf{0})} = 0.67 \rightarrow \kappa^R = \frac{u(\mathbf{0})}{u(\mathbf{0}) + (\lambda_0 + \lambda_1)\gamma(\mathbf{0})} = 0.53$$

• Uncond. matching probability $\kappa(\mathbf{0})m(\theta(\mathbf{0})) = 0.34 \rightarrow \kappa^R m(\theta^R) = 0.30$

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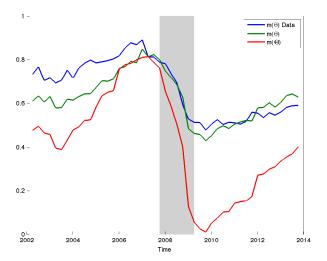
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⇒ Job-destructive Recovery

I. A SIMPLE EXERCISE

• Effective matching probability $m(\theta)$ drops (but less so than $m(\Theta)$)



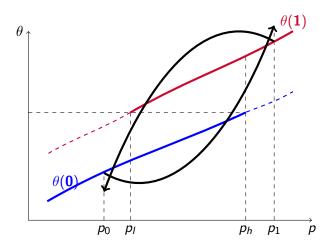
II. Productivity-Induced Dynamics

- Multiplicity selection criterion: history-dependent beliefs (Cooper 1994)
- Aggregate productivity p follows Markov process
- Agents are forward-looking
- Experiment: Economy has been in the recession for a while and positive shock p↑ induces unique equilibrium with OJS

II. PRODUCTIVITY-INDUCED DYNAMICS

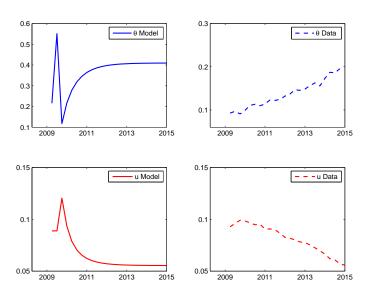
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- · Agents are forward-looking
- Experiment: Economy has been in the recession for a while and positive shock p↑ induces unique equilibrium with OJS
- Limitations: saddle-path stability + linear approximation dynamic system

II. PRODUCTIVITY-INDUCED DYNAMICS



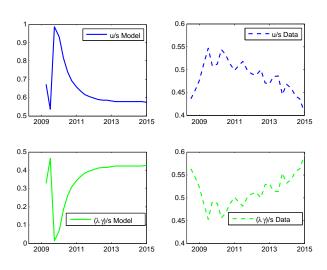
Jobless Recovery: Transition Paths

MARKET TIGHTNESS AND UNEMPLOYMENT



Jobless Recovery: Transition Paths

Composition of New Jobs



SUMMARY OF QUANTITATIVE RESULTS

- Fluctuations
 - Model generates sizable fluctuations v, u, EE without shift fundamentals
 - Small additional fluctuations from productivity change
- Jobless recovery
 - Unemployment initially grows during the recovery
 - Composition of *u*-jobs is initially higher in recovery

CONCLUSION

The labor market by itself can generate cycles

Unemployment Cycles

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WAGES

$$\underline{w}(\Omega) = pb\left(\frac{r + \lambda(\Omega)m(\theta(\Omega)) + \delta}{r + \delta}\right) - \frac{\lambda(\Omega)m(\theta(\Omega))}{r + \delta}p\underline{y} + \Omega pk$$

$$\overline{w}(\Omega) = p\underline{y}$$

▶ Back

PROOF OF LEMMA 1 PBack

1. No deviation when no one searches: $\underline{E}(0|\mathbf{0}) > \underline{E}(1|\mathbf{0})$.

$$\underline{\underline{F}}(1|\mathbf{0}) = \frac{1}{1 + rdt} \left[dt(\underline{\underline{w}}(\mathbf{0}) - pk) + (1 - \delta dt) dt \lambda(1) m(\theta(\mathbf{0})) \overline{\underline{F}} + (1 - \delta dt) (1 - dt \lambda(1) m(\theta(\mathbf{0})) \overline{\underline{F}} \right]$$
where $\overline{\underline{F}} = \overline{\underline{F}}(0|\mathbf{0})$.

 $E(0|\mathbf{0})(1+rdt) > dt(w(\mathbf{0})-pk) + dt\lambda(1)(1-\delta dt)m(\theta(\mathbf{0}))\overline{E} + (1-\delta dt - dt\lambda(1)m(\theta(\mathbf{0}))$

Subtracting $\underline{E}(0|\mathbf{0})$ from both sides and dividing by dt and take the limit $dt \to 0$:

 $rE(0|\mathbf{0}) > w(\mathbf{0}) - pk + \lambda(1)m(\theta(\mathbf{0}))\overline{E} + (-\delta - \lambda(1)m(\theta(\mathbf{0})))E(0|\mathbf{0}) + \delta U.$

Substituting the equilibrium values for $\underline{E}(0|\mathbf{0}), \overline{E}, U$ and $\underline{w}(\mathbf{0})$ we get:

$$(\underline{y}-b)[\lambda(1)-\lambda(0)]m(\theta(\mathbf{0}))-k(r+\delta)<0.$$

2. No deviation when all search: $\underline{E}(1|1) > \underline{E}(0|1)$ (proceed similarly). $(y-b)[\lambda(1)-\lambda(0)]m(\theta(1))-k(r+\delta)>0.$

Putting (1) and (2) together gives the condition in the Lemma.

(1)

MULTIPLE EQUILIBRIA: DYNAMICS

- Local stability around SS
- Our model can be reduced to a dynamic system in \mathbb{R}^3 : $\dot{u}(\Omega), \dot{\gamma}(\Omega), \dot{\theta}(\Omega)$.

$$\begin{split} \dot{u}(\Omega) &= \delta(1-u) - um(\theta(\Omega)) \\ \dot{\gamma}(\Omega) &= um(\theta(\Omega)) - (\delta + \lambda(\Omega)m(\theta(\Omega)))\gamma \\ \dot{\theta}(\Omega) &= \frac{m(\theta(\Omega))u}{(1-\eta(\theta(\Omega)))(u+\lambda(\Omega)\gamma)} \times \left[\frac{\lambda}{u}\left(-\frac{\theta(\Omega)c}{m(\theta(\Omega))} + \overline{J}\right)\left(-\dot{u}\frac{\lambda(\Omega)}{u} + \dot{\gamma}\right) \right. \\ &\left. - (p\underline{y} - \underline{w}(\Omega)) + \left(\frac{c}{q(\theta(\Omega))}\frac{u+\lambda(\Omega)\gamma}{u} - \frac{\lambda(\Omega)\gamma}{u}\overline{J}\right)(r+\delta + \lambda(\Omega)m(\theta(\Omega)))\right] \end{split}$$

CONDITION FOR MULTIPLE EQUILIBRIA PBack

Necessary and sufficient condition for existence of multiple steady states

$$-\frac{2(\phi\lambda_0+2r)}{4\alpha(\phi\lambda_0+r)} + \overline{y} - \alpha^2\rho\phi b + \frac{\sqrt{\alpha^2(-8cr^2(\phi\lambda_0+r)(2cr-\alpha\rho\phi(\overline{y}-b)) + (cr2(\phi\lambda_0+2r) + \alpha\rho\phi(-(\overline{y}-b)(\phi\lambda_0+r)))^2)}}{4\alpha^2cr(\phi\lambda_0+r)} \\ < \frac{kr}{\alpha\left(\phi\lambda_1(\underline{y}-b) - kr\right)} < -\frac{2(\phi(\lambda_0+\lambda_1)+2r) + kr}{4\alpha(\phi(\lambda_0+\lambda_1)+r)} + \overline{y} - \alpha^2\rho\phi b \\ (\text{ME}) \\ + \frac{\sqrt{\alpha^2(-8cr^2(\phi(\lambda_0+\lambda_1)+r)(2cr-\alpha\rho\phi(\overline{y}-b-k)) + (cr2(\phi(\lambda_0+\lambda_1)+2r) + \alpha\rho\phi(kr-(\overline{y}-b)(\phi(\lambda_0+\lambda_1)+r)))^2)}}{4\alpha^2cr(\phi(\lambda_0+\lambda_1)+r)} \\ = \frac{\sqrt{\alpha^2(-8cr^2(\phi(\lambda_0+\lambda_1)+r)(2cr-\alpha\rho\phi(\overline{y}-b-k)) + (cr2(\phi(\lambda_0+\lambda_1)+2r) + \alpha\rho\phi(kr-(\overline{y}-b)(\phi(\lambda_0+\lambda_1)+r)))^2)}}}{4\alpha^2cr(\phi(\lambda_0+\lambda_1)+r)} \\ = \frac{\sqrt{\alpha^2(-8cr^2(\phi(\lambda_0+\lambda_1)+r)(2cr-\alpha\rho\phi(\overline{y}-b-k)) + (cr2(\phi(\lambda_0+\lambda_1)+2r) + \alpha\rho\phi(kr-(\overline{y}-b)(\phi(\lambda_0+\lambda_1)+r)))^2)}}}{4\alpha^2cr(\phi(\lambda_0+\lambda_1)+r)} \\ = \frac{\sqrt{\alpha^2(-8cr^2(\phi(\lambda_0+\lambda_1)+r)(2cr-\alpha\rho\phi(\overline{y}-b-k)) + (cr2(\phi(\lambda_0+\lambda_1)+2r) + \alpha\rho\phi(kr-(\overline{y}-b)(\phi(\lambda_0+\lambda_1)+r)))^2)}}}{4\alpha^2cr(\phi(\lambda_0+\lambda_1)+r)} \\ = \frac{\sqrt{\alpha^2(-8cr^2(\phi(\lambda_0+\lambda_1)+r)(2cr-\alpha\rho\phi(\overline{y}-b-k)) + (cr2(\phi(\lambda_0+\lambda_1)+2r) + \alpha\rho\phi(kr-(\overline{y}-b)(\phi(\lambda_0+\lambda_1)+r)))^2)}}}$$

CONDITION FOR MULTIPLE EQUILIBRIA PBack

Necessary and sufficient condition for existence of multiple steady states

$$-\frac{2(\phi\lambda_0+2r)}{4\alpha(\phi\lambda_0+r)}+\overline{y}-\alpha^2\rho\phi b+\frac{\sqrt{\alpha^2(-8cr^2(\phi\lambda_0+r)(2cr-\alpha\rho\phi(\overline{y}-b))+(cr2(\phi\lambda_0+2r)+\alpha\rho\phi(-(\overline{y}-b)(\phi\lambda_0+r)))^2)}}{4\alpha^2cr(\phi\lambda_0+r)}\\ <\frac{kr}{\alpha\left(\phi\lambda_1(\underline{y}-b)-kr\right)}<-\frac{2(\phi(\lambda_0+\lambda_1)+2r)+kr}{4\alpha(\phi(\lambda_0+\lambda_1)+r)}+\overline{y}-\alpha^2\rho\phi b\\ (\text{ME})\\ +\frac{\sqrt{\alpha^2(-8cr^2(\phi(\lambda_0+\lambda_1)+r)(2cr-\alpha\rho\phi(\overline{y}-b-k))+(cr2(\phi(\lambda_0+\lambda_1)+2r)+\alpha\rho\phi(kr-(\overline{y}-b)(\phi(\lambda_0+\lambda_1)+r)))^2)}}{4\alpha^2cr(\phi(\lambda_0+\lambda_1)+r)}$$

Multiplicity bounds in terms of p:

$$\begin{array}{lcl} \rho^{l} & = & \frac{2c\lambda_{1}r(\underline{y}-b)[k(\lambda_{0}+\lambda_{1})+\lambda_{1}(\underline{y}-b)]}{\alpha[\lambda_{1}\phi(\underline{y}-b)-kr][b^{2}\lambda_{1}+k(\lambda_{0}+\lambda_{1})\overline{y}+\lambda_{1}(\overline{y}-k)\underline{y}-b(k\lambda_{0}+\lambda_{1}\overline{y}+\lambda_{1}\underline{y})]}\\ \rho^{u} & = & \frac{2c\lambda_{1}r(\underline{y}-b)}{\alpha(\overline{y}-b)[\lambda_{1}\phi(\underline{y}-b)-kr]}, \end{array}$$

Decomposition of EE Flows: ξ



Beveridge Curves

Steady state flow equations:

$$u = \frac{\delta}{\delta + m(\theta(\Omega))}$$

$$\gamma = \frac{\delta m(\theta(\Omega))}{[\delta + m(\theta(\Omega))][\delta + \lambda(\Omega)m(\theta(\Omega))]}.$$

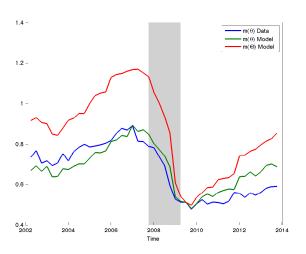
Beveridge Curves BC and BCs:

$$v = \frac{\delta u(1-u)[2\lambda(\Omega)(1-u)+u]}{\alpha[u(\delta+\phi)-\delta][\lambda(\Omega)(1-u)+u]}$$

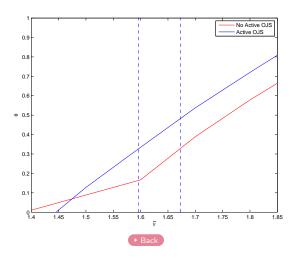
$$v = -\frac{(\delta s(2\delta(-1+s)+\phi(\lambda(-2+s)+s-\sqrt{\lambda^2(-2+s)^2+s^2-2\lambda s^2}))}{-2\alpha\delta(\delta+2\lambda\phi)+2\alpha(\delta+\phi)(\delta+\lambda\phi)s}$$

(BC

Jobless Recovery



Multiplicity Bounds: \overline{y} ($\underline{y} = 1$)

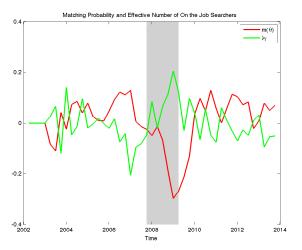


Decomposition of EE Flows: $EE = \lambda \gamma m(\theta)$

$$m(\theta) = \frac{UE}{u}$$
 and $\lambda \gamma = \frac{EE \cdot u}{UE}$

Decomposition of EE Flows: $EE = \lambda \gamma m(\theta)$

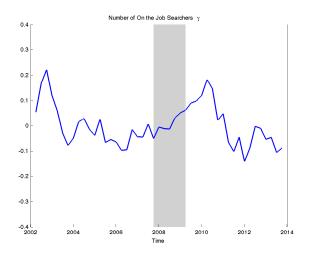
$$m(\theta) = \frac{UE}{u}$$
 and $\lambda \gamma = \frac{EE \cdot u}{UE}$



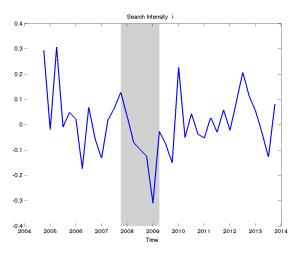
Decomposing $\lambda \gamma$

- ullet Problem: No direct measure of search intensity λ
- Use CPS micro-data panel structure
- Check whether individuals was unemployed before current job or transited from another job
- Construct γ (employed after UE transition) and ξ (after EE transition)
- Then, search intensity is computed as: $\lambda = \frac{EE}{m(\theta)\gamma}$

Decomposition of EE Flows: γ



Decomposition of EE Flows: $\lambda = \frac{EE}{m(\theta)\gamma}$



⇒ Pro-cyclical search intensity!

