Competing Teams

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SED
June, 2014

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  - R&D competition
  - Oligopoly
  - Auctions

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  - R&D competition
  - Oligopoly
  - Auctions
- Competing teams
  - Optimal and equilibrium matching
  - Inefficiency
  - Policy

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  - In our setup $\longrightarrow$ match output depends also on matching
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  - Oligopoly
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- Competing teams
  - Optimal and equilibrium matching
  - Inefficiency
  - Policy
- Related literature:
  - Small (to the best of our knowledge): Koopmans and Beckmann (1957); Sasaki and Toda (1996)

The Setup

Overview of the model:

- Large number of heterogeneous workers (and firms)
- Two stages:
  - Matching stage: Workers form teams of size two (or firms hire them) in a competitive labor market
  - Competition stage: Teams compete pairwise in output market
- Second stage induces matching with externalities in first stage
  - Match payoff of a team depends on composition of other teams
- Analysis of sorting patterns:
  - Planner v. Competitive Market
  - Wedge between them due to externalities

The Setup

- Continuum of agents
- Each has a characteristic ('type') $x \in \{\underline{x}, \overline{x}\}$, $\overline{x} > \underline{x}$
- Workers form teams of size 2
  - $\overline{X}$: team with two $\overline{x}$-type agents
  - $\underline{X}$: team with two $\underline{x}$-type agents
  - $\hat{X}$: team with one $\underline{x}$ and one $\overline{x}$-type agents
  - $\underline{X} < \hat{X} < \overline{X}$
- Transferable utility
- Matching μ partitions population in pairs:
  - PAM μ_+: half of the teams are $\overline{X}$ and half $\underline{X}$
  - NAM μ_-: all the teams are $\hat{X}$

The Setup

- Teams compete pairwise in downstream interaction (e.g., output market) against a randomly drawn team
  - $V(X_i|X_j)$: match output of team X_i when competing with X_j
  - V symmetric in components of X_i, and similarly in components of X_j
  - $\mathcal{V}(X_i|\mu_+) = \mathbb{E}_{\mu_+}[V(X_i|\tilde{X}_j)] = \frac{1}{2}V(X_i||\overline{X}) + \frac{1}{2}V(X_i|\underline{X})$
  - $\mathcal{V}(X_i|\mu_-) = \mathbb{E}_{\mu_-}[V(X_i|\tilde{X}_j)] = V(X_i|\hat{X})$

The Setup

An example of $V(X_i|X_j)$:

- Research: uncertainty about the exact outcome v_i
  1. Form R&D teams
  2. Draw uncertain research output v_i:
     - $v_i \in \{0, v\}$
     - probability to get v given team composition X_i: $p_i = p(X_i)$ (with $\overline{p} > \hat{p} > \underline{p}$)
  3. Winner takes all: $\max\{v_i, v_j\}$ (half in case of a tie)
- Expected payoff:

$$V(X_i|X_j) = p_i p_j \frac{v}{2} + p_i(1 - p_j)v = vp_i - \frac{v}{2} p_i p_j$$

$$\Rightarrow \quad \text{e.g. } V(\overline{X}|\underline{X}) = v\overline{p} - \frac{v}{2}\overline{p}\underline{p} \quad \text{and} \quad V(\overline{X}|\overline{X}) = v\overline{p} - \frac{v}{2}\overline{p}\overline{p}$$

$$\Rightarrow \quad \mathcal{V}(\overline{X}|\mu_+) = +\frac{1}{2}\left(v\overline{p} - \frac{v}{2}\overline{p}\underline{p}\right) + \frac{1}{2}\left(v\overline{p} - \frac{v}{2}\overline{p}^2\right)$$

The Setup

- **Planner:** Takes as given output market competition and chooses μ that maximizes sum of teams' outputs
  - PAM optimal if
$$\mathcal{V}(\overline{X}|\mu_+) + \mathcal{V}(\underline{X}|\mu_+) \geq 2\mathcal{V}(\hat{X}|\mu_-)$$
  - NAM optimal if
$$\mathcal{V}(\overline{X}|\mu_+) + \mathcal{V}(\underline{X}|\mu_+) \leq 2\mathcal{V}(\hat{X}|\mu_-)$$
  - Reduce to super or submodularity without externalities
$$\mathcal{V}(\overline{X}) + \mathcal{V}(\underline{X}) \quad \text{v.} \quad 2\mathcal{V}(\hat{X})$$

The Setup

- **Competitive Equilibrium:** Agents take market wages and matching as given when they choose partners
  - Textbook notion; large market assumption justifies belief that they do not affect the allocation
  - $(\underline{w}, \overline{w}, \mu)$ such that (i) each type maximizes his payoff given wages; and (ii) choices are consistent with μ (market clearing)
  - PAM if

$$\begin{aligned} \mathcal{V}(\overline{X}|\mu_+) - \overline{w} &\geq \mathcal{V}(\hat{X}|\mu_+) - \underline{w} \\ \mathcal{V}(\underline{X}|\mu_+) - \underline{w} &\geq \mathcal{V}(\hat{X}|\mu_+) - \overline{w} \end{aligned}$$

  - This implies $\mathcal{V}(\cdot|\mu_+)$ supermodular, or

$$\mathcal{V}(\overline{X}|\mu_+) + \mathcal{V}(\underline{X}|\mu_+) \geq 2\mathcal{V}(\hat{X}|\mu_+)$$

  - Wages given by $\overline{w} = 0.5\mathcal{V}(\overline{X}|\mu_+)$ and $\underline{w} = 0.5\mathcal{V}(\underline{X}|\mu_+)$
  - Analogous construction for NAM
  - Reduces to super or submodularity without externalities
  - Two interpretations: partnerships, firms hiring teams

Sorting and Inefficiency

Proposition

There is an equilibrium with PAM allocation while there is NAM in the planner's solution if and only if

(i) $\mathcal{V}(X|\mu_+)$ *supermodular in* X*;*

(ii) $\mathcal{V}(\overline{X}|\mu_+) + \mathcal{V}(\underline{X}|\mu_+) - 2\mathcal{V}(\hat{X}|\mu_+) \leq 2[\mathcal{V}(\hat{X}|\mu_-) - \mathcal{V}(\hat{X}|\mu_+)]$

- Intuition:
  - "Supermodularity" (modified)
  - *Differential* externality NAM outweighs "supermodularity"
- Conditions for uniqueness
- Similar conditions for NAM equilibrium, PAM planner
  - Replace (i) by submodular $\mathcal{V}(X|\mu_-)$; reverse inequality in (ii)

Sorting and Inefficiency

- Additively Separable Payoffs
  - $\mathcal{V}(X_i|\mu) = g(X_i) + h(\mu)$
  - $h(\mu_+) = \frac{1}{2}h(\overline{X}) + \frac{1}{2}h(\underline{X})$ and $h(\mu_-) = h(\hat{X})$
  - PAM (NAM) equilibrium and NAM (PAM) planner iff

$$g \text{ supermodular (submodular)}$$

$$g(\overline{X}) + g(\underline{X}) - 2g(\hat{X}) \leq (\geq) 2[h(\mu_-) - h(\mu_+)]$$

- Multiplicatively Separable Payoffs
  - $\mathcal{V}(X_i|\mu) = g(X_i)h(\mu)$
  - PAM (NAM) equilibrium and NAM (PAM) planner iff

$$g \text{ supermodular (submodular)}$$

$$g(\overline{X}) + g(\underline{X}) - 2g(\hat{X}) \leq (\geq) 2g(\hat{X})\frac{h(\mu_-) - h(\mu_+)}{h(\mu_+)}$$

  - Need h 'sufficiently submodular' in X

Sorting and Inefficiency

We can also provide sufficient conditions in terms of V:

- PAM equilibrium and NAM planner if
  - $V(X|\overline{X}) + V(X|\underline{X})$ supermodular in X
  - $V(X_i|X_j)$ supermodular in (X_i, X_j)
  - $V(X|X)$ concave in X
- NAM equilibrium and PAM planner if
  - $V(X|\hat{X})$ submodular in X
  - $V(X_i|X_j)$ submodular in (X_i, X_j)
  - $V(X|X)$ convex in X
- Interpretation of NAM equilibrium and PAM planner:
  - Competition 'strategic substitutes' $\Rightarrow$ V submodular in (X_i, X_j)
  - PAM planner (with convexity condition)
  - Submodular in $X_i \Rightarrow$ NAM equilibrium (firms do not internalize externalitities)

Uncertainty

- Many economic environments involve uncertainty
- Patent race between research teams; Knowledge spillovers; Auctions between competing teams; Sports competitions;...
- Important for estimation
- Set up:
  1. Team composition X_i: labor market competition
  2. Team generates stochastic product v_i, from $F(v_i|X_i)$
  3. Output market competition $z(v_i, v_j)$
- Expected output of team X_i:

$$V(X_i|X_j) = \int \int z(v_i, v_j) dF(v_i|X_i) dF(v_j|X_j)$$

Uncertainty

- The value is 'additively separable' as follows:

$$V(X_i|X_j) = g(X_i) + h(X_j) + k(X_i, X_j).$$

Proposition

Let $S_i = S(v|X_i) = 1 - F(v|X_i)$ denote the survival function. The expected value $V(X_i|X_j)$ can be written as

$$\underbrace{z(\underline{v},\underline{v}) + \int \frac{\partial z(v_i,\underline{v})}{\partial i} S_i dv_i + \int 2\frac{\partial z(\underline{v},v_j)}{\partial j} S_i dv_j}_{g(X_i)} + \underbrace{\int \frac{\partial z(\underline{v},v_j)}{\partial j} S_j dv_j}_{h(X_j)} + \underbrace{\int\int \frac{\partial^2 z}{\partial i \partial j} S_i S_j dv_i dv_j}_{k(X_i,X_j)}$$

- The expressions for $\mathcal{V}(\cdot|\mu_+)$ and $\mathcal{V}(\cdot|\mu_-)$ easily follow from V

COROLLARY

Let $z(v_i, v_j) = av_i + bv_j + cv_iv_j$ where a, b, c are constants and $\underline{v} = 0$. Then the value of the firm can be written as

$$V_i = (a + 2b)m(X_i) + bm(X_j) + cm(X_i)m(X_j),$$

where $m(X) = \mathbb{E}[v|X]$.

Uncertainty

Corollary

Let $z(v_i, v_j) = av_i + bv_j + cv_iv_j$ *where* a, b, c *are constants and* $\underline{v} = 0$. *Then the value of the firm can be written as*

$$V_i = (a + 2b)m(X_i) + bm(X_j) + cm(X_i)m(X_j),$$

where $m(X) = \mathbb{E}[v|X]$.

- From $\int S_i dvi = \int [1 - F(v|X_i)]dv = \mathbb{E}[\tilde{v}|X_i]$
- Value only depends only on mean
- It easily follows that

$$\begin{aligned}\mathcal{V}(X_i|\mu_+) &= (a + 2b)m(X_i) + \frac{1}{2}(b + cm(X_i))\left(m(\overline{X}) + m(\underline{X})\right) \\ \mathcal{V}(X_i|\mu_-) &= (a + 2b)m(X_i) + (b + cm(X_i))m(\hat{X})\end{aligned}$$

Economic Applications

I Spillovers

II Patent Race

III Auctions between Teams

IV Oligopolistic Competition

I. Spillovers

- Spillovers can be positive or negative
  - Positive: Development of a product by a firm helps another firm when developing a competing product
  - Negative: Development of a product by a firm adversely affects prospects of the other firm
- Assume $z(v_i, v_j) = v_0 + av_i + bv_j$, $a \geq 0$, $v_0 > 0$ large
- Assume $m(X) \geq 0$ for all X
- Then $V(X_i|X_j)$ is given by

$$V(X_i|X_j) = v_0 + (a + 2b)m(X_i) + bm(X_j)$$

I. Spillovers

Proposition

Let $z = v_0 + av_i + bv_j$, with $a \geq 0$.

1. *If $b \notin \left(-\frac{a}{3}, -\frac{a}{2}\right)$, the equilibrium allocation is efficient;*
2. *If $b \in \left(-\frac{a}{3}, -\frac{a}{2}\right)$, the equilibrium is inefficient: if m is supermodular (submodular), the equilibrium exhibits PAM (NAM), while the planner's solution exhibits NAM (PAM).*

- Positive spillovers always yield efficiency
  - Positive externality cannot offset private benefits
- Inefficiency can arise with negative spillovers
  - It occurs when b is in a range where private benefit parameter a is not large enough
  - Hence externality can dominate private benefit effect

I. Spillovers

- 'Romer-Lucas-like' setup
- Output: $A(\mu)g(X)$ where $A(\mu) = A(\sum g)$
- Inefficiency:
  - PAM equilibrium: $A(\overline{g} + \underline{g})(\overline{g} + \underline{g} - 2\hat{g}) > 0$
  - NAM planner: $A(\overline{g} + \underline{g})(\overline{g} + \underline{g}) < A(2\hat{g})2\hat{g}$

$\Rightarrow$ whenever g supermodular and $A(x)x$ is decreasing, or $A'(x) < -\frac{A(x)}{x}$

- Analogous conditions for PAM planner, NAM equilibrium

II. Patent Race

- Interesting application of negative spillovers
- Research: uncertainty about the exact outcome v_i
- A simple stochastic setting:
  1. Form teams X_i and X_j
  2. Draw uncertain research output v_i:
     - $v_i \in \{0, v\}$
     - probability to get v given X_i: $p_i = p(X_i)$ (with $\overline{p} > \hat{p} > \underline{p}$)
  3. Winner takes all: $\max\{v_i, v_j\}$
- Expected payoff:

$$V(X_i|X_j) = vp_i - \frac{v}{2}p_i p_j$$

- Planner maximizes $[1 - (1 - p_i)(1 - p_j)]v$

II. Patent Race

Proposition

Equilibrium is efficient. The allocation has PAM if p is supermodular, NAM if p is submodular.

- Depends on large market assumption
  - Random matching with opponents in a large market
  - External effect of meeting a high type team is negative
  - External effect of meeting a low type team is positive
  - These effects cancel out
- Inefficiency can arise in small markets (known opponent)

III. Auctions between Teams

- Team composition matters in auction: better estimates of value/cost of timber; make efficient use of bandwidth;...
- Uncertainty about outcomes: team-dependent
- Consider independent private values second price auction
- Order of events
  1. Teams are formed in a competitive labor market
  2. Valuation v_i from distribution of valuations $F(v_i|X_i)$
  3. Random pairwise matching of teams
  4. The two teams simultaneously submit their bids
- As usual, it is a dominant strategy for each bidder to submit a bid equal to the true valuation
- Large market with anonymous participants: e.g., eBay, telephone auctions, etc.

III. Auctions between Teams

- The value of an auction to team X_i when facing X_j is

$$V(X_i|X_j) = \int_{\underline{v}}^{\overline{v}} F(v|X_j)(1 - F(v|X_i)dv$$

III. Auctions between Teams

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$$V(X_i|X_j) = \int_{\underline{v}}^{\overline{v}} F(v|X_j)(1 - F(v|X_i)dv$$

- Follows from

$$\begin{aligned}
V_i &= \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \max\{v_i - v_j, 0\} dF(v_i|X_i) dF(v_j|X_j) \\
&= \int_{\underline{v}}^{\overline{v}} \left(1 - v_j F_i(v_j) - \int_{v_j}^{\overline{v}} F_i dv_i - v_j(1 - F_i(v_j) \right) dF_j \\
&= \int_{\underline{v}}^{\overline{v}} \left(\int_{v_j}^{\overline{v}} (1 - F_i) dv_i \right) dF_j = \int_{\underline{v}}^{\overline{v}} n(v_j|X_i) dF_j \\
&= n(v_j|X_i) F_j(v_j)|_{\underline{v}}^{\overline{v}} - \int_{\underline{v}}^{\overline{v}} F_j n'(v_j|X_i) dv_j = \int_{\underline{v}}^{\overline{v}} F_j(1 - F_i) dv_j
\end{aligned}$$

where $n(v_j|X_i) = \int_{v_j}^{\overline{v}} (1 - F_i) dv_i$

III. Auctions between Teams

- It easily follows from V that

PAM $$\mathcal{V}(X_i|\mu_+) = \int_{\underline{v}}^{\overline{v}} \frac{F(v|\overline{X}) + F(v|\underline{X})}{2}(1 - F(v|X_i))dv$$

NAM $$\mathcal{V}(X_i|\mu_-) = \int_{\underline{v}}^{\overline{v}} F(v|\hat{X})(1 - F(v|X_i))dv$$

III. Auctions between Teams

Proposition

The equilibrium allocation is PAM while planner's solution is NAM if F is submodular in X for each v and

$$\int_{\underline{v}}^{\overline{v}} \mathcal{F}(1-\mathcal{F}) \leq \int_{\underline{v}}^{\overline{v}} \hat{F}(1-\hat{F})$$

where $\mathcal{F} = \frac{\overline{F}+\underline{F}}{2}$.

- F submodular: PAM equilibrium
- The expected value of $F(1-F)$ under NAM dominates PAM
- $\int_{\underline{v}}^{\overline{v}} F(1-F)dv = \mathbb{E}_{F^2}[v|X] - \mathbb{E}[v|X]$ larger under NAM than PAM. For example: same mean but $\hat{F}$ has higher variance

IV. Oligopolistic Competition

- Cournot duopoly with linear demand $P = a - bQ$.

$$q_i = \frac{a - 2c_i + c_j}{3b} \quad \text{and} \quad V_i = \frac{(a - 2c_i + c_j)^2}{9b}$$

- Costs depend on team composition $c_i = c(X_i)$ with $\overline{c} < \hat{c} < \underline{c}$

Proposition

If c is supermodular, there is an interval of a, $\underline{x}$, and $\overline{x}$, such that the equilibrium is NAM while the planner is PAM. Equilibrium is efficient if c is submodular or the planner's allocation is NAM.

- Only inefficiency: planner PAM, equilibrium NAM.
  - This occurs when c is supermodular
  - Set of $\underline{x}$ and $\overline{x}$ limits extent of complementarities
  - Intermediate levels of a: if very low enough, externality not strong enough to overturn the NAM equilibrium; if very high profits and the planner's objective are aligned
- We have results for Bertrand and consumer surplus

Policy Implications

- Sports competitions: US vs. Europe
  - US: intervention for balanced competition: PAM $\rightarrow$ NAM
  - Europe: laissez-faire: PAM
- We use the model with negative spillovers $z_i = v_0 + av_i + bv_j$
- Need to calculate wages
- Effects of policies:
  1. Taxes
     - Suitable taxes for hiring same type changes PAM to NAM
  2. Salary Cap
     - Bound on wage of high type cannot change PAM to NAM
  3. Rookie Draft
     - Senior and rookie high and low types
     - Sequential hiring at set type dependent wages
     - Low type seniors choose first
     - Equilibrium with NAM
     - Both senior types prefer it to PAM

Variations

We check the robustness of the results along three dimensions:

- Continuum of types
  - Example with uniformly distributed types on the unit interval and supermodular V
  - Derive conditions for NAM planner/PAM equilibrium
- 'Mixed matching'
  - With externalities, planner may want to match a fraction α as PAM and $1 - \alpha$ as NAM
  - Not true without externalities
  - $\alpha = 1$ or 0 if planner's objective function is convex in α
  - We provide sufficient conditions, met in all of our applications
- Small markets
  - Analogous results for small number of agents
  - They take as given the allocation in a competitive equilibrium
  - Planner has similar conditions for PAM/NAM as well

Conclusion

- Assortative matching with externalities
  - Difficult problem in general (Koopmans and Beckmann (1957))
  - We analyze a tractable framework
- Competing Teams
  - Allocation problems with externalities/strategic interaction
  - If inefficient: discontinuous reallocation
- Complementarities in allocation problems:
  - Without externalities: correctly priced

  → no efficiency grounds for intervention
  - With externalities

  → role for intervention
- Extensions:
  - More than two types: Interesting mathematical problem
  - Stability and core

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