

# PRICES AS OPTIMAL COMPETITIVE SALES MECHANISMS

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# BROAD MOTIVATION

Matching

Bilateral  
Matching

## BROAD MOTIVATION

Meeting + Selection = Matching

Meeting  
Technology

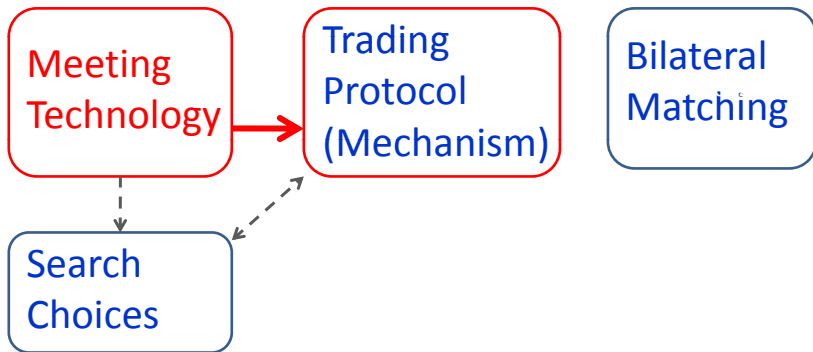
Trading  
Protocol  
(Mechanism)

Bilateral  
Matching

Search  
Choices

# BROAD MOTIVATION

Meeting + Selection = Matching



How are goods/labor sold *depending on the frictions*? (fixed prices/auctions/bargaining)

How is competing mechanism design *affected by the meeting process*?

# BROAD MOTIVATION

## MEETING FUNCTION EXAMPLE

### Example Meeting Technology:

- urnball application process
- N applications can be opened

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(no externality in meeting – ***no crowding out***)

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#### “Directed Search”:

Peters (1984, 1991, 1997a,b, ...);  
McAfee (1993),  
Burdett, Shi, Wright (2001);  
Shi (2002); Shimer (2005),...

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#### “Competitive Search”:

Shi (2001; Guerrieri (2008),  
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How does this affect the type  
of trading protocol (mechanism)?

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# THIS PAPER'S APPROACH

The approach in this paper:

- Lay out multilateral meeting function
- Specify mechanism space
- Analyze which mechanisms sellers use to attract buyers
  - homogeneous buyers
  - heterogeneous buyers with private values

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Focus on price posting (relative to auctions, bargaining...):

- When is price posting an equilibrium? When is it efficient?
- What is the relationship to random search?
- What is the relationship to the meeting technology?  
[Difference: "competitive" vs "directed" search]

# COMPETITION IN MECHANISMS

The game:

- 1 each buyer draw private value (if heterogeneous).
- 2 each seller posts mechanisms  $m$ .
- 3 each seller decides which mechanisms  $m$  to seek.
- 4 this gives buyer-seller ratios  $\lambda_i(m)$  at each mechanism.
- 5 meeting function: how many buyers of each type arrive at seller.
- 6 mechanisms are being played.

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Open questions even for standard urnball ( $N = \infty$ ):

- McAfee '93: auctions are always best reply, and strictly help under uncertainty about buyer types.
- Under price posting each seller only faces one buyer types (no uncertainty), and prices satisfy some planners problem.
- Are auctions only a weak best reply; are prices equally good?



## RESULTS

### Homogeneous Sellers:

- equivalence of many selling mechanisms  
(generalizes Camera and Selcuk 2009, justifies Kultti 1999)
- random search is socially efficient and equilibrium outcome

### Heterogeneous Sellers: Price Posting

- is constrained efficient if planner *can only use mechanisms that give the good away at random*
- is is constrained efficient and equilibrium outcome under bilateral matching (and under multilateral matching with strong crowding out, auctions still also weak best reply)
- is is not socially efficient and no equilibrium outcome under (but auctions are)

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### More search than mechanism design. Trade-offs:

- random search leads to most matches
- but crowding might make separation of types preferable

# COMPETITIVE PRICE POSTING

## Competitive Search / Directed Search

(Peters ('84,'91,'00,'05); Montgomery ('91); Moen ('97); Acemoglu & Shimer ('99a); Burdett, Shi & Wright ('01); Julien, Kennes & King ('00); Albrecht, Gautier & Vroman ('06); Galenianos & Kircher ('06); Shi ('07)...)

- competitive price setting with frictions
- alternative to random search
- good efficiency properties  
(Moen ('97); Acemoglu & Shimer ('99b); Shi ('01,'02); Mortensen & Wright ('03); Shimer ('05); Kircher ('06), Moen & Rozen ('06)...)

Question: When can we restrict attention to price posting?

- When is price posting an equilibrium? When is it efficient?
- What is the relationship to random search?
- What is the relationship to the meeting technology?  
[Difference: "competitive" vs "directed" search]

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Large measure of (homogeneous) risk-neutral sellers

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The market interaction:

- Sellers post prices
  - Seller's max. problem when buyers can get utility  $U$  elsewhere

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general  
questions:

1. Is price posting as efficient as other mechanisms?
2. Is price posting an equilibrium?
3. Is efficiency achievable by random search?

Importance: Can we restrict attention to “simple” mechanisms?  
What is the role of the meeting technology?

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heterogeneous buyers  
(high and low valuations):

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(screens perfectly between buyers)

(efficient in class of non-discriminatory mech.)



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a) efficient, equivalent to other mech.  
Not b) random search not efficient



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Not a) rather: auctions are efficient  
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What we know from competing mechanism design under "urn-ball" matching: (McAffee ('93), Peters ('97, '99), Peters & Severinov ('97))

- second price auctions are always a best reply
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On the other hand:

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Deeper Question:

- Are auctions only a weak best reply?
- Are prices also a best reply, as they "screen" ex ante?

# OUTLINE

- The price model
- The meeting function
- The general mechanism model

## 1 Price posting

- ① homogeneous buyers
- ② heterogeneous buyers

## 2 General Mechanisms

- ① "Directed Search" (multilateral matching, no crowding out)
- ② "Competitive Search" (bilateral matching, crowding out)  
Comment: multilateral matching with crowding out

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  - Seller's maximization problem

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An equilibrium consists of distributions of trading strategies, buyer-seller ratios and buyer utilities such that

- 1 **Seller Optimality:** sellers solve their maximization problem.
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- 3 **Market Clearing:** buyer-seller ratio arises from trading.

For any measurable subset  $\mathcal{P}$  of prices:

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For definition with arbitrary anonymous mechanisms:

- A mechanism describes expected payoff for low buyer
- Similar for high buyer type and for seller.
- Has to obey **resource constraint** and **incentive compatibility**

## THE MEETING FUNCTION

- $\lambda$ : expected number of buyers
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  - 2 Kiyotaki-Wright matching:  $P_1(\lambda) = \alpha \lambda / (1 + \lambda) = 1 - P_0(\lambda)$
- All types have equal meeting chances in a market.

# PRICE POSTING AND MECHANISMS

## 1.1) HOMOGENEOUS BUYERS

### PROPOSITION (PRICE POSTING W/ HOMOG. BUYERS )

*Under price posting, in equilibrium one price is offered, buyers select at random, and randomness is efficient*

Def.: A class of mechanisms is *payoff complete* if it has some dimension (like the reserve price in an auction) to shift payoffs between buyers and sellers.

### PROPOSITION (EQUIVALENCE )

*In any class of pay-off complete (full-trade) mechanisms*

- *an equilibrium mechanism exists*
- *remains equilibrium mech. when other mech. are added*
- *equilibrium payoffs are identical as under price posting*
- *search is (essentially) random.*

Second price auctions:  $r^* = 1 + \lambda^* P'_0(\lambda^*)/P_1(\lambda^*)$

# PRICE POSTING

## 1.2) HETEROGENEOUS BUYERS

### PROPOSITION (PRICE POSTING W/ HETEROG. BUYERS )

*Price Posting leads in equilibrium to*

- two prices, one for each type*
- buyers separate by "voting with their feet"*
- constrained efficient given frictions and within the class of non-discriminatory mechanisms (Hosios' Condition).*

Sketch of separation argument:

- low types want low price more than good matching probability
- single crossing property
- pricing effectively separates the types.

# MECHANISM POSTING

## 2.1) "DIRECTED" (MULTILATERAL MATCHING - NO CROWDING OUT)

### DEFINITION (NO CROWDING OUT)

The meeting technology exhibits "no crowding out" if the meeting probability for one buyer type is independent of choices by the other.

### PROPOSITION (MECHANISM POSTING)

- *Identical auctions are more efficient than price posting*
- *Price posting is not an equ. when auction are available.*

Sketch of Proof:

- Random search yields most matches [ $1 - P_0$  concave]
- More matches with identical auctions than w/ price posting
- High types choose randomly and get the object first
- $\Rightarrow$  most matches for high types.
- Most matches & most matches for high types  $\Rightarrow$  efficiency.
- Individual deviation to auction mechanisms is profitable.

# MECHANISM POSTING

## 2.2.) "COMPETITIVE" (BILATERAL MATCHING - CROWDING OUT)

Bilateral matching has "crowding out":  $Q_0(\underline{\lambda} + \bar{\lambda})$  increases in  $\underline{\lambda}$ .

$[1 - Q_0(\lambda) = Q_1(\lambda) = P_1(\lambda)/\lambda = (1 - P_0(\lambda))/\lambda, P'_1 < 0.]$

### PROPOSITION (MECHANISM POSTING)

*Under bilateral matching*

- *Price posting is constrained efficient.*
- *Price posting is an equilibrium.*
- *Random search is never constrained efficient.*

Sketch of Proof:

- The presence of low types "crowds out" high types.
- Sellers never see high types when a low type is present.
- All "selection" before the seller can intervene.
- Best not to mix types.
- Under separation: prices do a good job.

# MECHANISM POSTING

## 2.2) COMMENT: MULTILATERAL MATCHING - CROWDING OUT

Multilateral Matching w/o Crowding Out and Bilateral Matching are extremes.

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- a seller can only show up to  $N$  buyers the good (his house/car...)
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$N = 1$ : Bilateral Meetings

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Surplus under separate markets:  $S^{sep}(\underline{b}, \bar{b})$

Surplus under separate markets:  $S^{joint}(\underline{b}, \bar{b})$ .

Conjecture:

- $S^{sep}(\underline{b}, \bar{b}) > S^{joint}(\underline{b}, \bar{b}) \forall (\underline{b}, \bar{b})$  : Price Posting optimal and equilibrium
- $S^{sep}(\underline{b}, \bar{b}) < S^{joint}(\underline{b}, \bar{b}) \forall (\underline{b}, \bar{b})$  : Auctions optimal and equilibrium
- Otherwise: Possibly “partial pooling”

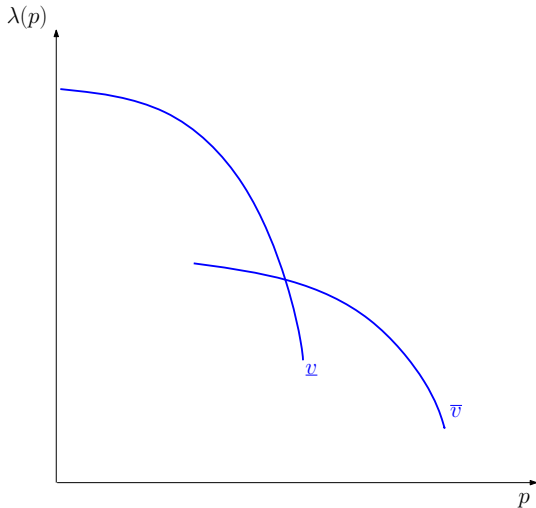
# CONCLUSION

- “Competitive” (Homog. agents or bilateral matching):
  - Prices are constrained efficient  
(other mechanisms only replicate the pricing outcome)
  - Random search is not efficient under buyer heterogeneity.
- “Directed” (multilateral matching w/o crowding out):
  - Prices are not constrained efficient  
(when discriminatory mechanisms are available).
  - Random search is efficient  
(when discriminatory mechanisms are available – Caveat: only when sellers are homogeneous).
- Larger relevance:
  - Clarifies when prices do a "good job".
  - Shows relevance of the specific meeting technology.
  - Highlights when we can focus on one buyer type (even under additional problems such as moral hazard ect.)

# PRICE POSTING

## 1.2) SINGLE CROSSING - SEPARATION "BY FEET"

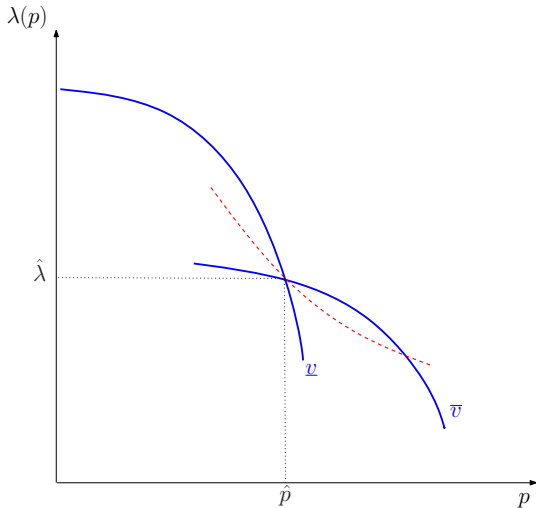
Buyer's indifference curves



# PRICE POSTING

## 1.2) SINGLE CROSSING - SEPARATION "BY FEET"

Iso-profit curve at a single market price



# PRICE POSTING

## 1.2) SINGLE CROSSING - SEPARATION "BY FEET"

Equilibrium with two prices

