Spatial Sorting

Jan Eeckhout (University College London and GSE-UPF)
Roberto Pinheiro (Colorado)
Kurt Schmidheiny (Basel)

University of Mannheim
April 16, 2013

complementarities

complementarities

complementarities

synergies

(knowledge) spillovers
complementarities
synergies

(knowledge) spillovers

complementarities

synergies

substitutes

supermodularity

(knowledge) spillovers

complementarities

synergies

substitutes

supermodularity
(knowledge) spillovers
complementarities
synergies
assortative matching substitutes

supermodularity

(knowledge) spillovers

complementarities

synergies

assortative matching

substitutes

choice of partner

large groups

supermodularity

(knowledge) spillovers

complementarities

synergies

assortative matching

substitutes

choice of partner

large groups

supermodularity
(knowledge) spillovers
complementarities
synergies
assortative matching
substitutes
choice of partner

firms, teams, class rooms

large groups

supermodularity

(knowledge) spillovers

complementarities

synergies

assortative matching

substitutes

choice of partner

firms, teams, class rooms

peer effects

large groups

supermodularity

(knowledge) spillovers

complementarities

synergies

assortative matching

substitutes

choice of partner

firms, teams, class rooms

peer effects

large groups **Sorting**

supermodularity

(knowledge) spillovers

complementarities

synergies

assortative matching substitutes

choice of partner

firms, teams, class rooms

peer effects

large groups **Sorting**

Who is on which team?

supermodularity

(knowledge) spillovers

complementarities

synergies

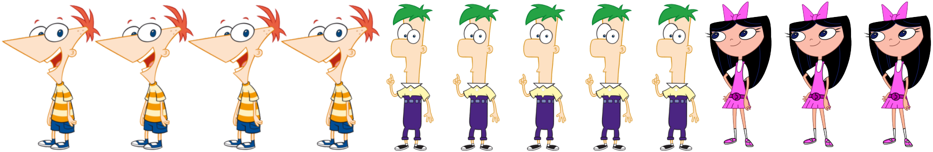
assortative matching

choice of partner

substitutes

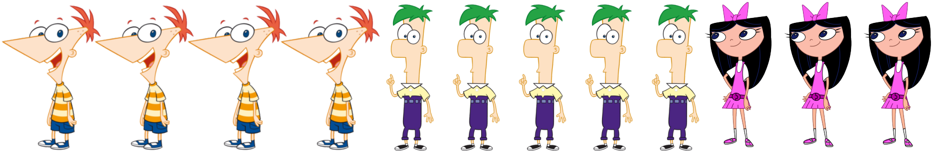
Sorting

Who is in which team?



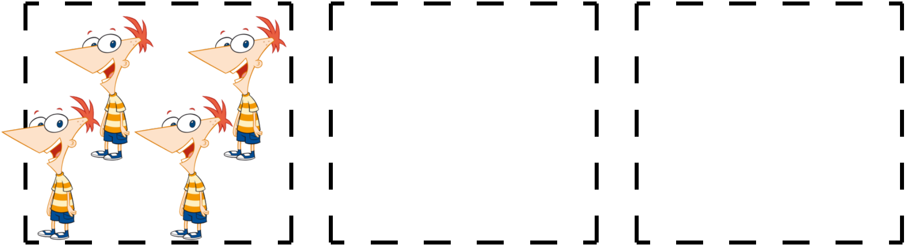
Sorting

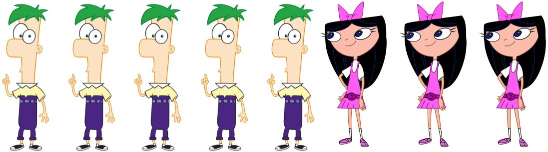
Who is in which team?



Sorting

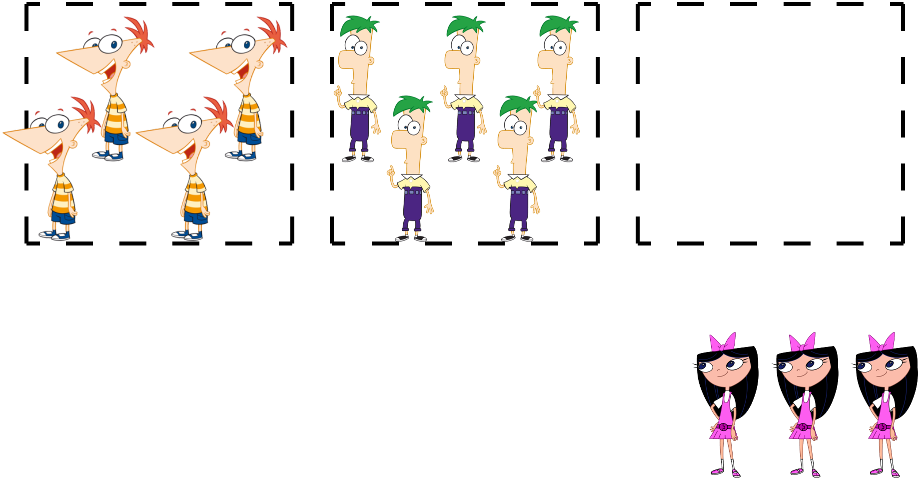
Who is in which team?





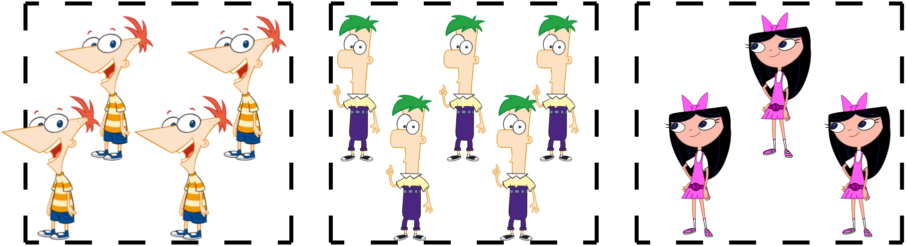
Sorting

Who is in which team?



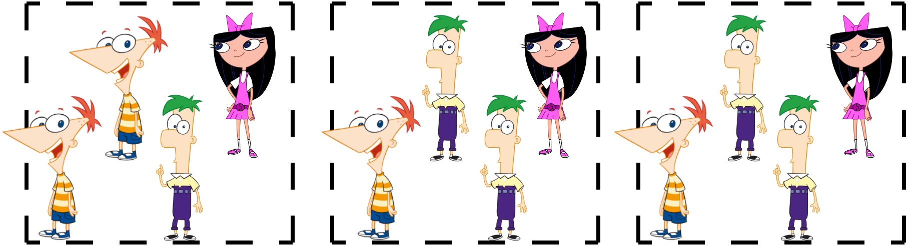
Sorting

Who is in which team?



Sorting

Who is in which team?



City as a team

Sorting across Space

- From team to city:
  - Agg. technology w/ *specific* complementarities
  - Additional economic forces: housing prices

City as a team

Sorting across Space

- From team to city:
  - Agg. technology w/ *specific* complementarities
  - Additional economic forces: housing prices
- Objective: derive compl. from location choice

$\Rightarrow$ Spatial Sorting

City as a team

Sorting across Space

The model

- J locations (cities) $j \in \mathcal{J} = \{1, ..., J\}$
- Fixed amount of land (housing) H_j

Citizens

- Citizens (workers) with heterogenous skills x_i
- Preferences over consumption and housing (price p):

$$u(c, h) = c^{1-\alpha} h^{\alpha}$$

- Worker mobility $\Rightarrow$ utility equalization across cities:

$$u(c_{ij}, h_{ij}) = u(c_{ij'}, h_{ij'}), \quad \forall j' \neq j$$

Technology

- Cities differ exogenously in TFP A_j
- Representative firm in city j produces

$$A_j F(m_{1j}, ..., m_{Ij})$$

  m_{ij}: employment level of skill i; given wages w_{ij}
- Nested CES $\sim$ Krusell-Violante-Ohanian-Rios (2000)

Technology: Nested CES

3 skill types ⇒ 5 configurations

0. Benchmark CES:

$$A_j F = A_j \left(m_{1j}^{\gamma} y_1 + m_{2j}^{\gamma} y_2 + m_{3j}^{\gamma} y_3 \right)^{\beta} \;\; \gamma \in [0,1], \beta > 0$$

Technology: Nested CES

3 skill types ⇒ 5 configurations

0. Benchmark CES:

$$A_j F = A_j \left(m_{1j}^{\gamma} y_1 + m_{2j}^{\gamma} y_2 + m_{3j}^{\gamma} y_3 \right)^{\beta} \;\; \gamma \in [0,1], \beta > 0$$

1. Extreme-Skill Complementarity

$$A_j F = A_j \left[m_{2j}^{\gamma} y_2 + \left({m_{1j}}^{\gamma} y_1 + {m_{3j}}^{\gamma} y_3 \right)^{\lambda} \right]^{\beta}$$

   A. $\lambda > 1$: skills 1 and 3 are (relative) complements;
   B. $\lambda < 1$: skills 1 and 3 are (relative) substitutes;
   C. $\lambda = 1$: CES

Technology: Nested CES

3 skill types ⇒ 5 configurations

0. Benchmark CES:

$$A_j F = A_j \left(m_{1j}^{\gamma} y_1 + m_{2j}^{\gamma} y_2 + m_{3j}^{\gamma} y_3 \right)^{\beta} \quad \gamma \in [0,1], \beta > 0$$

1. Extreme-Skill Complementarity

$$A_j F = A_j \left[m_{2j}^{\gamma} y_2 + ({m_{1j}}^{\gamma} y_1 + {m_{3j}}^{\gamma} y_3)^{\lambda} \right]^{\beta}$$

   A. $\lambda > 1$: skills 1 and 3 are (relative) complements;
   B. $\lambda < 1$: skills 1 and 3 are (relative) substitutes;
   C. $\lambda = 1$: CES

2. Top-Skill Complementarity

$$A_j F = A_j \left[m_{1j}^{\gamma} y_1 + ({m_{2j}}^{\gamma} y_2 + {m_{3j}}^{\gamma} y_3)^{\lambda} \right]^{\beta}$$

   A. $\lambda > 1$: skills 2 and 3 are (relative) complements;
   B. $\lambda < 1$: skills 2 and 3 are (relative) substitutes;
   C. $\lambda = 1$: CES

Technology: Nested CES

3 skill types ⇒ 5 configurations

0. Benchmark CES:

$$A_j F = A_j \left(m_{1j}^{\gamma} y_1 + m_{2j}^{\gamma} y_2 + m_{3j}^{\gamma} y_3 \right)^{\beta} \quad \gamma \in [0,1], \beta > 0$$

1. Extreme-Skill Complementarity

$$A_j F = A_j \left[m_{2j}^{\gamma} y_2 + \left(m_{1j}{}^{\gamma} y_1 + m_{3j}{}^{\gamma} y_3 \right)^{\lambda} \right]^{\beta}$$

   A. $\lambda > 1$: skills 1 and 3 are (relative) complements;
   B. $\lambda < 1$: skills 1 and 3 are (relative) substitutes;
   C. $\lambda = 1$: CES

2. Top-Skill Complementarity

$$A_j F = A_j \left[m_{1j}^{\gamma} y_1 + \left(m_{2j}{}^{\gamma} y_2 + m_{3j}{}^{\gamma} y_3 \right)^{\lambda} \right]^{\beta}$$

   A. $\lambda > 1$: skills 2 and 3 are (relative) complements;
   B. $\lambda < 1$: skills 2 and 3 are (relative) substitutes;
   C. $\lambda = 1$: CES

3. Bottom-Skill Complementarity: see 2.

Market clearing

- Housing market: $\sum_{i=1}^{I} h_{ij} m_{ij} = H_j$
- Labour market: $\sum_{j=1}^{J} m_{ij} = M_i \quad (M_i\text{: total \# of skill } i)$
- City population: $S_j = \sum_{i=1}^{I} m_{ij}$
- Two types of cities, C_1, C_2 of each type

Citizen's problem

- Optimal consumption

$$c_{ij}^{\star} = (1-\alpha) w_{ij} \quad \text{and} \quad h_{ij}^{\star} = \alpha \frac{w_{ij}}{p_j}$$

- Indirect utility function

$$U_i = \alpha^{\alpha} (1-\alpha)^{1-\alpha} \frac{w_{ij}}{p_j^{\alpha}}$$

$\Rightarrow$ From mobility, utility equalization:

$$\frac{w_{i1}}{p_1^{\alpha}} = \frac{w_{i2}}{p_2^{\alpha}}$$

Extreme-Skill Complementarity

Equilibrium conditions ($\beta = 1$)

$$\begin{aligned}
\lambda A_j \left[m_{1j}^{\gamma} y_1 + m_{3j}^{\gamma} y_3\right]^{\lambda-1} \gamma m_{1j}^{\gamma-1} y_1 - w_{1j} &= 0 \\
\gamma A_j m_{2j}^{\gamma-1} y_2 - w_{2j} &= 0 \\
\lambda A_j \left[m_{1j}^{\gamma} y_1 + m_{3j}^{\gamma} y_3\right]^{\lambda-1} \gamma m_{3j}^{\gamma-1} y_3 - w_{3j} &= 0
\end{aligned}$$

Extreme-Skill Complementarity

Equilibrium Demand ($\beta = 1$)

- Equilibrium demand for middle skills m_{21}:

$$m_{21} = \frac{\left[\left(\frac{p_1}{p_2}\right)^{\alpha} \frac{A_2}{A_1}\right]^{\frac{1}{\gamma-1}} \frac{M_2}{C_2}}{1 + \frac{C_1}{C_2}\left[\left(\frac{p_1}{p_2}\right)^{\alpha} \frac{A_2}{A_1}\right]^{\frac{1}{\gamma-1}}}$$

Extreme-Skill Complementarity

Equilibrium Demand ($\beta = 1$)

- Equilibrium demand for middle skills m_{21}:

$$m_{21} = \frac{\left[\left(\frac{p_1}{p_2}\right)^{\alpha} \frac{A_2}{A_1}\right]^{\frac{1}{\gamma-1}} \frac{M_2}{C_2}}{1 + \frac{C_1}{C_2}\left[\left(\frac{p_1}{p_2}\right)^{\alpha} \frac{A_2}{A_1}\right]^{\frac{1}{\gamma-1}}}$$

- and extreme skills

$$m_{11} = \frac{\left[\left(\frac{p_1}{p_2}\right)^{\alpha} \frac{A_2}{A_1}\right]^{\frac{1}{\lambda\gamma-1}} \frac{M_1}{C_2}}{1 + \frac{C_1}{C_2}\left[\left(\frac{p_1}{p_2}\right)^{\alpha} \frac{A_2}{A_1}\right]^{\frac{1}{\lambda\gamma-1}}}$$

likewise for m_{31}

Main Results

Theorem 1. City Size and TFP

The more productive city is larger, $S_1 > S_2$

Main Results

Theorem 1. City Size and TFP

The more productive city is larger, $S_1 > S_2$

Theorem 2. Extreme-Skill Complementarity and Fat Tails

The skill distribution in the larger city has fatter tails

Main Results

Theorem 1. City Size and TFP

The more productive city is larger, $S_1 > S_2$

Theorem 2. Extreme-Skill Complementarity and Fat Tails

The skill distribution in the larger city has fatter tails

$\rightarrow$ Mechanism: skill complementarity also in small cities, but demand for extreme skills is higher in big cities due to TFP (A_j)

Main Results

Corollary 1. CES technology

If $\lambda = 1$, then the skill distribution across cities is identical

Main Results

Corollary 1. CES technology

If $\lambda = 1$, then the skill distribution across cities is identical

Corollary 2. Extreme-Skill Substitutability and Thin Tails

The skill distribution in the larger city has thinner tails

Main Results

Corollary 1. CES technology

If $\lambda = 1$, then the skill distribution across cities is identical

Corollary 2. Extreme-Skill Substitutability and Thin Tails

The skill distribution in the larger city has thinner tails

Theorem 3. Top-Skill Complementarity and FOSD

The skill distribution in the larger city first-order stoch. dominates

Main Results

5 technologies → 5 distributions

1. Extreme-Skill Complementarity ⇒ fat tails
2. Extreme-Skill Substitutability ⇒ thin tails
3. Top-Skill Complementarity ⇒ FOSD of big city
4. Top-Skill Substitutability ⇒ FOSD of small city
5. Constant Elasticity (CES) ⇒ identical distributions

Empirical evidence

Empirical evidence

- Use theory to obtain a measure for skills

$$U_i = \alpha^{\alpha} (1-\alpha)^{1-\alpha} \frac{w_{ij}}{p_j^{\alpha}}$$

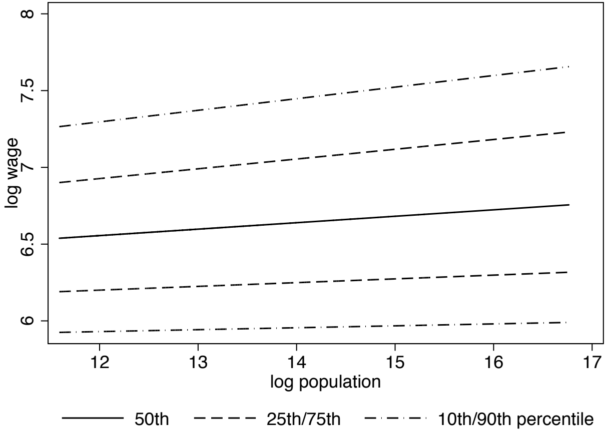
- Need to observe:
  - wage distribution w_{ij} by city
  - housing price level p_j
  - budget share of housing α
    $\widehat{\alpha} = 0.24$ from Davis and Ortalo-Magné (RED 2010)

Wages

CPS 2009

10th percentile: pop < 1m = 5.93, pop > 2.5m = 5.99, diff = 0.065*** (0.007)
90th percentile: pop < 1m = 7.36, pop > 2.5m = 7.56, diff = 0.198*** (0.007)

Wages and city size



10th percentile: slope = 0.013*** (0.002)
90th percentile: slope = 0.076*** (0.003)

Wages and city size

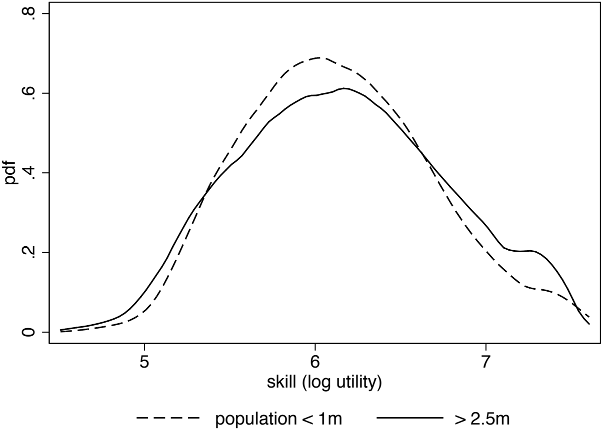
Housing prices

- American Community Survey (ACS) 2009
- Rental prices (robust: sales)

$\Rightarrow$ Hedonic price schedule: to obtain housing price index

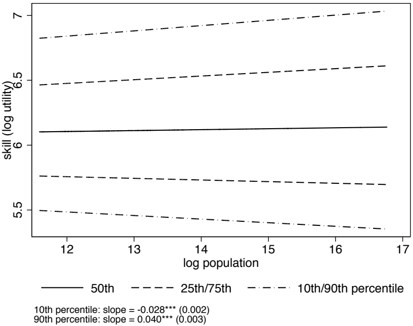
Skills and city size

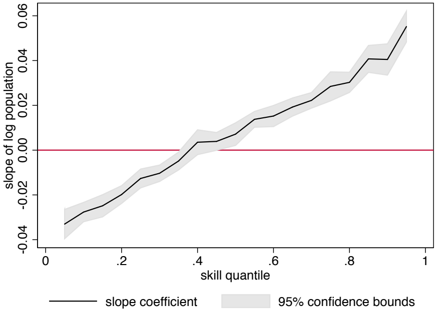
Skill measure: $\frac{w_i}{p_i^{\alpha}}$



10th percentile: pop < 1m = 5.44, pop > 2.5m = 5.36, diff = -0.074*** (0.006)
90th percentile: pop < 1m = 6.86, pop > 2.5m = 6.99, diff = 0.132*** (0.009)

Skills and city size





Skills and city size

1. Constant mean:

   housing cost increases 4 $\times$ faster than wages
   $\Rightarrow 1.169^{0.24} = 1.038 \approx 1.042$

2. Variance increases in city size

$\therefore$ Skill distribution has fat tails

Skills and city size

1. Constant mean:

   housing cost increases 4 $\times$ faster than wages
   $\Rightarrow 1.169^{0.24} = 1.038 \approx 1.042$

2. Variance increases in city size

$\therefore$ Skill distribution has fat tails $\rightarrow$ extreme-skill complementarity

$$A_j F = A_j \left[m_{2j}^{\gamma} y_2 + \left({m_{1j}}^{\gamma} y_1 + {m_{3j}}^{\gamma} y_3 \right)^{\lambda} \right]^{\beta}, \quad \lambda > 1$$

Skills and city size

1. Constant mean:

   housing cost increases 4 $\times$ faster than wages
   $\Rightarrow 1.169^{0.24} = 1.038 \approx 1.042$

2. Variance increases in city size

$\therefore$ Skill distribution has fat tails $\rightarrow$ extreme-skill complementarity

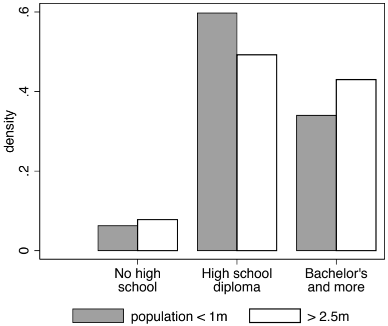
$$A_j F = A_j \left[m_{2j}^{\gamma} y_2 + ({m_{1j}}^{\gamma} y_1 + {m_{3j}}^{\gamma} y_3)^{\lambda} \right]^{\beta}, \quad \lambda > 1$$

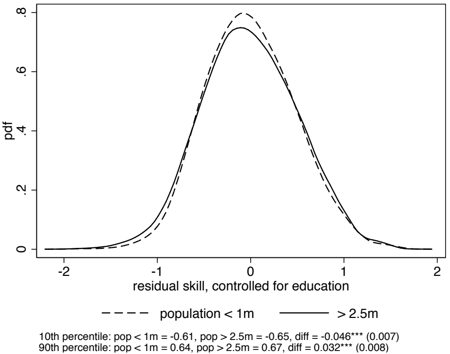
$\rightarrow$ Interpretation: high skilled workers need low-skilled services for production

- administrative/sales help
- household help and child care
- food services, restaurants,...

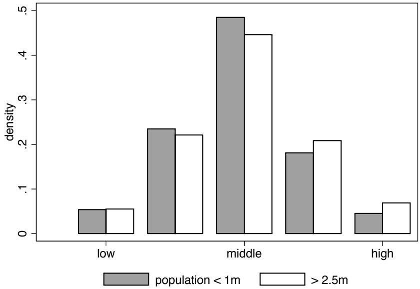
Robustness: Observables

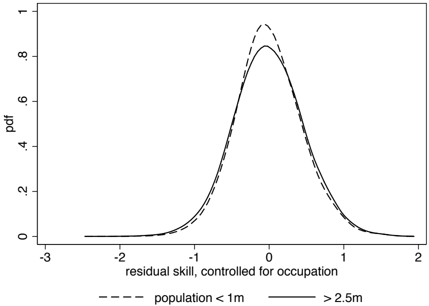
Education: A Direct Measure of Skill





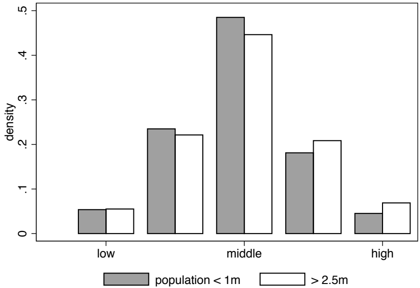
Occupation

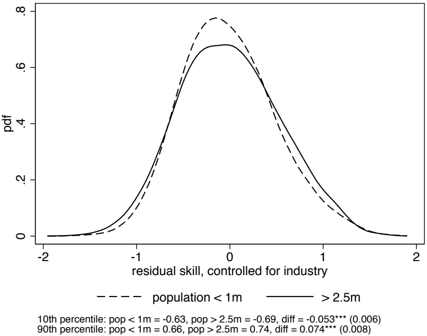




10th percentile: pop < 1m = -0.55, pop > 2.5m = -0.59, diff = -0.042*** (0.006)
90th percentile: pop < 1m = 0.56, pop > 2.5m = 0.60, diff = 0.040*** (0.007)

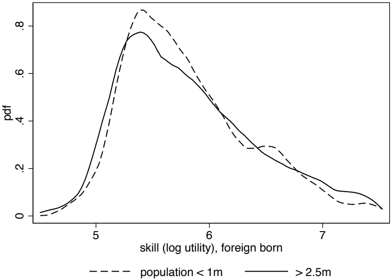
Industrial Composition





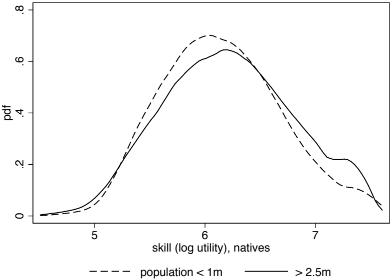
Migration

Foreign Born



10th percentile: pop < 1m = 5.23, pop > 2.5m = 5.14, diff = -0.085*** (0.017)
90th percentile: pop < 1m = 6.61, pop > 2.5m = 6.70, diff = 0.083** (0.046)

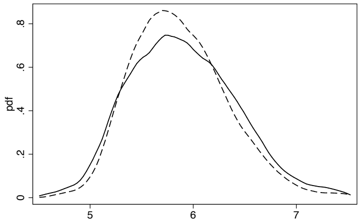
Natives



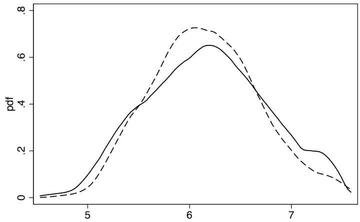
10th percentile: pop < 1m = 5.47, pop > 2.5m = 5.45, diff = -0.014** (0.007)
90th percentile: pop < 1m = 6.87, pop > 2.5m = 7.02, diff = 0.151*** (0.010)

Age

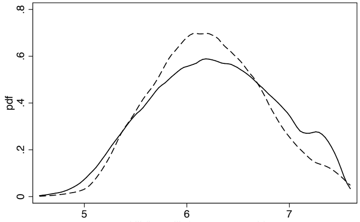
20-29 year old



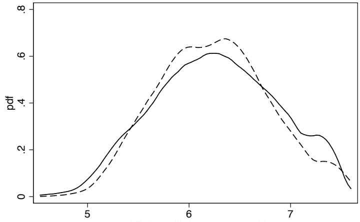
30-39 year old



40-49 year old



50-59 year old

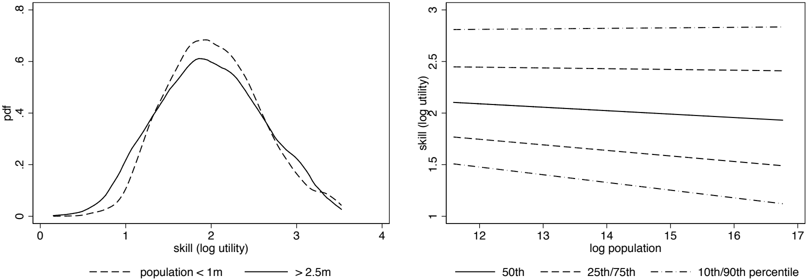


Decomposing the skill distributions

Small vs. big cities

	10% Quantile			90% Quantile		
Observed Quantiles:						
- Large cities	5.365	(0.004)	***	6.994	(0.006)	***
- Small cities	5.439	(0.005)	***	6.862	(0.007)	***
- Difference	-0.074	(0.006)	***	0.132	(0.009)	***
Firpo, Fortin, Lemieux (2009)						
Predicted Quantiles:						
- Large cities	5.387	(0.005)	***	7.022	(0.005)	***
- Small cities	5.454	(0.004)	***	6.878	(0.008)	***
- Difference	-0.068	(0.007)	***	0.144	(0.009)	***
Explained by observables:						
- Education (16 categories)	0.003	(0.002)	**	0.052	(0.002)	***
- Occupation (22 categories)	0.004	(0.002)	*	0.025	(0.003)	***
- Industry (51 categories)	-0.001	(0.002)		0.013	(0.002)	***
- Race (4 groups)	-0.004	(0.001)	***	-0.015	(0.001)	***
- Sex	-0.001	(0.001)	*	-0.002	(0.001)	*
- Foreign born	-0.020	(0.002)	***	-0.004	(0.001)	***
- Age (2nd order polynomial)	0.000	(0.001)		-0.002	(0.001)	*
Total explained by observables	-0.018	(0.004)	***	0.067	(0.005)	***
Not explained by observables	-0.049	(0.006)	***	0.077	(0.008)	***
Chernozhukov, Fernández-Val, Melly (2012)						
Predicted Quantile difference	-0.068	(0.006)		0.113	(0.009)	
Explained by observables	-0.019	(0.004)		0.064	(0.005)	
Not explained by observables	-0.050	(0.007)		0.049	(0.007)	

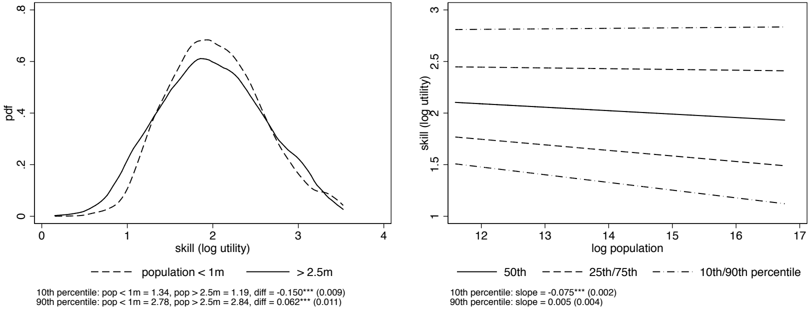
Variation in *all* consumption prices



10th percentile: pop < 1m = 1.34, pop > 2.5m = 1.19, diff = -0.150*** (0.009)
90th percentile: pop < 1m = 2.78, pop > 2.5m = 2.84, diff = 0.062*** (0.011)

10th percentile: slope = -0.075*** (0.002)
90th percentile: slope = 0.005 (0.004)

Variation in *all* consumption prices



- Prices for grocery items (sausage), housing (rent), utilities (phone call), transportation (gasoline), health care (Lipitor) and services (haircut).
- Does not correct (enough) for quality differences

→ Likely to overstate price differentials

⇒ We see above figure as *upper bound*

Discussion

- Sorting within Cities
- Non-linear Engel Curves
- Quantifying Production Technology

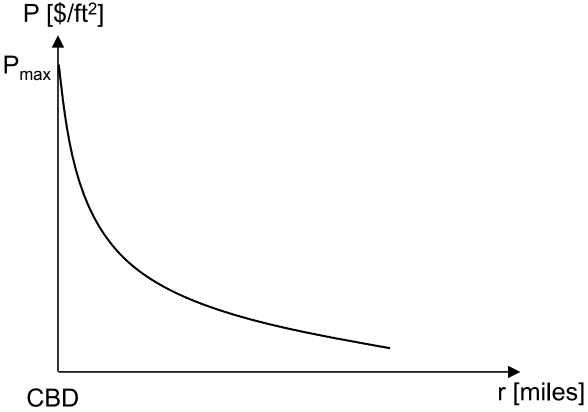
Sorting within Cities

What is the relevant housing price?

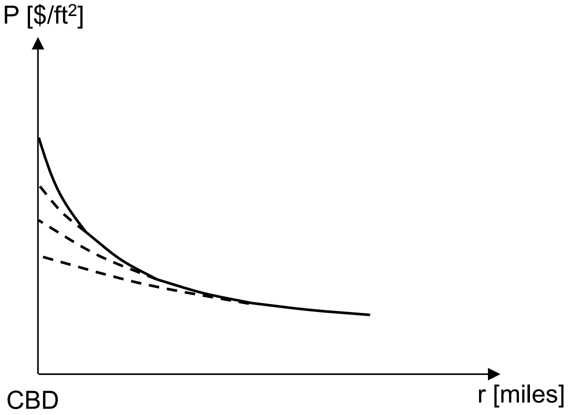
Sorting within Cities

What is the relevant housing price?

Monocentric city without sorting



Monocentric city with sorting

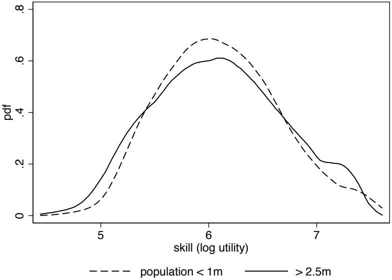


Sorting within Cities

Utility based on highest price in CBSA

Sorting within Cities

Utility based on highest price in CBSA

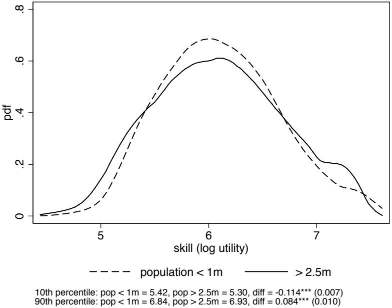


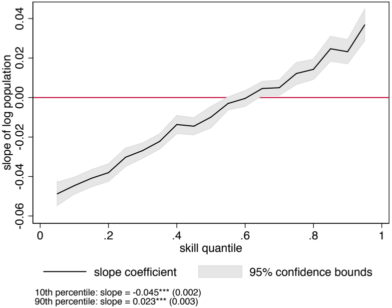
10th percentile: pop < 1m = 5.42, pop > 2.5m = 5.30, diff = -0.114*** (0.007)
90th percentile: pop < 1m = 6.84, pop > 2.5m = 6.93, diff = 0.084*** (0.010)

10th percentile: slope = -0.045*** (0.002)
90th percentile: slope = 0.023*** (0.003)

Sorting within Cities

Utility based on highest price in CBSA

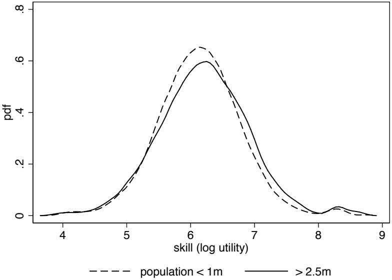




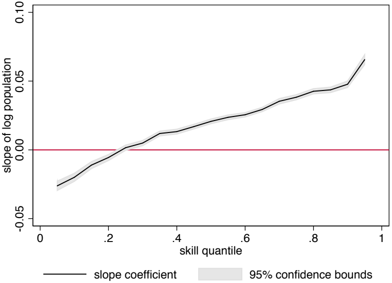
- Upper bound of relevant price

Sorting within Cities

Utility based on price of neighbourhood (PUMA)



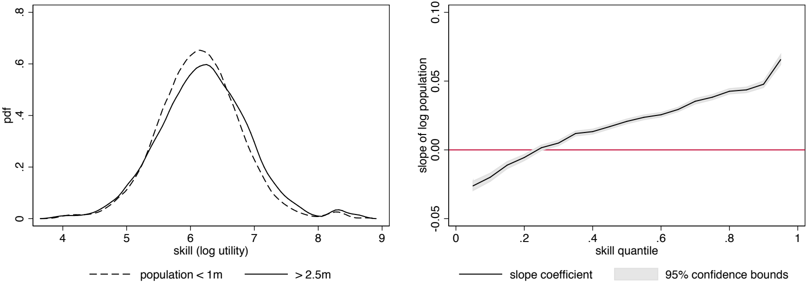
10th percentile: pop < 1m = 5.35, pop > 2.5m = 5.33, diff = -0.026*** (0.003)
90th percentile: pop < 1m = 6.92, pop > 2.5m = 7.06, diff = 0.141*** (0.004)



10th percentile: slope = -0.020*** (0.001)
90th percentile: slope = 0.048*** (0.001)

Sorting within Cities

Utility based on price of neighbourhood (PUMA)



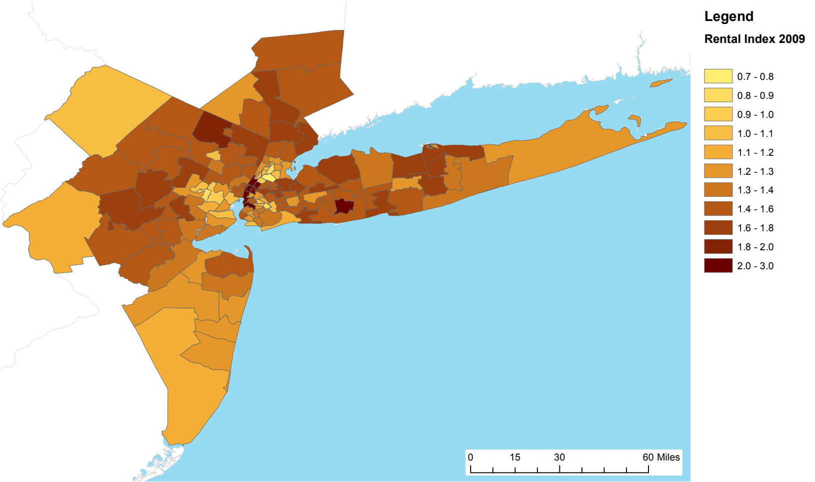
10th percentile: pop < 1m = 5.35, pop > 2.5m = 5.33, diff = -0.026*** (0.003)
90th percentile: pop < 1m = 6.92, pop > 2.5m = 7.06, diff = 0.141*** (0.004)

10th percentile: slope = -0.020*** (0.001)
90th percentile: slope = 0.048*** (0.001)

- Lower bound of relevant price

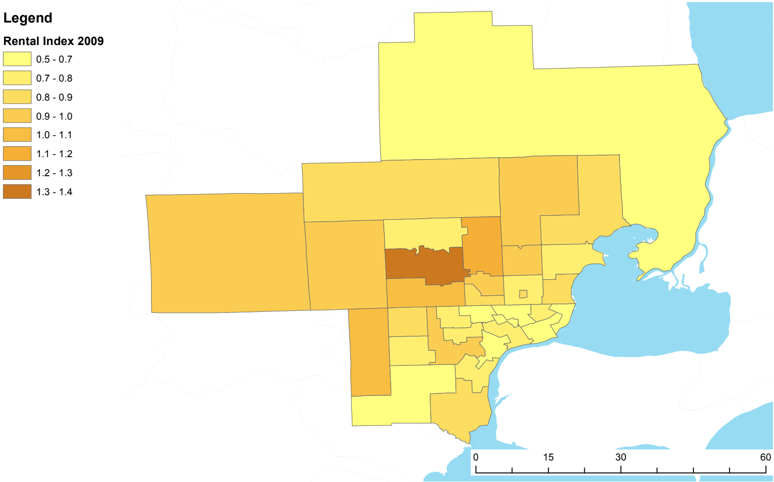
Sorting within Cities

New York City



Sorting within Cities

Detroit



Non-linear Engel Curves

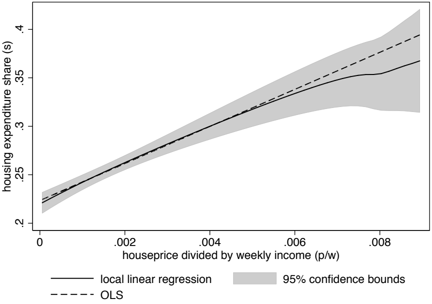
- Stone-Geary utility function

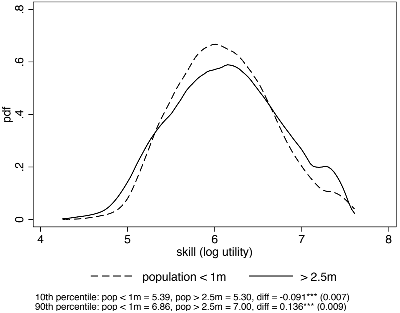
$$u(c,h) = c^{1-\alpha}(h - \underline{h})^{\alpha} \quad \Rightarrow \quad \frac{ph^\star}{w} = \alpha + (1-\alpha)h^\star \frac{p}{w}$$

- Using CEX, estimate $\widehat{\underline{h}} = \widehat{\beta}/(1-\widehat{\alpha})$ from

$$s_i = \alpha + \beta \frac{p_j}{w_i} + \varepsilon_i$$

Non-linear Engel Curves





$\widehat{\alpha} = 0.224$ (s.e.$= 0.005$), $\widehat{\underline{h}} = 27.7$ (3.8)

Quantifiying Production Technology

$$\lambda = \frac{1}{\gamma}\left[1 + \frac{(\gamma-1)\log\left(\frac{C_2 m_{21}}{M_2 - C_1 m_{21}}\right)}{\log\left(\frac{C_2 m_{11}}{M_1 - C_1 m_{11}}\right)}\right]$$

$$A_1 = \frac{w_{21}}{\gamma y_2 m_{21}^{\gamma-1}}, \quad A_2 = A_1 \left(\frac{p_2}{p_1}\right)^{\alpha} \left(\frac{C_2 m_{21}}{M_2 - C_1 m_{21}}\right)^{\gamma-1}$$

$$y_1 = \left(\frac{w_{11}}{\lambda\gamma A_1 \left[m_{11} + m_{31}\frac{w_{31}}{w_{11}}\right]^{\lambda-1} m_{11}^{\lambda(\gamma-1)}}\right)^{\frac{1}{\lambda}}$$

$$y_3 = \left(\frac{w_{31}}{\lambda\gamma A_1 \left[m_{31} + m_{11}\frac{w_{11}}{w_{31}}\right]^{\lambda-1} m_{31}^{\lambda(\gamma-1)}}\right)^{\frac{1}{\lambda}}$$

Quantifiying Production Technology

Observed model outcomes:

city j	w_{1j}	w_{2j}	w_{3j}	m_{1j}	m_{2j}	m_{3j}	C_j
1	416	844	1923	730,509	1,953,303	730,509	21
2	354	717	1634	30,900	105,516	30,900	204

Implied production technology for different values of γ:

γ	λ	A_1	A_2	y_1	y_2	y_3
0.655	1.0407	190,228	59,107	0.2329	1	1.0762
0.8	1.0193	19,118	9,065	0.3189	1	1.4733
0.9	1.0086	3,992	2,534	0.3964	1	1.8317

Open Question

City-specific Optimal Taxation

- Progressive tax: affects worker of same skill more in big city
- Average tax rate: 3% points difference *at median*:

Open Question

City-specific Optimal Taxation

- Progressive tax: affects worker of same skill more in big city
- Average tax rate: 3% points difference *at median*:

	Population	Wage level	Avg. Tax Rate
New York	19 million	1.20	16%
Janesville, WI	160,000	1.00	13%

Open Question

City-specific Optimal Taxation

- Progressive tax: affects worker of same skill more in big city
- Average tax rate: 3% points difference *at median*:

	Population	Wage level	Avg. Tax Rate
New York	19 million	1.20	16%
Janesville, WI	160,000	1.00	13%

- Due to mobility: no redistribution! Same skills, same utility
- Policy: city-specific progressive tax: adjust for city-level wages
  - Net wages in large cities $\uparrow$
  - Move from small cities to large cities: average city size $\uparrow$
  - GDP and Utility $\uparrow$ everywhere

Spatial Sorting

Jan Eeckhout (University College London and GSE-UPF)
Roberto Pinheiro (Colorado)
Kurt Schmidheiny (Basel)

University of Mannheim
April 16, 2013