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The Effect of Wealth on Worker Productivity

Jan Eeckhout
UPF Barcelona, BSE-ICREA-CREI, Spain

and

Alireza Sepahsalari
University of Bristol, UK

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We propose a theory that analyzes how a workers' asset holdings affect their job productivity. In a labor market with uninsurable risk, workers choose to direct their job search trading off productivity and wages against unemployment risk. Workers with low asset holdings have a *precautionary job search motive*, they direct their search to low productivity jobs because those offer a low risk at the cost of low productivity and a low wage. Our main theoretical contribution shows that the presence of consumption smoothing can reconcile the directed search model with negative duration-dependence on wages, a robust empirical regularity that the canonical directed search model cannot rationalize. We calibrate the infinite horizon economy and find this mechanism to be quantitatively important. We evaluate a tax financed unemployment insurance (UI) scheme and analyze how it affects welfare. Aggregate welfare is inverted U-shaped in benefits: the insurance effect UI dominates the incentive effects for low levels of benefits and vice versa for high benefits. In addition, when UI increases, total production falls in the economy while worker productivity increases.

Key words: Unemployment risk, Precautionary savings, Precautionary job search, Sorting, Unemployment insurance, Directed search, Duration dependence

JEL codes: C6, E2

1. INTRODUCTION

Unemployment is arguably the biggest risk workers face in their lifetime. Even without a perfect insurance market, workers nonetheless self-insure. They accumulate assets while employed, which they consume when unemployed. But workers with few assets can also use the labor market to self-insure by directing their job search toward lower-productivity jobs that are easier to find. These two self-insurance strategies of wealth accumulation and directed search interact. We ask the following basic questions: what is the role of wealth in determining worker productivity and how does it affect unemployment? We then study the welfare implications of government mandated Unemployment Insurance (UI) benefits. In addition to the tension between the unemployed who receive UI benefits and the employed who pay for them through taxes, we find there is now also a tension within the pool of unemployed workers. We show that the unemployed

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with low asset levels benefit considerably more from a UI increase than the rich unemployed. We also evaluate the effect that such benefits have on total productivity.

We model the worker's savings and job search decision in a labor market where workers can direct their search towards jobs of different levels of productivity, with firms posting wages to attract applicants. The worker's incentives thus trade off wages and job productivity against the probability of finding a job. Asset holdings crucially affect this tradeoff because each worker faces less exposure to the consumption risk inherent in joblessness. In addition to the standard *precautionary savings motive* with asset-contingent consumption smoothing à la Bewley-Huggett-Aiyagari, workers now also counter unemployment risk by directing their search to jobs with a high matching probability and low productivity, call it a *precautionary job search motive*. Sorting workers with different asset holdings into different productivity jobs proves essential to our analysis. Our main objective is to analyze how the inequality inherent in a labor market with heterogeneous productivity jobs interacts with the inequality that results from asset accumulation, *i.e.* how the two precautionary motives interplay and affect jobs productivity.

Our paper contributes to the literature on three fronts. First, our model reconciles the directed search model with the evidence on unemployment duration, both theoretically and quantitatively. One of the most robust facts regarding unemployment is the *negative duration dependence* of unemployment on wages. Workers with higher wages tend to have shorter unemployment duration.[1] A major weakness of the canonical directed search model—and therefore a fundamental criticism of its broader applicability in explaining labor market frictions—includes that the model predicts the opposite, a positive duration dependence of unemployment on wages. Higher wages attract more applicants and therefore result in lower matching probabilities, *i.e.* longer unemployment duration. One of the contributions of this paper is to show negative duration dependence under directed search, due to the presence of consumption smoothing. We establish the negative duration dependence both analytically and in the quantitative exercise. Workers who remain unemployed longer run down their assets, and end up applying for low wage jobs, which induces negative duration dependence. The model therefore combines the moral hazard aspect of UI with a changing job finding probability due to asset decumulation. We believe that this paper provides a new insight and offers an important empirical justification for the applicability of directed search models.

The second contribution shows that workers with heterogeneous asset holdings sort into firms with heterogeneous productivities. This implies that equally skilled workers have different productivities, depending on their wealth holdings. The sorting happens despite the fact that no inherent technological complementarity (supermodularity) exists between job productivity and worker skill. Nonetheless, a natural *preference complementarity* arises between firm productivity and worker assets because risk aversion generates different preferences for self-insurance, with high asset holders trading off lower insurance for a higher productivity job. We solve the model as an allocation problem with risk aversion and therefore imperfectly transferable utility (ITU) as well as search frictions. The selection or sorting of workers into different productivity jobs that is responsible for the different matching probabilities of different asset holders occurs under fairly common conditions (we can derive condition related to Absolute Relative Risk Aversion in the two-period model). While directed search in the presence of risk aversion has been analyzed

1. See amongst many others, Heckman and Singer (1984a, 1984b), Honoré (1993), Van den Berg and Van Ours (1996), and Baley *et al*. (2021). One of the most challenging research questions there is the extent to which the negative duration dependence is driven by genuine duration dependence, such as the depreciation of skills, or selection, where high wage workers have different job finding rates.

in the literature (most notably Acemoglu and Shimer, 1999 and more recently Golosov *et al*., 2013), these are representative agent models without a non-degenerate distribution of assets.[2]

Our third contribution consists of the *quantitative* analysis of the model. While we do derive analytical results for the two-period version of the model and establish duration dependence theoretically in the infinite horizon, the quantitative analysis provides a main focus of our paper. The interaction between the distribution of assets and the incentives to search for different productivity jobs as well as smoothing consumption is not merely a theoretical artifact; we show its importance quantitatively. We also analyze the steady state of an infinite horizon version where workers and firms sort in each period. In the steady state, unemployed workers run down their assets, while at the same time moving their target from high-to-low productivity jobs. Employed workers run up their assets anticipating the eventual job loss resulting in necessity to insure against income loss while unemployed. Workers continuously move up and down the asset distribution, but the aggregate distribution of assets remains stationary. We derive the ergodic distribution in this steady state as well as wages, savings, jobs search decisions (and unemployment), and the vacancy posting decision for every asset and productivity level. Unlike most existing work on UI, we incorporate the endogenous savings decision of the employed.[3] The novelty of our computational model is the solution of a sorting problem, with risk-aversion, in infinite horizon and with search frictions. In the process, we solve for the ergodic asset distribution as a state variable.

We calibrate our model to the U.S. economy and find its features to be quantitatively important. Workers direct their search towards jobs with different bundles of productivity and job finding probabilities. We find that the job finding probability of the low asset holders is 7% higher than that of the high asset holders, which establishes the importance of endogenous job finding rates and their interaction with the distribution of asset holdings.

In this setting, we analyze the role of government mandated unemployment benefits. We have no pretense of analyzing a general mechanism design question where agents submit messages about their private asset holdings and receive benefits depending on their and all other agents' messages. This turns out to be an immensely complex problem with an infinite horizon and a continuum of heterogeneous agents. Rather, we analyze a realistic UI institution where ex ante homogeneous workers with ex post heterogeneous (but time varying) asset holdings receive a constant benefit while unemployed and pay a constant tax rate on wage income while employed.

Multiple channels exist through which benefits affect the equilibrium allocation and therefore welfare. In this paper, we single out five equilibrium effects that result from an increase in benefits: (1) The unemployed worker is better insured and enjoys smoother consumption; (2) Because of better insurance prospects, workers with more wealth tend to sort into more productive jobs; both of these effects affect welfare positively. The next effects are negative. (3) Higher wages reduce the firm's benefits and therefore job creation; (4) Higher benefits affect the sorting pattern with more workers applying for high productivity jobs, which uniformly leads to lower job finding probabilities and therefore higher unemployment; (5) Higher benefits increases the productivity of jobs but reduces the total production (extensive margin) and therefore lead to

2. Acemoglu and Shimer (1999) do consider a non-degenerate distribution when analyzing the case of constant absolute risk aversion (CARA), which, as we show in this paper, is a knife-edge case with no sorting and where the asset distribution is indeterminate.

3. The standard assumption in the literature is that employed workers values are constant (see for example Hopenhayn and Nicolini, 1997; Shimer and Werning, 2007, 2008). This is typically achieved by assuming that once employed, they do not face job separation, in conjunction with the assumption that discounting is exactly proportional to the return on assets. All this implies that workers in each period consume the return on their assets, keeping their asset holdings invariant.

lower dividends. Some of these five equilibrium effects are present in other models, but the sorting mechanism and its effect on productivity distinguishes the mechanism here.

We are interested in the net effect of these countervailing forces on welfare, but are reveal the conflicts of interest extant between different agents. Broadly speaking the unemployed are not only better off from higher benefits than the employed, but their benefits also have higher welfare effects for those with low assets. Overall, we find that nearly all workers, including those with high asset levels and those employed, have a preference for relatively high benefits. Depending on their asset holdings, the optimal benefit for the unemployed is between 62% and 46% of wages whereas the optimal benefit is less than 45% for the employed. When we aggregate the value functions across all agents, we find that welfare has an inverted U-shape in benefits where the optimal benefit level is higher for unemployed workers compared to the employed. A rise in UI from the laissez-faire economy increases welfare for all workers, but especially for the asset poor and the unemployed. In contrast, when the UI level moves closer to the full replacement rate, welfare falls for all workers, in particular the asset rich employed workers.

A novel feature of our model consists of the sorting between workers with different asset holdings and firms with different productivities. This implies that UI benefits affect the productivity of workers in the economy, through the allocation of workers to jobs of different productivities as well as through the firms' entry decision. This contrasts with models of homogeneous firms where a change in benefits leaves the average firm's productivity unaffected. We find that when UI benefits increase, average worker productivity increases, even though total production decreases. Higher benefits result in workers applying to more productive jobs, due to better insurance. But, at the same time, these benefits decrease the job finding probability, and as a result, fewer workers find jobs. Firms do not respond opening more vacancies because they see their profits reduced as benefits push up wages.

Related literature. We are intellectually indebted to earlier work that has shaped our thinking on this topic. This paper builds on an abundant literature on unemployment risk and consumption smoothing. Danforth (1979) is one of the first to analyze search with risk averse workers in a partial equilibrium setting. Hopenhayn and Nicolini (1997) and Shimer and Werning (2007, 2008) analyze optimal UI in a similar setting. Our paper employs a general equilibrium search model with risk averse agents, closely related to Acemoglu and Shimer (1999). Acemoglu and Shimer (1999) either assume asset holdings are identical for all agents— thus they cannot address the role of inequality in assets—or that preferences satisfy CARA—in which case the asset distribution is indeterminate. The latter is a knife-edge special case in our paper. The properties of equilibrium change completely when moving away from CARA, without sorting and with and indeterminate allocation. We derive all of our results from the fact that workers sort on assets and job productivity. Under CARA none of the implications for welfare or the impact of unemployment benefits would hold. We are able to endogenize asset holdings and move beyond Acemoglu and Shimer (1999) only because we had the benefit of the results on sorting with risk averse agents in Legros and Newman (2007). The result is a directed search model with general preferences and an ergodic distribution of assets where we find substantial effects on wages and the value functions (a feature that is hard to obtain in the most basic random search model, *i.e.* without search intensity or endogenous match formation).

Golosov *et al.* (2013) consider a similar setup to Acemoglu and Shimer (1999) with identical agents and analyze optimal taxation and benefits. Here, we focus on the distribution of assets and where the distribution of those assets is non-degenerate.

Our model follows in the footsteps of Krusell *et al.* (2010), who analyze the relation between asset dependent consumption-savings decisions and unemployment risk. We focus on directed rather than random search, which is not merely a semantic distinction. Directed search allows for the fact that asset holdings affect the job finding probability. While Krusell *et al.* (2010) obtain a welfare function that is decreasing in benefits for asset rich workers, we obtain the opposite because in the basic random search model the probability of job finding is exogenous for workers. When UI goes up, rich workers become disadvantaged: they pay higher taxes yet, they find jobs at lower rates and their consumption smoothing does not change much. Instead in our framework, all workers endogenously adjust their probability of job finding depending on the UI level, which leads to an increase of welfare in benefits. Similar to the endogenous directed search, we also find that equilibrium job finding rates are increasing in assets and varying considerably, while they remain constant in the basic random search without endogenous search intensity.

Our paper extolls the advantages of directed search framework to study unemployment. Given the difficulties analyzing sorting in the random search model (Shimer and Smith, 2000), there is little hope to address sorting on assets in random search with risk aversion. Recent work by Krusell *et al*. (2019), Chaumont and Shi (2022) and Baley *et al*. (2021) extends our setting (most notably with on-the-job search, absent in our paper), a testament to the virtues of the directed search model compared to random search to study the asset distribution and the consumption-savings decision. The directed search model can take the results in Krusell *et al*. (2010) a step further with the aim of building a model versatile enough to address canonical macroeconomic questions with the properties that the random search model lacks.

Our directed search setup is complex—it has risk averse agents, involves a consumption-savings decision, sorting, and a dynamic economy (infinite horizon)—and the block-recursivity property (Menzio and Shi, 2011) does not apply due to its two-sided heterogeneity, with firm productivity and worker asset holdings. Nonetheless, from the combination of directed search with two-sided heterogeneity (as in Eeckhout and Kircher, 2010), we can solve an assignment problem with risk aversion. We extend the analysis in Legros and Newman (2007) to derive the conditions for sorting. The novelty of our approach allows us to analyze an economy where the asset distribution is endogenous and where both savings and job search decisions depend on the worker's asset holdings. We can thus analyze how unemployment benefits affect workers' asset holdings and in turn the productivities of jobs they search for.

In the matching literature, our paper further relates to models with types endogenous to investment (see amongst others Peters and Siow, 2002; Cole *et al*., 2001), and where matching incentives are derived from preferences coupled with market incompleteness rather than built into the technology (for example Legros and Newman, 1996).

This paper also relates to the larger literature that looks at the welfare impact of a change in UI in search and matching models with risk averse agents. Merz (1995), Andolfatto (1996) and den Haan *et al*. (2000) study the macroeconomic implications of search frictions in business cycle models, in an economy where a worker's idiosyncratic income shocks are fully insured. Krusell *et al*. (2010) nests the Diamond–Mortensen–Pissarides framework with asset dependent consumption savings decisions as in Bewley (1977), Huggett (1993) and Aiyagari (1994). This setting of job search with risk averse workers allows them to analyze the interaction of search frictions with the precautionary savings motive. Our paper takes this approach one step further. We introduce how endogenous job search allows workers to implicitly insure unemployment risk, the precautionary job search motive. We find this distinction is important quantitatively, and as a result, a change in unemployment insurance changes the workers' welfare by affecting their job search decisions as well as the productivity of jobs they choose.

Direct evidence in the literature supports the main mechanism of our model, namely that higher asset holdings leads to prolonged job search. Card *et al*. (2007) find that a lump sum transfer of two months of salary reduces the job finding rate by 8–12%. These numbers align with what we find for our benchmark economy.[4] Chetty (2008) shows that the elasticity of the job finding rate with respect to unemployment benefits decreases with liquid wealth. And Browning and Crossley (2001) show that UI improves consumption smoothing for poor agents, but not for rich ones. Herkenhoff (2019) and Herkenhoff *et al*. (2015) provide evidence for the effect of better credit access on lower job finding rates. Herkenhoff (2019) shows that through this channel, increased credit access leads to longer recessions and slower recoveries, while Herkenhoff *et al*. (2015) exploit credit tightening over the business cycle increases employment and decreases output and productivity. We believe our model is novel in providing a theoretical framework where this observed relation between asset holdings and job finding rates stems from a precautionary job search motive and firm heterogeneity.

Finally, in an interesting study, Michelacci and Ruffo (2015) analyze a related question where workers are heterogeneous: how does optimal UI vary over the life-cycle. Because workers accumulate human capital, young workers have strong incentives to find a job, yet they do not have the means to smooth consumption. Instead, older workers have less incentives and can smooth consumption better. They focus on the role of human capital accumulation and to that end, assume that matching probabilities are exogenous.

We organize the paper as follow. In Section 2 we lay out the model. In Section 3 we derive the equilibrium allocation and the conditions under which there exists positive (negative) assortative matching. In Section 4, we compute and quantitatively analyze the full infinite horizon model. We perform a benchmark calibration, and evaluate the effects of different benefit levels as well the welfare analysis. We conclude in Section 5.

2. THE MODEL

Time horizon. This is a T-period economy in which agents make a joint consumption-savings and job search decision. Endowed with assets, in each period $t < T$, unemployed workers choose their consumption-savings level, as well as for which job to search. Our interest focuses on analyzing the infinite horizon setting $T \to \infty$ (Section 3.2). To gain insights into the mechanism and in order to derive analytical results, we first analyze the two-period model $T = 2$ (Section 3.1), in which workers make decisions only once at $t = 1$.

Agents. There is a measure one of workers. When they are unemployed, they are indexed by their heterogeneous asset holdings in period t, $a_t \in \mathcal{A} = [\underline{a}, \overline{a}] \subset \mathbb{R}_+$.[5] Let $F_u(a)$ denote the measure of unemployed workers with asset levels weakly below $a \in \mathcal{A}$ (with positive derivative $f_u(a)$). When they are employed, workers are indexed by both assets a and a wage w. Let $F_e(a, w)$ be the measure of employed workers with asset levels below a and wages below w. We denote the marginal over w by $F_e(a)$ (with positive derivative $f_e(a)$).[6] In order to reduce notation, we denote $F = (F_u(a), F_e(a, w))$. The distribution of asset holdings amongst unemployed and employed workers is endogenous. In the infinite horizon model we derive the ergodic distribution of assets. Each worker supplies her labor and can only apply to one job at a time. Firms are heterogeneous in their productivities y and each has one job. Let $y \in \mathcal{Y} = [\underline{y}, \overline{y}] \subset \mathbb{R}_+$ and assume the firm type is observable. $H(y)$ denotes the measure of firms in the economy and with a type weakly below y. The total measure of firms $H(\overline{y})$ is assumed large. H is assumed C^2 with

4. See also Rendon (2006) and Lentz (2009) for related findings.

5. For much of the paper, we will drop the subscript t and in the recursive (two-period) formulation we refer to $a_t = a$ ($a_1 = a$) and $a_{t+1} = a'$ ($a_2 = a'$).

6. These are not distributions since their total measure is not equal to one. Because the measure of workers is equal to one and all are either employed or unemployed, it is the case that $F_u(\overline{a}) + F_e(\overline{a}) = 1$ and $F_u(\overline{a})$ is equal to the unemployment rate.

strictly positive derivative h. Not all firms enter the market, nor are all firms searching for workers. The measure of firms that post vacancies is endogenous and denoted by $G(y)$ (with positive derivative $g(y)$).

Preferences and technology. Workers are risk averse and their preferences are represented by the von Neumann-Morgenstern utility function $u(c)$ over consumption level c, where $u : \mathbb{R}_+ \to \mathbb{R}_{++}$. We assume that u is increasing and concave: $u' > 0, u'' < 0$. Agents discount utility with factor $\beta < 1$. Savings can be invested in a risk free bond at a fixed rate $R = 1 + r > 1$. We assume that firms are owned by entrepreneurs who are risk neutral and who do not participate in the labor market.[7] Firms have one job and can post a vacancy at cost k. Output produced at a firm of type y is equal to y.

Search technology. Job search is directed. Firms post a wage w and there is a search technology that governs the frictions. These frictions crucially depend on the degree of competition for jobs, as captured by the ratio of vacancies to unemployed workers, denoted by $\theta \in [0, \infty]$. This ratio represents the relative supply and demand for jobs, as it determines the probability of a match for an unemployed worker denoted by $m(\theta)$, where $m : [0, \infty] \to [0, 1]$: the higher the value of θ, the easier it is for a worker to find a job, so m is a strictly increasing function: $m' > 0$. In contrast, the higher the ratio of firms to workers, the harder it is for a firm to fill its vacancy. We denote the probability that a firm gets matched by $q(\theta)$, where $q : [0, \infty] \to [0, 1]$ is a strictly decreasing function, $q' < 0$. Since matching is always in pairs, the matching probability of workers must be consistent with those of firms, in particular, it must be the case that $q(\theta) = m(\theta)/\theta$. We also require the standard assumptions hold: m is twice continuously differentiable, strictly concave and has a strictly decreasing elasticity. The fact that we express the matching probability in terms of the ratio of firms to workers θ and not the number of unemployed workers and vacancies effectively means that we assume a matching technology that is constant returns. As the number of workers and firms doubles, the number of matches doubles, yet the matching probabilities remain unchanged.

Inherent to the nature of directed search, there is a separate submarket for each firm-worker type pair. Heterogeneous firms and workers operate in different markets, while identical agents share a common market, which permits workers to direct their search to those firms offering optimal terms (matching probability and wages), enabling firms with vacancies to influence the search decision of workers by changing the terms of the wage offer. Whenever unemployed, a worker searches to find a job, and once employed she holds the job until the match is separated with exogenous probability λ.

Unemployment benefits. We assume that all unemployed workers receive unemployment benefit b. The benefit b is financed by a budget balancing proportional tax τ on wages. This requires that the sum of all benefits b over the unemployed agents is equal the sum of all taxes levied on wage income τw. We also assume that the entire income for the unemployed comes from UI. For a given b, the government sets τ to balance its period-by-period budget constraint:

$$ub = \tau \int w(a) f_e(a) da. \tag{1}$$

7. This approach does not affect any of the results since the dividend deterministically increases the workers' asset holdings and merely shifts the asset distribution. However in the infinite horizon version of the model we assume that profits are distributed as the risk free dividend of a mutual fund owned by all workers and that has all firms in its portfolio as in Golosov *et al*. (2013). This closes the model in order to analyze the welfare implications of changes in UI.

Profits and dividends. Due to the sorting with firms of heterogenous productivity, all firms except the marginal firm make profits. The hedonic profit schedule that clears the market increases in the firm type: higher productivity firms make higher profits.[8] We assume that consumers own an equal share of the equity of all firms.[9] This assumption implies that all workers regardless of their employment status receive a dividend d every period and enables the welfare analysis to take into account the impact of a change in unemployment benefits on profitability of firms.[10]

Actions. In period $t < T$, workers choose their consumption-savings bundle as well as the job search decision.[11] A worker enters period t with assets Ra_t chosen in period $t-1$. The worker then chooses the assets a_{t+1} saved. In period t, firms y post wages w_{t+1}, and the worker chooses in which submarket (y_{t+1}, w_{t+1}) to search. Even if firms with different productivity y_{t+1} offer the same wage w_{t+1}, in directed search they operate in different markets. Given the behavior of all other firms and applying workers, this market has a tightness $\theta_t(y_{t+1}, w_{t+1})$. As customary in directed search literature, we drop the argument and write θ_t for notational simplicity.

Period t's consumption is contingent on the saved assets and on the labor market outcome. A worker carries over last period's assets with return R. If unemployed, her income is thus $Ra_t + b$ and $Ra_t + (1-\tau)w_t$ if employed. Her consumption is equal to this income net of here savings for the next period a_{t+1}: $c_{e,t} = Ra_t - a_{t+1} + (1-\tau)w_t$ when employed and $c_{u,t} = Ra_t - a_{t+1} + b$ when unemployed. Within the same period t, a directed search game determines the labor market outcome for unemployed workers and vacant firms.

The firms' wages w_{t+1} set in period t will be paid starting in the next period: $w_{t+1} \in \mathcal{W} = [\underline{w}, \overline{w}] \subset \mathbb{R}_+$. We restrict the contract space to invariant wages. Denote by $P(y_{t+1}, w_{t+1})$ and $Q(a_t, a_{t+1}, y_{t+1}, w_{t+1})$ the distribution of actions by firms and workers: $P(y_{t+1}, w_{t+1})$ is the measure of firms that offer a productivity-wage pair below (y_{t+1}, w_{t+1}) and $Q(a_t, a_{t+1}, y_{t+1}, w_{t+1})$ is the measure of workers with assets below a_t who save less than a_{t+1} and who match with firms that have productivity-wage pairs below (y_{t+1}, w_{t+1}). We impose that those distributions of actions remain consistent with the initial distributions of types $G(y)$ and $F_u(a)$, *i.e.* and that market clearing holds. In particular, it must be the case that $P_{\mathcal{Y}}(\cdot) = G(\cdot)$ and $Q_{\mathcal{A}} = F_u(\cdot)$, where $P_{\mathcal{Y}}$ and $Q_{\mathcal{A}}$ are the marginal distributions. This consistency ensures that the allocation is measure preserving.

Value functions and equilibrium. Denote by $U(a_t)$ the value of being unemployed in period t with asset level a_t and by $E(a_t)$ the value of being employed.[12] The unemployed worker

8. This feature could be reconciled with zero profits by adding an earlier stage: firms pay an entry cost before the realization of their type. In equilibrium, the expected profits equate the entry cost.

9. That is, no consumer holds the claim to the profit of an individual job but she holds the claim to an identical share of the aggregate profit. This assumption avoids that an employed worker holds a short position in her own job in order to hedge against the risk of separation. Also, in Appendix D we consider alternative distributions of profits, including profits that are taxed in order to finance UI benefits and profits that are redistributed to workers in proportion to their asset holdings.

10. For the remainder of the theory section, we set $d = 0$ and drop d from the equations. In the quantitative analysis, we reintroduce d.

11. Firm types y are invariant, and in what follows, we therefore use no time subscript for firm types y. All worker decisions change over time, even in the ergodic equilibrium where aggregate distributions are stationary, because individual behavior changes. We therefore use time subscripts. Because the choice of the worker is a submarket y_t, w_t which evolves over time with changing asset holdings, we have a time subscript on the worker's choice of firm type y_t.

12. More precisely, $U(a_t, a_{t+1}, y_{t+1}, w_{t+1}, P, Q, F)$ is the value of an unemployed worker with assets a_t who saves a_{t+1}, who applies to a job y_{t+1} with wage w_{t+1} and who anticipates a distribution of offers P and a distribution of jobs Q, and when the asset distributions are given by F. Of course the worker does not care about the productivity y_{t+1} and only about the wage w_{t+1}, but the submarket is indexed by the bundle y_{t+1}, w_{t+1} because different firm types y_{t+1} may offer the same wage, y_{t+1} formally enters in the value function of the worker. For notational convenience,

simultaneously chooses how many assets a_{t+1} to save for next period, and in which submarket y_{t+1}, w_{t+1} to search. The choice of the submarket determines the wage w_{t+1} but also the market tightness θ_t and hence the matching probability $m(\theta_t)$; recall that we use the shorthand notation for tightness that depends on the wage and the productivity of the firm that posts the wage: $\theta_t(y_{t+1}, w_{t+1})$. The employed worker with assets a_t chooses how much to save a_{t+1}.

We can then write (where θ is shorthand for $\theta_t(y_{t+1}, w_{t+1})$)

$$U(a_t) = \max_{a_{t+1}, y_{t+1}, w_{t+1}} \left\{u(c_{u,t}) + \beta\left[m(\theta_t)E(a_{t+1}, w_{t+1}) + (1-m(\theta_t))U(a_{t+1})\right]\right\}$$

$$\text{s.t.} \quad c_{u,t} = Ra_t - a_{t+1} + b + d \quad \text{and} \quad a_{t+1} \geq \underline{a} \tag{2}$$

$$E(a_t, w_t) = \max_{a_{t+1}} \left\{u(c_{e,t}) + \beta[\lambda U(a_{t+1}) + (1-\lambda)E(a_{t+1}, w_{t+1})]\right\}$$

$$\text{s.t.} \quad c_{e,t} = Ra_t - a_{t+1} + (1-\tau)w_t + d \quad \text{and} \quad a_{t+1} \geq \underline{a}. \tag{3}$$

All workers' savings are limited by a borrowing constraint, $\underline{a}$, which measures the incompleteness of the credit market.

The continuation value to the firm of productivity y that posts a vacancy is denoted by $V(y)$:[13]

$$V(y) = -k + \max_{w_{t+1}} \beta[q(\theta_t)J(y, w_{t+1}) + (1-q(\theta_t))V(y)]\}, \tag{4}$$

where $J(y, w_{t+1})$ as well as the market tightness θ_t (shorthand for $\theta_t(y_{t+1}, w_{t+1})$) depend on the firm's choice w_{t+1}. $V(y)$ is the steady state continuation value when the job is not filled. At a cost k, the firm announces a vacancy and commits to a wage w_{t+1} that it will pay starting next period in the case of a match. Like workers, firms discount the future at rate β. $J(y, w_t)$ is the value of a filled job for a firm with productivity y_t when paying a wage w_t:

$$J(y, w_t) = y - w_t + \beta[\lambda V(y) + (1-\lambda)J(y, w_{t+1})]. \tag{5}$$

In the infinite horizon version of the model, we focus on the ergodic steady state where the distribution is time invariant, but individual workers' assets, consumption, and labor market choices evolve. The firm's choices are time-invariant ($w_t = w_{t+1}$, so $J(y, w_t) = J(y, w_{t+1})$. In the two-period version of the model, we will use the shorthand notation $U_2 = u(c_{u,2})$ and $E_2 = u(c_{e,2})$, and where for all $t > 2$, $a_t = 0$, $U(a_t) = 0$, $E(a_t, w_t) = 0$, $V = 0$ and $J = 0$.

The matching of asset holders to firms can now be fully described by the optimization decision of firms of type y with wages w and of unemployed workers of asset holdings a_t who choose which submarkets to enter, together with market clearing. Next, we formalize these properties in the equilibrium concept. Just like in the standard Becker (1973) assignment problem, a hedonic price schedule mediates the competition by heterogenous agents on both sides of the market. In the assignment game, that price is the wage schedule. Here, this equilibrium object reveals the tightness of each submarket, which in turn is determined by the wage. Like in the sorting problem with directed search in Eeckhout and Kircher (2010), we adjust the market clearing condition for the fact that match formation is stochastic and dependent on the tightness in each market.

We adopt the equilibrium concept used by Acemoglu and Shimer (1999). To accommodate the two-sided heterogeneity of firm productivity and worker assets, we will use the version of

we restrict the argument of the value function to the variable that indexes the type $U(a_t)$, *i.e.* the heterogeneity that is relevant for sorting: a_t. Likewise $E(a_t) = E(a_t, a_{t+1}, y_{t+1}, w_{t+1}, P, Q, F)$.

13. Recall that to the firm, its type y is time-invariant and therefore has no time subscript.

their equilibrium adjusted by Eeckhout and Kircher (2010) to allow for two-sided heterogeneity and a continuum of agents, who consider the Acemoglu and Shimer (1999) setup as a large game where each individual's payoff is determined only by her own action and the distribution of actions in the economy, which consists of the optimal choices of each of the individuals in the distribution.[14]

In line with the literature on directed search (see for example McAfee, 1993; Acemoglu and Shimer, 1999), we impose restrictions on the beliefs about off equilibrium path behavior. In the current setup, beliefs about the queue length corresponding to firm or worker choices that do not occur in equilibrium are not defined. Therefore, we define those off equilibrium path beliefs as corresponding to the notion of subgame perfection.[15] Firms expect workers to queue up for jobs as long as it proves profitable for them to do so given the options they have on the equilibrium path. Formally, this defines the queue length over the entire domain as: $\theta(y, w) = \sup\{\theta \in \mathbb{R}_+ : \exists a; U(a) \geq \max_{y', w' \in \text{supp} P} U(a, y', w', P, Q)\}$, where $U(a)$ satisfies Equation (2). In all other cases, the queue length is zero.

This description of the economy now permits us to define equilibrium. When time is finite, the equilibrium can be defined recursively starting from an initial asset distribution. In the infinite horizon economy, we solve for the stationary asset distribution. In each period, an equilibrium is a pair of distributions (P, Q) such that the following conditions hold: (1) Worker optimality: $(a_t, a_{t+1}, y_t, w_t) \in \text{supp}\, Q$ only if it maximizes (2) and (3) for a_t; (2) Firm optimality: $(y_t, w_t) \in \text{supp}\, P$ only if w' maximizes (4) and (5) for y.

This is a matching problem with a non-linear pairwise Pareto frontier. Legros and Newman (2007) and Kaneko (1982) establish existence. Jerez (2014) goes further and establishes the existence of an equilibrium in a directed search model with a continuum of agents and a general matching technology.

The (measure preserving) market clearing condition becomes particularly transparent when matching is monotone, in which case there is one-to-one matching of a to y, represented by a function $\mu : \mathcal{A} \to \mathcal{Y}$. Under positive assortative matching (PAM), $\mu'(y)$ is positive and negative under negative assortative matching (NAM). Under PAM high asset workers match with high productivity jobs, and the market clearing condition can be written as:

$$\int_{a}^{\overline{a}} \theta(s) f_u(s) ds = \int_{\mu(a)}^{\overline{y}} g(s) ds. \tag{6}$$

3. THE EQUILIBRIUM ALLOCATION

We first analyze a simple two-period model. We aim to provide insights into how the per period allocation of asset holders to firms works. We then turn to the infinite horizon model, where we focus our attention on the steady state and lay the ground for the calibration and policy exercise. For the purpose of the theory results in this section, we assume that benefits, vacancy posting costs, and dividends are zero: $b = 0, k = 0, d = 0$.[16] Benefits, vacancy posting costs, and dividends remain important for the calibration in

14. The queue length θ is a function of the distribution of offers P and visiting decisions Q. Written explicitly, $\theta_{PQ} : \mathcal{Y} \times \mathcal{W} \to [0, \infty]$ is the expected queue length at each productivity-wage combination (y, w). Then along the support of the firms' wage setting distribution, $\theta_{PQ} = dQ_{\mathcal{YW}}/dP$ is given by the Radon–Nikodym derivative, where $Q_{\mathcal{YW}}$ is the marginal distribution of Q with respect to $\mathcal{Y}$ and $\mathcal{W}$.

15. Peters (1997, 2000) provide micro foundations for a version of this model where this assumption is indeed justified as the limit of deviations in a finite game.

16. Zero dividends to consumers implicitly means that firms are owned by absentee investors.

the infinite horizon model, but do not add any insights into understanding the mechanism of the equilibrium allocation.

3.1. *The two-period model*

We first analyze the decentralized equilibrium allocation in the two period model where all workers are initially unemployed. Let there be an exogenously given initial distribution of assets $G(a_1)$, which carries over to Ra_1 in period 1. With $T=2$, there is only a consumption/savings-search $(a_2; w_2, y_2)$ decision in period 1. In the final period, consumption is determined by the period's savings decision and the outcome of the job search. The value of both employment and unemployment are therefore equal to the utility of consumption in the respective states: $E(a_2)=u(Ra_2+w_2)$, where $a_3=0$ and $U(a_2)=u(Ra_2)$. Then we can then rewrite (2) after substituting for (3) as:[17]

$$U(a_1)=\max_{a_2,y_2,w_2}\left\{u(Ra_1-a_2)+\beta\left[m(\theta_1)u(Ra_2+w_2)+(1-m(\theta_1))u(Ra_2)\right]\right\}, \tag{7}$$

where the market tightness θ_1 is a function of the choice of submarket (y_2, w_2). The consumption is thus completely pinned down by the savings choice a_2 and the labor market choice (y_2, w_2), *i.e.* which submarket to search in, resulting in a matching probability $m(\theta_1)$. The expected payoff to a firm y posting a vacancy with posted wage w_2 is

$$V(y)=\max_{w_2}\beta q(\theta_1)\left(y-w_2\right), \tag{8}$$

from (4) since $J_2=y-w_2$, and the continuation value is zero, and where, again, the market tightness θ_1 is a function of the posted wage w_2.

The firm sets wages w to maximize expected profits $V(y)$. The consumer's problem is to maximize expected utility from consumption while simultaneously making an optimal search decision. We can therefore summarize the joint worker and firm optimization as:

$$\max_{a_2,y,w_2}\left\{u(Ra_1-a_2)+\beta\left[m(\theta_1)u(Ra_2+w_2)+(1-m(\theta_1)\,u(Ra_2)\right]\right\} \tag{9}$$

$$\text{s.t. } V=\max_{w_2}\beta q(\theta_1)\left(y-w_2\right). \tag{10}$$

Given $w_2=y-\frac{V}{\beta q(\theta_1)}$ from (10) we can write this joint optimization problem as a single optimization after substituting for w_2, the standard solution method for directed search problems. With the wage w_2 substituted out, optimality now follows from the optimal choice of the queue length θ_1, since the posted wage directed determines the queue length. With risk averse preferences, we can write this problem as a matching problem with a non-linear Pareto frontier denoted by $U(a_1, y, V)$. This denotes the value to the worker when matched with a firm y to which it leaves the value V, and where the optimal choice is now over (a_2, θ_1):

$$U(a_1,y,V)=\max_{a_2,\theta_1}u(Ra_1-a_2)+\beta\left[m(\theta_1)u\left(Ra_2+y-\frac{V}{\beta q(\theta_1)}\right)+(1-m(\theta_1))\,u(Ra_2)\right] \tag{11}$$

17. We drop the time subscript $t=1$ of the value functions. The period 2 values are either zero or we substitute them by the period payoff.

Then the solution to the maximization problem is $a_2^\star, \theta_1^\star$ and satisfies:

$$-u'(Ra_1 - a_2) + \beta R \left[m(\theta_1) u' \left(Ra_2 + y - \frac{V}{\beta q(\theta_1)} \right) + (1 - m(\theta_1))\, u'(Ra_2) \right] = 0 \tag{12}$$

$$\beta m(\theta_1)' \left[u \left(Ra_2 + y - \frac{V}{\beta q(\theta_1)} \right) - u(Ra_2) \right] + \beta u' \left(Ra_2 + y - \frac{V}{\beta q(\theta_1)} \right) \frac{\theta_1 q'(\theta_1) V}{\beta q(\theta_1)} = 0. \tag{13}$$

The optimal savings behavior and optimal job search simultaneously imply a matching decision. That is, a worker a effectively chooses a firm y. We can now analyze this allocation problem with a non-linear frontier $U(a_1, y, V)$, where a_2 and θ_1 are chosen endogenously. We use the standard solution method for an assignment problem. The worker takes the firm payoff $V(y)$ as given (typically called the hedonic price schedule) and chooses the firm type y that maximizes her expected utility. From the first-order condition, the optimal y therefore satisfies $U_y + U_V \frac{\partial V}{\partial y} = 0$. This implies:

$$\beta m u' \left(Ra_2 + y - \frac{V}{\beta q(\theta_1)} \right) \left(1 - \frac{V'}{\beta q} \right) = 0. \tag{14}$$

where the effect of y and V on U through a_2 and θ_1 is zero from the envelope theorem: $\frac{\partial U(a_2)}{\partial a} = 0$, $\frac{\partial U(\theta_1)}{\partial \theta} = 0$ imposed by Equations (12) and (13). The details of the derivation of the partial derivatives can be found in the Appendix.

We want to ascertain under which circumstances monotone matching of asset holdings a_1 in job productivities y exists. This is now a matching problem $U(a_1, y, V)$ where a type a_1 chooses the optimal y, given optimizing behavior regarding a_2 and θ_1. The allocation is denoted by $a_1 = \mu(y)$. Then the total cross derivative of U with respect to a_1 and y is positive provided[18]

$$\frac{d^2 U}{da_1 dy} = U_{a_1 y} + U_{a_1 V} \frac{\partial V}{\partial y} = U_{a_1 y} - U_{a_1 V} \frac{U_y}{U_V} > 0, \tag{16}$$

where we use the first order condition to substitute for $\frac{\partial V}{\partial y}$. This sorting condition can be derived from the second-order condition, and therefore ensures that this solution is also a global maximum. In addition, for a *given distribution of types* this solution also ensures uniqueness (see Legros and Newman, 2007; Chade *et al.*, 2017). Therefore, there will be Positive Assortative Matching in types a_1, y provided $U_{a_1 y} > \frac{U_y}{U_V} U_{a_1 V}$. The next Proposition establishes under which conditions on the primitives (preferences and technology) this property is satisfied:

18. As is conventional, we use subscripts for partial derivatives, for example $U_{a_1 y} = \frac{\partial^2 U(a_1, y, V)}{\partial a \partial y}$ is the cross-partial derivative of the value function U with respect to a and y, where $a = a_1$. In other words, Equation (16) is short for:

$$\begin{aligned}\frac{d^2 U(a_1, y, V)}{dady} &= U_{ay}(a_1, y, V) + U_{aV}(a_1, y, V) \frac{\partial V}{\partial y} \\ &= U_{ay}(a_1, y, V) - U_{aV}(a_1, y, V) \frac{U_y(a_1, y, V)}{U_V(a_1, y, V)} > 0. \end{aligned} \tag{15}$$

Proposition 1. *Workers with higher initial asset levels a_1 will apply for higher productivity jobs y provided*

$$\frac{u'(c_{e,2})-u'(Ra_2)}{u(c_{e,2})-u(Ra_2)} < \frac{u''(c_{e,2})}{u'(c_{e,2})}, \tag{U}$$

for all y, where $c_{e,2}=Ra_2+y-\frac{V}{\beta q(\theta_1)}$. This condition is implied by decreasing absolute risk aversion (DARA). Moreover, the equilibrium is unique.

Proof. In Appendix. □

Proposition 1 establishes under which conditions on the utility function, agents with higher levels of assets will choose riskier jobs. In addition to the solution being positively assorted under the condition, the allocation is unique in the two-period version. This follows from the fact that the inequality in condition (**U**) is strict and the fact that this is effectively a static problem with exogenous types.

The condition is satisfied for any utility function that exhibits DARA.[19] To further illustrate how the condition is equivalent to DARA, in the next Corollary, we focus on a well-known class of utility functions, namely the class of Hyperbolic Absolute Risk Aversion (HARA) utility functions. The corollary illustrates that a number of results for special cases of the HARA preferences immediately follow, including DARA, constant relative risk aversion (CRRA), logarithmic, CARA, risk neutrality and the quadratic.

Corollary 1. *Consider the class of utility functions with HARA:*

$$u(c)=\frac{1-\gamma}{\gamma}\left(\frac{\alpha c}{1-\gamma}+\beta\right)^{\gamma} \quad \text{where } \alpha>0,\ \beta+\frac{\alpha c}{1-\gamma}>0.$$

Then condition (**U**) *holds:*

(1) whenever there is DARA: $\gamma<1$.
(2) under CRRA $u(c)=\frac{1-\gamma}{\gamma}c^{\gamma}$ $(\alpha=1-\gamma, \gamma<1, \beta=0)$ and Log utility: $u(c)=\log c$ $(CRRA, \gamma\to 0)$;
(3) with equality under CARA $u(c)=1-e^{-\alpha c}$ $(\beta=1, \gamma\to-\infty)$ and Risk Neutral $u(c)=\alpha c$ $(\gamma=1)$;
(4) with opposite inequality under Quadratic utility: $u(c)=-\frac{1}{2}(-\alpha c+\beta)^2$ $(\gamma=2)$.

Proof. In Appendix. □

Finally, condition (**U**) establishes that there are complementarities in the match value between a firm type y and a worker with assets a_1. In other words, the match value $U(a_1,y,V)$ between types a_1 and y is supermodular, and therefore the equilibrium allocation matches high asset workers with high productivity firms. Even without inherent technological complementarities (all workers are identically skilled), risk aversion and two-sided heterogeneity generates a *natural preference* complementarity between assets and job productivity.

This condition implies that when high asset workers apply for high productivity jobs, they earn higher wages, have higher unemployment, consume more and have higher expected utility. Likewise, when high productivity firms post higher wages, they attract higher asset workers, have higher expected profits and fill vacancies faster.

19. We are grateful to Xiaoming Cai for pointing this out to us.

3.2. *Infinite horizon*

We now consider the stationary equilibrium allocation in the infinite horizon version of the model. The per period allocation problem in the labor market is similar to the one analyzed for the two-period model, with the exception of the continuation value. We derive a condition similar to the (**U**) condition, but now for the infinite horizon economy. Note that this condition now involves value functions, *i.e.* endogenous objects and not just primitives, such as utilities and consumption bundles.

Quantitatively we analyze the parameter configuration $\beta R < 1$. This implies that while employed, the consumption-savings decision varies with time, and we can thus incorporate precautionary savings by the employed who anticipate the possibility of becoming unemployed.[20] We show the following result:

Proposition 2. *Then workers with higher initial asset levels a will apply for higher productivity jobs provided*

$$\frac{E_{a_{t+1}}(a_{t+1},y)-U_{a_{t+1}}(a_{t+1})}{E(a_{t+1},y)-U(a_{t+1})} < \frac{E_{w_{t+1}a_{t+1}}(a_{t+1},y)}{E_{w_{t+1}}(a_{t+1},y)} \tag{U_∞}$$

Proof. In Appendix. □

The result in Proposition 2 thus generalizes to the case with an infinite horizon, albeit with two important caveats. The first caveat is that we cannot derive conditions on the primitives. In the next section, we compute the equilibrium allocation with the corresponding ergodic distributions, and we verify whether along the equilibrium allocation condition $\mathbf{U}_\infty$ is satisfied.

The second caveat is that even though the equilibrium allocation satisfies positive sorting, we cannot guarantee the uniqueness of the positively assorted equilibrium allocation.[21] While condition (16) guarantees uniqueness of the match surplus for a *given* surplus, the match surplus is *endogenous* and depends on the distribution of assets. Potentially there could therefore be multiple distributions of assets that give rise to different equilibrium actions—most notably different savings decisions—resulting in multiple ergodic asset distributions. This type of multiplicity of steady states proves common in other models with endogenous inequality (see amongst others Banerjee and Newman, 1993; Mookherjee and Ray, 2002, 2003) and models of random search with two-sided heterogeneity (see Burdett and Coles, 1997; Shimer and Smith, 2000).

Unfortunately, with a continuous distribution of assets, there is no hope to find analytical solutions.[22] In the absence of analytical solutions, in the quantitative analysis we therefore perform different exercises to ascertain whether the quantitative solution is unique or whether there is multiplicity. First, we start the numerical exercise from initial values of the asset distribution at opposite extremes. If multiple steady states exist, those extreme initial values are more likely to converge to different allocations. Second, we perturb the parameter estimates around the estimated equilibrium values to verify whether the equilibrium allocation is locally unique. In none of these robustness exercises have we found evidence of multiple steady states.

Duration dependence. Our model has novel implications for duration dependence on wages. Under the canonical directed search model without precautionary savings, identical workers who

20. Traditionally, models such as ours with infinitely lived agents have been solved assuming $\beta R = 1$ together with $\lambda = 0$ (see amongst others Acemoglu and Shimer, 1999; Shimer and Werning, 2008; Hopenhayn and Nicolini, 1997; the notable exception is Krusell *et al.*, 2010). Under the assumptions of this special case, the continuation value of employment can be derived the closed form, which we do in the Appendix.

21. We are grateful to one of the Referees for pointing out that multiple steady-state equilibria are possible.

22. Even in the case of simple examples with two types as in Burdett and Coles (1997) it is extremely hard to find analytical conditions for uniqueness.

apply to high wage jobs necessarily face longer unemployment duration to make them indifferent with low wage jobs that have shorter unemployment. This positive duration dependence on wages is considered counterfactual as data on wages exhibit negative duration dependence: workers with lower unemployment duration have higher wages.

In our directed search model with precautionary savings, we find that high wages jobs have shorter unemployment duration, the opposite duration dependence compared to the canonical model of directed search because the hazard rate of finding a job is not constant but increasing. As workers are unemployed longer, they run down their assets and therefore apply to jobs with higher matching probability (and lower wages and lower productivity). In other words, for a given worker, our model now illustrates negative duration dependence: workers with shorter unemployment duration tend to have higher wages (and also higher productive jobs). We state this finding formally, which follows immediately from Proposition 2:

Proposition 3 (Negative Duration Dependence)**.** *In the directed search model with consumption smoothing, for a given cohort there is negative duration dependence under positive sorting provided* $\lambda < 1 - m(\theta)$*: workers of a given cohort deplete their assets while unemployed, and their wages w are lower the longer they are unemployed.*

Proof. In Appendix. □

In the proof, we first show that unemployed workers deplete assets to smooth consumption. We closely follow Huggett (1993) and show that this is the case provided the job separation probability is lower than the probability of remaining unemployed, which is always satisfied in the data. The intuition for this condition is that the unemployed worker knows she has better odds at staying in the job once employed, than staying unemployed when unemployed, and therefore chooses to deplete assets while unemployed. Second, due to directed search, the wage decreases as assets decrease. This is the standard logic of sorting: workers with low asset levels apply for low wage, low productivity jobs.

This result addresses one of the main shortcomings of the canonical directed search model, namely the counterfactual prediction of positive duration dependence. The result establishes that precautionary savings is a force towards negative duration dependence.

This negative duration dependence holds for a given cohort of workers. Since within the cross-section, there is positive duration dependence for the same reason as in the canonical directed search model without precautionary savings, the net effect is ambiguous. In the quantitative exercise that we analyze below, we set out to answer the quantitative importance of each of the two effects.

4. QUANTITATIVE EXERCISE

We will now analyze the full model with ergodic asset and firm productivity distributions as well as with non-stationary savings by individual workers while unemployed and employed with the objective of studying welfare and the impact of unemployment benefits. The key feature of the model consists of the sorting of unemployed workers into different productivity jobs depending on their assets, just as in the simplified two-period model. Now, with an infinite horizon, unemployed workers run down their assets while searching for a job in order to smooth consumption. In the process, as their assets decrease, they apply to the low productivity jobs with higher matching probability as a precautionary search motive. When on the job, they face a probability of exogenous separation. Anticipating the possibility of unemployment, workers therefore accumulate assets while working, which gives rise to a pattern of individual asset fluctuations in order to endogenously insure against unemployment risk.

Computationally, we derive the ergodic distribution of assets in this economy, which is the time-invariant aggregate distribution where the asset holdings of individual workers are time-varying. In other words, individual changes cancel out in the aggregate.[23] It should be pointed out that a major technical innovation of our computation is the fact that the employed have a non-stationary policy function that reflects their precautionary savings behavior, *i.e.* $\beta R < 1$, unlike much of the exiting literature.[24] The non-stationarity of the savings decision of the employed is particularly demanding considering the endogenous sorting of workers to jobs with different productivities. Non-stationarity implies that the endogenous distribution of asset is a state variable and unlike most directed search models with savings, our problem is therefore not Block Recursive.

Explained in detail in Appendix C, the algorithm works as follows: an efficient algorithm for any given level of benefit initially guesses (i) the dividend, (ii) the tax rate, (iii) the distribution of workers' assets (both employed and unemployed) and firms posting vacancies, (iv) the value of employment and unemployment and (v) the labor market clearing condition determining the productivity cutoff level of entry as well as its measure. We then take the following six steps: (1) given the distribution of unemployed workers and vacancies, the algorithm first sorts the top workers and firms in the first submarket and finds the job finding rate and wage for this submarket; then it finds the value of a vacancy in the next submarket, using the first order condition of the allocation problem with respect to productivity, and again calculates the job finding rate and the wage. (2) We continue at each subsequent submarket until we reach the boundary of at least one distribution and check if the labor market clears. If not, we change the cutoff point of firm entry. (3) We solve the consumer's dynamic non-linear programing problem. (4) We check the convergence of the distribution of firm types and worker assets (both employed and unemployed) and update them. (5) We check if the total tax revenue and benefits paid are equal. (6) We check whether our guess on the dividend is correct and update accordingly.

Our objective is to study the role of policy on the equilibrium allocation and welfare. As UI changes, both the incentives to save and accumulate assets and the job search behavior change. This change also affects the allocation of workers to jobs of different productivities. Different asset holders have different preferences for insurance and therefore for benefits. We decompose the channels through which UI affects the workers' welfare across the distribution.

The remainder of this section has five parts. First, we calibrate the baseline model with suitably chosen parameters and report its basic properties. Second, we analyze the equilibrium effect of UI benefits. Third, we perform the welfare analysis and find the optimal UI policy. Fourth, we analyze the effect of UI benefits on worker productivity.

4.1. *Benchmark calibration*

We choose the following functional forms: the utility function is $u(c) = \log(c)$, the output produced is y and the numeraire in this economy is one unit of output. Following Menzio and Shi (2011), we use the constant elasticity of substitution matching rate function, $m(\theta) = \theta(1+\theta^{\gamma})^{\frac{-1}{\gamma}}$ where $q(\theta) = \frac{m(\theta)}{\theta}$. Next, we set the value of some parameters. These parameters have a direct counterpart in the data, or have been widely used by other studies. We then

23. As we stated following Proposition 2, the uniqueness of the ergodic distribution cannot be guaranteed. Below, we report robustness checks to help establish that under our parameter configurations no other ergodic steady-state exists.

24. For computational reasons, we assume that there is no capital investment and that the interest rate is exogenous. Introducing both aspects neither affects the basic features of the directed search mechanism nor its interaction with the consumption-savings decision, the heart of our paper. In Appendix E, we analyze a version of our model with capital investment and an endogenous interest rate.

set the value of another group of parameters to match a set of long-term statistics for the U.S. economy. Although the model-generated steady-state variables jointly affect these parameters, a close connection exists between these individual parameters and model predictions.

We set one period to be a month to capture the flow rates in the U.S. labor market. In the computational exercise, we set the borrowing limit $\underline{a} = -1$ and the upper bound of the asset distribution to be larger than the equilibrium support of the asset distribution. Following Krusell *et al*. (2019) we set the discount factor (β) to be 0.9976 which corresponds to approximately 3% annual discounting. The monthly interest rate (r) is 0.002, equivalent to an annual interest rate of 2.5%. In the model, we use the separation rate (λ) reported by Eeckhout and Lindenlaub (2019) and calculated from the Current Population Survey (CPS). This is a monthly separation rate of 1.7%, implying an average employment duration of nearly 5 years.

We jointly set the rest of parameters to match the long run statistics in the U.S.; we can detect which parameters affect which moment the most. We set the flow value of the unemployment benefit $b = 0.85$, which in equilibrium equals approximately 40% of average wages at steady state, similar to Shimer (2005). We pick the elasticity of the matching function (γ) to target the average job-finding probability of 25% observed in the CPS and reported by Eeckhout and Lindenlaub (2019). This probability implies an average unemployment duration of four months in the model which is similar to an average of 4.5 months in the data over the last four decades.[25] We set the cost of posting a vacancy (k) at 0.52, which means that at steady state the cost of a vacancy is 21% of the average productivity of active firms (see Shimer, 2005).

We assume productivity y of potential entrants is uniformly distributed over $\mathcal{Y} = [2, 2.5]$. In equilibrium, only those firms with productivity $y \geq y^\star$ enter the market, where $y^\star \in \mathcal{Y}$ is determined endogenously.[26] The output split between workers and firms implies a median of the annual wealth to wage ratio of 0.49,[27] comparable with the same figure from Panel Study of Income Dynamics (PSID) 2013 which is 0.47. The choice to parameterize the measure of firms at each productivity level as uniform is motivated by the employment distribution over industries in the PSID. Once we control for the probability of filling vacancies at steady states for different levels of productivities, this distribution results approximately uniform.[28]

Table 1 summarizes the externally chosen parameters, and Table 2 reports the key endogenous moments in the ergodic steady-state equilibrium. This unemployment rate at the steady state is 6.1%, similar to the average U.S. unemployment rate between 1980 and 2020.[29] Also, at the benchmark economy, unemployment duration is four months.

Further, the elasticity of the job finding rate to the market tightness at 0.40 lies within the range of empirical estimates by Pissarides (2009) and Shimer (2005).[30] At the steady state the total dividend pay-out is 5.3% of total production similar to the same figure in the U.S. between

25. https://fred.stlouisfed.org/series/UEMPMEAN.

26. In the case with homogenous firms, the free entry condition implies that the value of posting a vacancy is zero at steady state. However, with two-sided heterogeneity, free entry implies that the value of posting a vacancy for firms at the threshold is zero and that firms above the threshold have positive values for posting vacancies. Moreover, a change in unemployment benefit with no heterogeneity among firms only affects the number of vacancies created by firms while with heterogenous firms, benefits affect the number of vacancies as well as the quality of vacancies by shifting the productivity threshold.

27. In Appendix C, we explain how we calculated this figure from PSID.

28. To choose the domain of the productivities, we have been conservative and have chosen a limited domain. Ours is a model with identical workers, so we are modeling productivity for workers with the same education, experience and demographic characteristics (see Bonhomme *et al*., 2019). A wider domain exacerbates our results even further.

29. The U.S. unemployment rate between 1980 and 2020 is 6.1%: https://fred.stlouisfed.org/series/UNRATE.

30. Shimer (2005) reports 0.27 and Pissarides (2009) finds 0.50.

TABLE 1
Externally calibrated parameters

	Definition	Value	Source
β	Discount factor	0.9976	Krusell *et al.* (2019)
r	Interest rate	0.002	Yearly risk-free rate of 2.5%
λ	Exogenous separation	0.017	Eeckhout and Lindenlaub (2019) (CPS)
$\underline{a}$	Borrowing constraint	-1	
b	Replacement rate	0.85	Average $\frac{b}{w}=0.4$, Shimer (2005)
k	Cost of vacancy	0.52	$\frac{k}{y}=21\%$, Shimer (2005)
γ	Elasticity of matching function	0.41	Job finding probability—CPS

TABLE 2
Endogenous outcomes

Moments	Model	Data	Source
$u\%$	6.1%	6.1%	Federal Reserve Bank of St. Louis (FRED)
avg. $(\frac{b}{w})$	0.40	0.40	Shimer (2005)
unemp. dur.	4	4.50	Federal Reserve Bank of St. Louis (FRED)
avg. $m(\theta)$	0.25%	0.25%	CPS from Eeckhout and Lindenlaub (2019)
med $(\frac{a}{w})$	0.49	0.46	PSID
$\frac{\text{tot. dividend}}{\text{tot. output}}$	5.3%	4.8%	Federal Reserve Bank of St. Louis (FRED)
elasticity of $m(\theta)$ to θ	0.40	[0.27–0.50]	Pissarides (2009) & Shimer (2005)
avg. $(\frac{k}{y})$	0.21%	0.21%	Shimer (2005)

1990 and 2010.[31] Moreover, the correlation between assets and the probability of job-finding is –0.77, indicating the lower hazard of job finding for richer unemployed workers. On average, a 1% rise in assets is associated with a 0.025% decline in the probability of job finding.

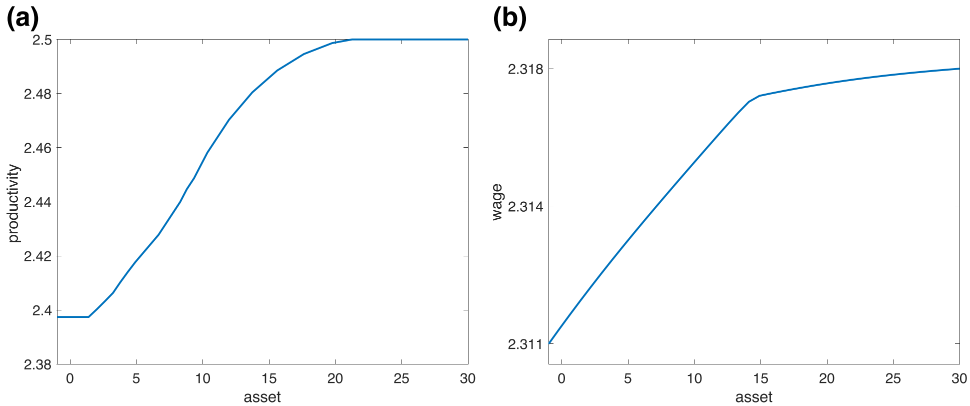
4.2. *Properties of the steady state equilibrium*

In this section, we study the key features of the steady-state equilibrium. We quantify the sorting of wealth to jobs of different productivities through precautionary search decisions of workers. We also provide model validation demonstrating the ability of the model to: (i) replicate the negative relationship between asset holding and job finding rate; (ii) capture the negative duration dependence.

There is positive assortative matching between workers' asset holdings and firms' productivity: in equilibrium workers with a higher level of assets are matched with more productive firms.[32] Figure 1(a) shows the allocation of workers to firms in the labor market. There is relatively more mass at the bottom of the asset distribution for unemployed workers compared to employed workers, and the less mass at the bottom of the productivity distribution, consistent with Equation (6). The market clearing condition implies that all workers are allocated to submarkets while firms below a productivity threshold are staying out of the market, a threshold obviously sensitive to different parameterizations of the model. In particular, below we study

31. The U.S. dividend pay-out ratio between 1990 and 2010 is 4.8%: https://fred.stlouisfed.org/graph/?g=t8O.

32. From Proposition 1, we know that under log preferences there is indeed positive sorting in the two period model. Because we cannot solve the general model analytically, we guess the allocation is positively assorted and verify ex-post that the match surplus along the equilibrium allocation is indeed supermodular, and the condition in Proposition 2 is satisfied.



Allocation of firms and workers in the labor market

Wages by asset level at hiring

FIGURE 1
Equilibrium allocation and wages

the impact of a change in unemployment benefits on the threshold and therefore on job creation. A higher threshold means more firms stay out of the market and hence fewer jobs are created.

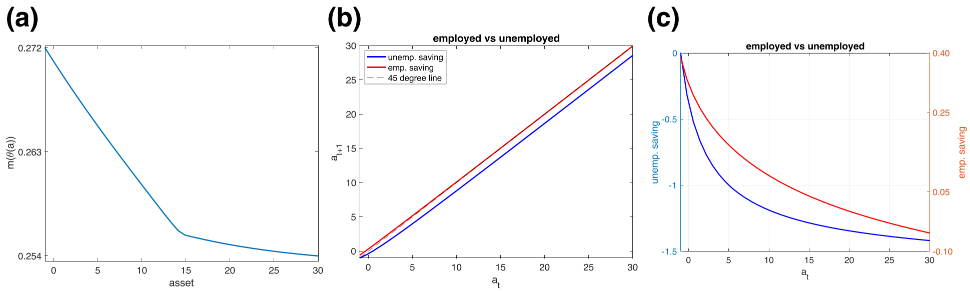
Figure 1(b) depicts equilibrium wages for different asset levels. Firms with more productive jobs post higher wages which decreases the vacancy to unemployment ratio θ and allows them to fill the vacancy with higher probability. Workers with more assets apply for the high wage jobs because they can better insure against unemployment as their assets allow them to maintain a higher level of consumption so they can afford to apply for riskier jobs.

Asset holding and job finding. Quantitatively, assets play a key role in the productivity of *equally skilled* workers. Workers with higher assets apply for jobs with a substantially higher productivity than those with low assets (2.50 versus 2.39, Figure 1(a)). They secure those better jobs because they take a substantially longer time to find a job than those with low asset holdings. As shown in Figure 2(a), the monthly matching probability decreases from 27.2% for the low asset unemployed workers to just over 25% for those with high asset levels, or a 7% fall in the monthly job finding probability.[33] At any level of assets, unemployed workers deplete their assets and subsequently adjust their job search strategy. If they do not find a job during this period and deplete their asset stock further, during the next period they apply for lower productivity jobs which they can obtain with a higher probability. This dependence of the job search decision on assets is absent in the basic random search model without search intensity: with random search, the probability of finding a job is the same for all workers regardless of their asset holding.

Also absent in models with homogenous firms, this channel has important implications for the role of UI benefits. A change in unemployment benefits only affects the measure of vacancies when all firms have identical productivity. In contrast, in our framework, a change in UI not only affects the measure of jobs created but also the productivity distribution of filled jobs and the productivity level in the economy even when the production function has constant returns to scale.

The endogenous matching probability explains why the wage function is increasing whereas it is mostly flat in the basic random search model. At first glance, the derivative of the wage

33. The difference in job finding probabilities between low and high asset holders is higher for low levels of UI benefits that for high benefits. For instance, at $b = 0$ the monthly matching probability decreases from 39.6% to 28.5%.



Job finding probability $m(\theta(a))$ Savings decision a_{t+1} Savings decision $a_{t+1} - a_t$

FIGURE 2

Policy functions of the unemployed

function appears small. However, since the average duration of employment equates to around 59 periods (5 years), these small wage differences translate into big income differences over the duration of employment. In other words, workers choose submarkets with different probabilities of job finding, and different wages for the whole duration of employment, which is reflected in the fact that the equilibrium value of employment $E(a)$ shows large variation.

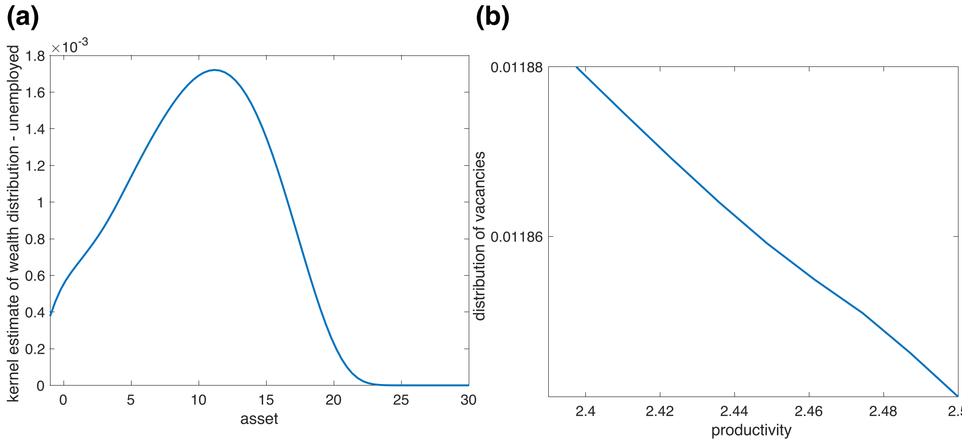
Interestingly, the dynamic nature of the problem now implies a time-varying job choice decision. A worker who fails to become employed sees her assets gradually deplete ($a_{t+1} < a_t$). But, the optimal search decision dictates application to less productive, lower wage jobs when assets are lower. As a result, over the duration of unemployment, workers will gradually apply for less productive, lower wage jobs that they get with higher probability. Instead, while employed, they gradually increase their assets. In Figure 2(b), we see that savings by the employed is higher than that of the unemployed, and that the unemployed always deplete their savings (a_{t+1} is below the 45 degree line). The employed with low asset levels accumulate assets. To make this crossing of the employed's policy function for savings with the 45 degree line more explicit, in Figure 2(c) we plot the difference between a_t and a_{t+1} for employed and unemployed workers.

The probability of job finding is significantly lower for high asset holders. Those who are unlucky and do not find a job run down their assets in order to smooth consumption. In this process, they gradually apply to lower productivity jobs. As a result of this endogenous job finding probability, workers move down in their asset holdings during unemployment which results in a stationary asset distribution for the unemployed, depicted in Figure 3(a).

On the firm side (Figure 3(b)), we observe a fatter left tail for the stationary distribution of firms posting vacancies compared to the distribution of firms in the population, which is uniform. High productivity firms have a higher option value of filling a vacancy, so they increase the probability of filling the vacancy by offering higher wages to the unemployed. More productive firms therefore leave the pool of firms with a vacancy faster than less productive ones. As a result, in the steady state fewer high productive firms are searching. In addition to the endogeneity of the vacancy distribution, the marginal firm $y^\star$ is also endogenous. This cutoff thus determines a measure of job creation. When we analyze the impact of UI policy below, we investigate how unemployment benefits affect the equilibrium allocation (including job creation).

To validate the important implication of the model on the negative relationship between asset holdings and probability of job findings in the data, we perform the following analysis. We run the cross-sectional regression

$$UE_i = \omega_0 + \omega_1 \tilde{a}_i + \nu_I.$$



kernel estimate of asset distribution unemployed workers $f_u(a)$

Productivity distribution vacant firms $g(y)$

FIGURE 3

Ergodic distributions (densities)

Where UE_i is the monthly transition rate from unemployment to employment for a given worker i. Following Burbidge *et al*. (1988), we apply a log-type transformation for wealth ($\tilde{a}$).[34] The estimated regression coefficient is statistically significant and negative ($\omega_1 = -0.0071$; see Appendix B for further details). The negative sign of this coefficient aligns with what the policy functions in our model prescribes. Richer workers apply for more productive jobs which offer them higher wages, but those jobs are more risky to secure. Lise (2013) runs the same regression on NLSY data after controlling for observables. He reports a regression coefficient of –0.0065 for high education workers and –0.0045 for low education workers using the same log-type transformation for wealth. Our model does a very good job at capturing this empirical relationship reported by Lise (2013).

Duration dependence. We now show quantitatively that the directed search model exhibits negative duration dependence. Workers with higher wages have shorter unemployment duration, as demonstrated in Figure 4. This finding reconciles the directed search model with one of the most robust facts regarding unemployment dynamics. Let us dig deeper and attempt to uncover how consumption smoothing leads to negative duration dependence, thus overturning the positive duration dependence inherent in the canonical directed search model without consumption smoothing.

In order to decompose the two opposing effects of negative duration dependence due to consumption smoothing and positive duration dependence due to directed search, we construct a simulation exercise that follows a cohort of newly unemployed workers with identical wealth. As we establish in Theorem 3, the likelihood of finding a job changes over time due to consumption smoothing. If two workers with the same wealth become unemployed at the same point in time, they have the same dissaving behavior but, the one who gets lucky and leaves unemployment first, has a shorter unemployment spell as well as higher reemployment wages. Why? When the first worker to leave finds a job, she has depleted less assets due to the shorter unemployment duration. Therefore, she can apply for higher productivity jobs with higher wages and lower job

34. In order to account for negative assets, we follow Burbidge *et al*. (1988) and Lise (2013) and transform liquid wealth a at the moment of falling into unemployment, by $\tilde{a} = \log(a + \sqrt{1 + a^2})$.

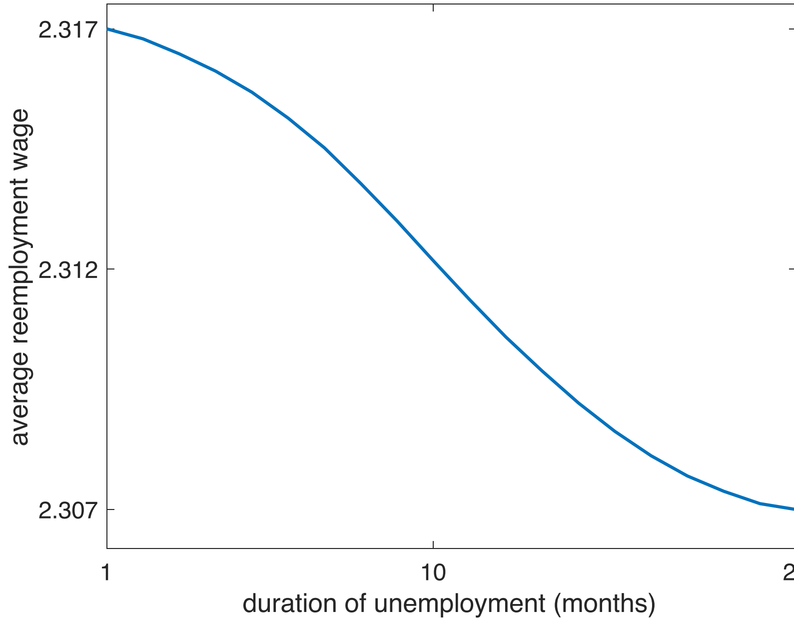


FIGURE 4
Unemployment duration & reemployment wages

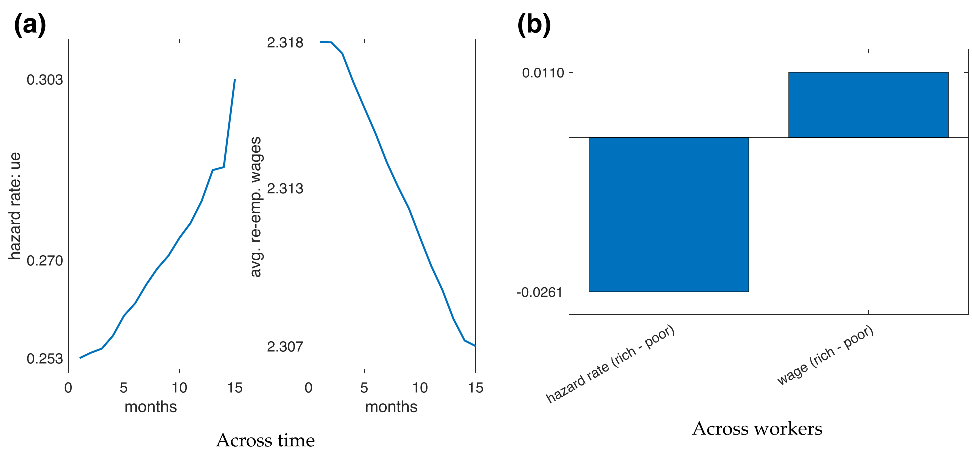


FIGURE 5
Duration dependence

finding probabilities. Figure 5(a) depicts the job finding rate and wage for the cohort of workers who all initially entered with the same amount of wealth. We then keep track of the unemployment duration as well as their reemployment wages and hazard rate of leaving unemployment (the *UE* transition rate). The correlation coefficient between the hazard rate (which is the inverse of unemployment duration) and reemployment wages is −0.87.

However, Figure 5(a) does not illustrate the full story. Just like in the canonical model of directed search, positive duration dependence across workers still exists in a given cohort, which implies that the high asset workers who seek higher wages also have a higher unemployment duration. This positive duration dependence comes from the workers' indifference condition implying that better paid jobs attract more applicants and therefore obtaining those jobs is more difficult for workers, while easier for firms to fill the open positions. Figure 5(b) shows the difference between the rich and the poor in the hazard rate and the wage in the cross section. The rich have lower job finding rates and higher wages, which leads to positive duration dependence.

As illustrated in Figure 4, we find that the overall duration dependence that combines these two effects is negative: unemployment duration is declining in wages, which establishes that in our calibration the consumption smoothing effect across time dominates the canonical directed search effect, a finding consistent with the econometric literature on duration dependence (for a recent paper see Ahn and Hamilton, 2020) that finds that selection plays a dominant role over direct duration dependence such as skill depreciation. Ahn and Hamilton (2020) find that typically selection can be explained by skill heterogeneity, in brief, that workers with higher productivity find jobs faster. Here, we show that heterogeneity in asset holdings is an alternative determinant for the job finding rate and hence selection.[35]

To validate the empirical content of this prediction of the model, we run a regression with the reemployment wage as the dependent variable and the duration of unemployment as the independent variable for each individual i.

$$w_i = \zeta_0 + \zeta_1 \mathrm{dur}_i + \psi_i.$$

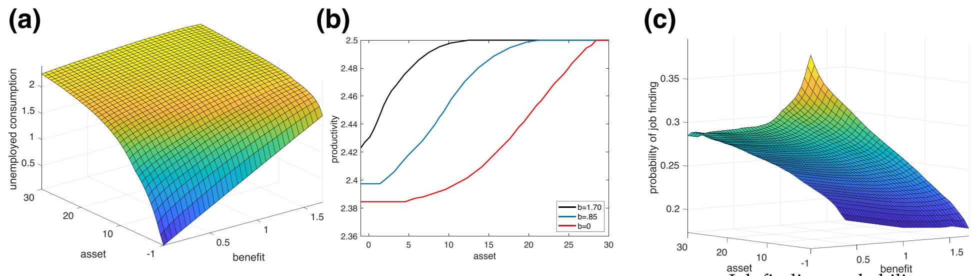
The estimated regression coefficient for duration ζ_1 is equal to –0.0006 (see Appendix B for further details on the data and the regression). This estimate captures the inverse relationship between duration of unemployment and reemployment wages. We estimate the same regression using NLSY after controlling for observables and the estimated coefficient is –0.0013. Our model captures half of this documented stylized fact. In the absence of unobserved heterogeneity in the model and given that the only source of heterogeneity in the model for workers is asset holding, the model does a good job of capturing this empirical relationship. In our model, the only channel for duration dependence is wealth depreciation during unemployment. Of course, in the data other determinants such as skill or network depreciation during joblessness also play important roles in strengthening the negative duration dependence.

4.3. *Equilibrium effects of UI*

We now study the impact of different unemployment benefits on the equilibrium allocation. In the absence of complete markets to insure employment risk, we ask how changes in the government-mandated UI policy affects welfare.[36] UI directly impacts workers by allowing them to smooth consumption and apply for more productive jobs with lower job finding probabilities. However, UI also affects welfare through various general equilibrium channels. In the first place, higher UI reduces the firm's share of the match surplus. With a higher outside option, workers command a higher wage, thereby reducing job creation as only firms with higher productivity enter the market to post vacancies. Similar to the mechanism discussed in Krusell *et al.*

35. Our mechanism of course does not preclude the role of other mechanisms in generating negative duration dependence. Prominent examples include statistical discrimination against the long-term unemployed as in Jarosch and Pilossoph (2019) which leads to negative duration dependence. Related, Gonzalez and Shi (2010) analyze the role of workers' beliefs about their employment prospects in the presence of learning and how this affects duration dependence.

36. Throughout we assume the budget is balanced.



Consumption of unemployed — Allocation for different benefits — Job finding probability

FIGURE 6

Consumption, equilibrium allocation, and job finding probability for different levels of benefits

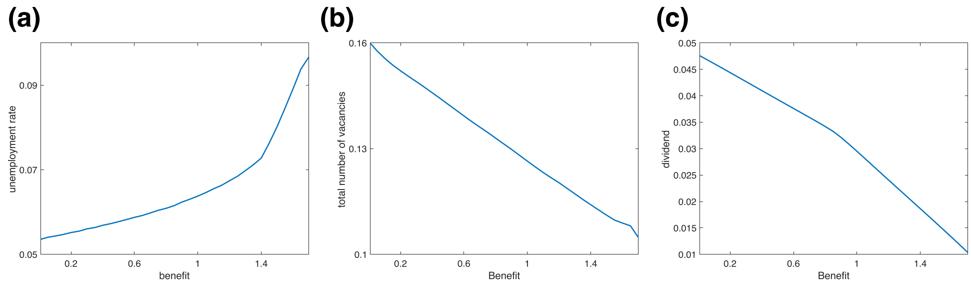
(2010) with random search, though now with two-sided heterogeneity, a change in unemployment benefits also moves the productivity threshold above which firms enter the market while with homogenous firms, this change only affects the measure of job openings but not the quality of jobs.

In our framework with heterogeneous productivity and sorting, direct insurance against unemployment affects the distribution of unemployed workers by influencing their saving decisions and therefore their allocation to jobs of different productivities. Guaranteed higher unemployment benefits, workers save less while employed, and as a result they hold fewer assets while unemployed. In addition to the savings decision, benefits also affect the workers' job search behavior. Since workers with different asset levels direct their search to firms of different productivity, higher benefits increase the unemployment rate (through a reduction in the matching probability) as well as increase the productivity of jobs to which workers apply. With less necessity to use their own assets for self-insurance because of higher benefits, workers more willingly take risks take and increasingly direct their search towards high productivity jobs that pay higher wages at the expense of lower matching probabilities.

The general equilibrium effect of unemployment benefits are made explicit in the following series of Figures wherein the benchmark economy we vary the benefits b between 0 and 1.70.[37] First, consider the impact on consumption of the unemployed (Figure 6(a)). For all asset holders, equilibrium consumption of the unemployed increases in benefits. The effect however is much more pronounced for the low asset holders. In fact, those with assets close to the borrowing constraint nearly exclusively consume the entire benefits. For the high asset holders, benefits have a much more moderate impact on consumption.

Figure 6(b) illustrates the impact of benefits on the job search behavior and the resulting equilibrium allocation. When benefits are higher, all workers direct their search to more productive, high paying jobs. As a result, for all asset levels, the allocation of assets to productivities shifts upwards as benefits increase. Moreover, higher levels of benefits not only shift the allocation function to the left (more productive jobs), but they also increase the entry threshold of firms. Moving from the laissez faire economy with zero benefits to an economy with a replacement rate of 80% increases the productivity threshold of jobs by 1.6%, and increases the aggregate productivity of jobs by 1.5%. As benefits increase the productivity of the jobs, they also increases the competition for jobs and hence decrease the job finding probability (Figure 6(c)). This decrease

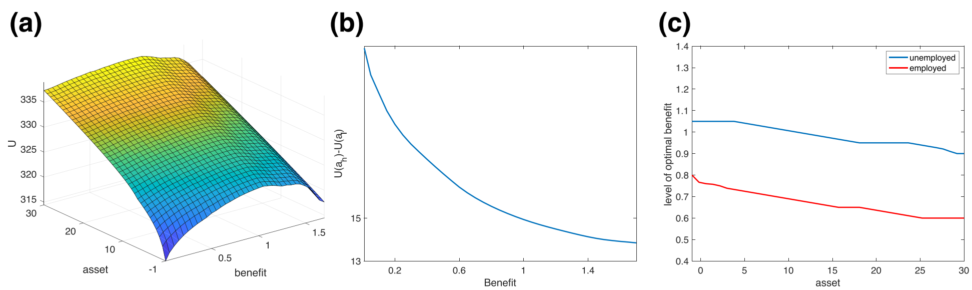
37. The average wage is endogenous, and in our simulated economies this range of benefits corresponds to the range of 0% and 80% of average wages. Recall that in all counterfactual economies the government budget is balanced (total benefit is equal to total tax) and firm dividends are distributed uniformly across all workers.



Unemployment rate — Total number of vacancies — Firms' dividend

FIGURE 7

Equilibrium unemployment, vacancy creation and dividends for different levels of benefits



Value of unemployment — Difference value unemployment — Optimal Benefit by Assets

FIGURE 8

Value of unemployment

is much more pronounced for the low asset holders. Benefits induce them to compete for higher productivity jobs.

Not surprisingly then, the impact of increased benefits is an increase in aggregate unemployment (Figure 7(a)). The unemployment rate goes up by almost 4 percentage points as benefits increase from 0 to 1.70. At the same time, the number of firms entering the market decreases only modestly: the cutoff $y^{\star}$ goes from 2.38 to 2.42. Note also the huge decline in the average number of vacancy rate in equilibrium, going from 0.16 to 0.11 (Figure 7(b)). Firms leave the market faster (because θ, the ratio of vacancies to unemployed searchers has fallen), hence the total equilibrium number of vacancies drops by half, resulting in a significant effect on job creation. Moreover, a rise in UI means that workers have higher outside options and that increases wages which in turn reduces workers' consumption from firms dividends. Figure 7(c) depicts the fall in the firms' dividend as a result of rise in UI.

Higher benefit levels clearly pull the value of unemployment in opposing directions: search for better jobs and more consumption smoothing on the one hand, but lower vacancy creation, lower job finding rates and lower dividends on the other hand. To evaluate the overall impact, we look at the option value of unemployment as a function of assets and benefits, illustrated in Figure 8(a).[38]

38. Given log preferences, the variation in utility is nominally small, even if assets and benefits drop to zero.

A rise in unemployment benefits affects the value of unemployment in the following ways: (i) increasing consumption; (ii) decreasing job finding probability; (iii) decreasing firm entry; (iv) decreasing dividends; and (v) increasing gross wages.[39] While effect (i) increases the value of unemployment, the next three generate the opposite effect. Higher benefits means more insurance and consumption smoothing during unemployment, but when UI goes up, workers tend to apply for better paying jobs with a lower job finding probability. Moreover, higher benefits imply less entry of firms which contributes further to the lower job finding rates in the aggregate, and they also imply higher wages because workers have a higher outside option which in turn reduces the surplus of firms and therefore the dividend workers receive.

The change in the value of unemployment depends on the level of asset holdings. While $U(a)$ has an inverted U-shape for all levels of asset holdings, the maximum value of unemployment is achieved at higher levels of UI for low asset holders compared with rich workers. However, the effect of benefits is much more pronounced for the asset poor unemployed workers. Because they have a high marginal utility of consumption, the insurance effect of unemployment benefits is much stronger. Figure 8(b) depicts the difference in value of unemployment for a high asset unemployed worker compared with a low asset holder for all benefit levels. When UI increases, this difference shrinks because high asset holders need less insurance while they suffer more from lower probabilities of job findings.

While not immediately obvious from inspecting the three-dimensional figure, the value of unemployment attains its maximum at an interior benefit for the entire asset domain of the unemployed. Figure 8(c) depicts the level of benefits at which the value of unemployment and employment is maximized for each level of assets.

Low asset holders prefer higher benefits, and those preferred benefits are decreasing in assets. The unemployed with the highest assets prefer lower levels of benefits because they have a relatively low marginal utility of consumption. The negative impact of being taxed after becoming employed as well as the lower probability of job finding dominate the little extra marginal utility of consumption during unemployment and therefore workers prefer lower levels of benefits. However, if these workers do not find jobs and deplete their asset during unemployment, they end up preferring higher levels of benefits when their assets run down and their marginal utility of consumption goes up. In other words, with fewer assets the relative importance of higher consumption increases compared to the probability of job finding. Figure 8(c) also plots the optimal benefit for the employed. At each levels of asset holdings, employed workers prefer lower levels of benefits. For them the negative impact of the taxes has a bigger impact relative to the positive insurance effect of the benefit (in the case of losing their jobs) compared to unemployed workers. Interestingly, as they build up their asset stock during employment, their need for external insurance falls and therefore they prefer lower levels of benefits.

Overall, this suggests that for high asset holders, the allocation and probability of job finding effect dominates the consumption smoothing effect. For workers who already have a high level of assets, an increase in UI does not affect their marginal utility of consumption much but it does considerably reduce workers' probability of job finding. In contrast, low asset holders have high marginal utility of consumption. As mentioned, the highest unemployed asset holders prefer some benefits because life becomes dire without any assets or benefits, and even the high asset holders have a positive probability of reaching that outcome.

39. All this implies a reduction in net wages when the threshold of entry for firms in not changed.

4.4. *Welfare*

To study the welfare impact of a change in unemployment benefits, we compare steady states with different levels of UI. We measure welfare gains or losses by computing the percentage change in life time consumption required to give workers the steady state average lifetime utility. In our welfare analysis, we follow Krusell *et al*. (2010) and fix the distribution of workers' asset holdings at the benchmark economy. This implies that to compare the counterfactuals economies with the benchmark, we move all workers to a different economy which has a different level of UI but is otherwise identical to our benchmark, and then measure the consumption losses or gains of workers across the asset distribution. We hold fixed the distribution of assets in the welfare calculation so that welfare is always compared from the perspective of the same agents. We can thus isolate the welfare effect from a change in the distribution of asset holding.[40]

Our welfare measure in these comparisons is denoted by ψ. c_t is the consumption in the benchmark economy and $\hat{c}_t$ is the consumption in any of the counterfactual experiments.[41] Welfare is calculated satisfying the following condition:

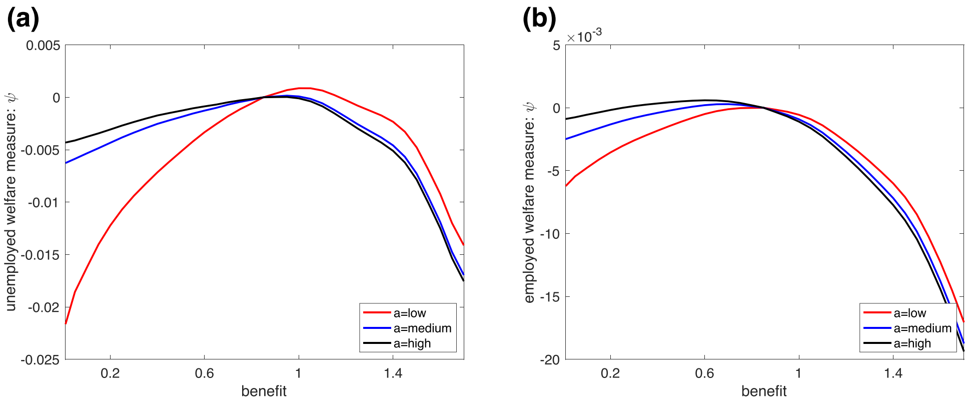
$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \log((1+\psi)c(a_t))\right]=\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \log(\hat{c}(a_t))\right]. \tag{17}$$

Figure 9 depicts the welfare measure for an unemployed and employed worker with low (P25), medium (P50), and high (P75) asset holdings at different economies. The net gain or loss of changing UI is heterogenous across the distribution of asset. The welfare of all unemployed workers is inverted U-shaped in benefits (Figure 9(a)). They all gain from moving to an economy with a higher level of benefit up to a certain level of UI and then their utility falls when UI increases further. In addition, maximal welfare is a decreasing function of asset holdings. Asset poor unemployed workers have a higher marginal utility of consumption and value insurance more. Therefore their welfare gain presents higher than that of workers with a medium or high level of asset holdings. The asset rich unemployed workers care more about their employment probability as they have enough assets to smooth consumption, while poor unemployed workers care more about the direct insurance effect of UI as they have high marginal utility of consumption. For this reason, asset richer unemployed workers' welfare gain is less than that of other groups and also why their welfare falls more when UI increases further.

The welfare gains or losses also differ across the distribution of asset holdings for employed workers (Figure 9(b)). Rich employed workers have lower welfare gains when UI goes down and face a higher welfare loss when UI increases. These workers have already high levels of asset holdings and can insure themselves well in case they lose their jobs. Instead, higher benefits also mean higher levels of taxes for them. In contrast, asset poor employed workers gain more welfare when UI increases from zero, since they value external insurance as they do not have enough assets to smooth their consumption in case they lose their jobs. However, their welfare is maximized close to the benchmark economy level of UI. Increasing UI more is detrimental to welfare for these workers, affecting their net wages through taxation, while not providing substantially more insurance at the margin.

40. We have repeated the entire welfare analysis with endogenous asset distributions, finding similar results. If anything, the qualitative findings are more pronounced. The results are available upon request.

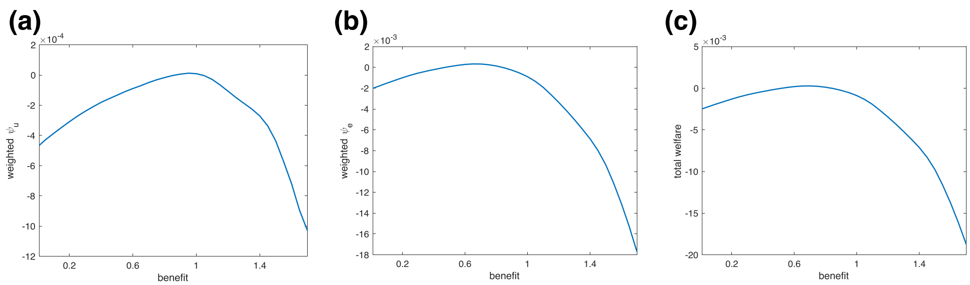
41. The value of consumption in the benchmark economy is $V_i=\mathbb{E}_0[\sum_{t=0}^{\infty}\beta^t\log(c_i(a_t))]$, and in the counterfactual economy is $\hat{V}_i=\mathbb{E}_0[\sum_{t=0}^{\infty}\beta^t\log(\hat{c}_i(a_t))]$ where $i\in\{u,e\}$, where the expectations operator is taken over the labor market uncertainty. The welfare gain or loss, ψ, can be calculated as $\psi_i=\exp[(1-\beta)(\hat{V}_i-V_i)]-1$.



unemployed workers   employed workers

FIGURE 9

Welfare measure: ψ



Average value unemployed   Average value employed   Total welfare

FIGURE 10

Welfare

To calculate the aggregate measure of the welfare change, we integrate the welfare measure ψ over the distribution of asset holdings in the benchmark economy, as depicted in Figure 10. Although the welfare function is inverted U-shaped for both employed and unemployed workers, the welfare maximizing level of benefits for unemployed workers is nearly 32% higher than that of the employed. This finding again highlights the value of direct insurance for unemployed workers. Since employment consists of 90–95% of the total labor force and unemployment only 5–10%, the aggregate welfare function has a similar shape to the one of the employed workers.

In Table 3, we report the welfare change for different benefit levels relative to the benchmark economy benefits of $b = 0.85$. Again, the net welfare gain is heterogenous across the distribution of workers. The last six columns aim to capture the heterogeneity for different asset holdings within the pool of employed and unemployed. A low asset unemployed worker gains more than 2.16% by moving from a laissez-faire economy to the benchmark while an asset rich unemployed worker only gains 0.45%. Increasing benefits further to 1.10 results in a 0.1% further rise for an asset poor unemployed worker compared to the benchmark economy while, it reduces the welfare of an asset rich unemployed worker.

A low asset employed worker gains 0.62% if they move from the Laissez-faire economy to the benchmark economy. An asset rich employed worker gain 15% of that amount of welfare in the same situation. For higher levels of benefits than the benchmark economy most employed

TABLE 3

Welfare change compared to benchmark economy (first three columns are welfare gains for all workers, the unemployed, and the employed. The last six columns illustrate the welfare gains by an unemployed and employed worker with low(l), medium(m) and high(h) asset holding

from $b=0.8$ to $b=$	Total %	Unemp. %	Emp. %	$a_{u,l}$ %	$a_{u,m}$ %	$a_{u,h}$ %	$a_{e,l}$ %	$a_{e,m}$ %	$a_{e,h}$ %
0	−0.25	−0.05	−0.20	−2.16	−0.63	−0.43	−0.62	−0.25	−0.09
0.10	−0.19	−0.04	−0.15	−1.58	−0.54	−0.39	−0.47	−0.19	−0.06
0.35	−0.06	−0.02	−0.04	−0.81	−0.29	−0.20	−0.22	−0.06	0.03
0.45	−0.02	−0.02	−0.01	−0.62	−0.22	−0.14	−0.15	−0.02	0.04
0.70	0.03	−0.01	0.04	−0.18	−0.07	−0.05	−0.01	0.04	0.06
0.85	0	0	0	0	0	0	0	0	0
0.95	−0.05	0.00	−0.05	0.08	0.02	0.01	−0.02	−0.05	−0.06
1.10	−0.18	−0.00	−0.18	0.10	−0.04	−0.07	−0.12	−0.18	−0.21
1.30	−0.52	−0.02	−0.50	−0.13	−0.32	−0.37	−0.42	−0.52	−0.57
1.55	−1.10	−0.05	−1.05	−0.60	−0.86	−0.92	−0.96	−1.11	−1.17
1.70	−1.87	−0.10	−1.77	−1.41	−1.70	−1.75	−1.71	−1.87	−1.94

workers start to experience a welfare loss. The losses are substantially higher for rich employed workers. They experience a welfare loss of more than 1.9% if they move from the benchmark to a counterfactual economy with twice more generous benefits, while the same loss is 11% lower for poor employed workers.

4.5. *The effect of benefits on worker productivity*

A novel feature of our model compared to the models with identical firms is the impact of UI changes on worker productivity. Homogenous firms produce using a linear production technology, so a rise in benefits only affects the measure of firms entering the market, and leaves job productivity unaffected. However, in this framework, a rise in UI affects worker productivity because the worker allocation to jobs changes, which in turn affects the firms' entry decision. Figure 11(a) depicts the percentage change in total output, and 11(b) shows percentage change in average output per worker for different levels of benefits compared to the benchmark economy. By construction, at the benchmark $b=0.85$, the change is zero.

We find that the total output is decreasing in benefits. When UI goes up, there are three countervailing forces: (1) all workers tend to apply to more productive jobs; (2) savings and therefore asset holdings are lower; and (3) the threshold of firms' entry moves up so lower productive firms do not find it profitable to enter the market. Since higher UI benefits are associated with less filled jobs, total production in the economy falls, as depicted in Figure 11(a). However, Figure 11(b) shows that when UI increases workers tend to apply to more productive jobs indicating that the incentive effect dominates the effect of lower asset holdings, exhibiting the impact of higher asset holding as well as a higher entry threshold of firms. Therefore, labor productivity increases by up to 1% compared with the benchmark economy. Benefits affect productivity positively at the intensive margin as each worker is more productive, and also affect output produced negatively at the extensive margin as fewer workers hold a job. The net effect on total production is negative.

This measure of output per worker is not equal to the measure of welfare because it does not take into account the employment probability, which is decreasing with benefits, nor the measure of the unemployed. We know from the welfare calculations that welfare is inverted U-shaped in benefits, indicating that benefits increase worker productivity of the asset rich, while the decrease

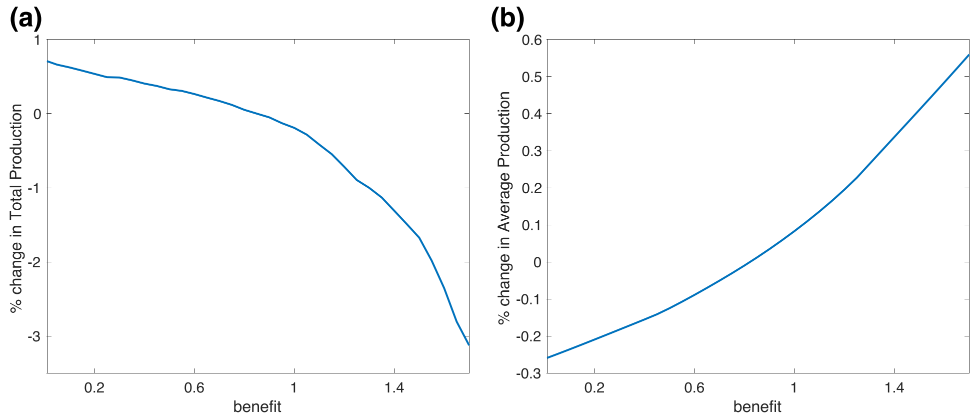


FIGURE 11
Total output and productivity

in overall employment (just over four percentage points) is limited. The incentive effect therefore remains key to understanding the change in welfare from an increase in benefits.[42]

5. CONCLUSION

We have analyzed the effect of asset holdings on worker productivity in the presence of frictional job search. In the absence of complete insurance markets, workers have a precautionary search motive: the job search decision provides an important source of self-insurance for those with low assets levels. To analyze this, we solve a model with directed search and consumption smoothing where workers with high asset holdings sort into more productive jobs. Because asset holdings allow workers to smooth consumption, they can afford to face a substantially lower job finding probability. The difference in the job finding probability depends on the benefit levels. For our benchmark calibration, we find that the job finding rate for those with high assets is 10% lower, and it is 18% lower if benefits are zero, consistent with independent findings that the poor without liquid assets find jobs faster (Chetty, 2008; Lise, 2013; Baley *et al.*, 2021).

An important insight of our analysis illustrates that the presence of consumption smoothing can address a major shortcoming of the canonical directed search model. When preferences are linear, the directed search predicts positive employment duration dependence on wages, which is counterfactual. Workers who direct their search to higher wage jobs face longer queues and thus stay unemployed longer. When workers are risk averse however, the unemployed deplete their assets while job searching which forces them to apply for low wage jobs, which in turn induces negative duration dependence on wages. In the quantitative exercise, we find that negative duration dependence dominates.

Key to the mechanism is that workers sort into firms with different levels of productivities based on their assets. Even if workers are identically skilled, there is nonetheless a preferences

42. In the online Appendix, we also compare per-period benefits to a one-off severance payment. Severance pay provides better job search incentives, but comes at the cost of poorer consumption smoothing. We find that per-period benefits dominate severance pay for low benefits but not for high benefits. At low benefits, workers value the insurance effect more than the search incentive effect.

complementarity between assets and productivity. We derive conditions under which the model exhibit PAM or NAM and use the sorting mechanism to solve for a non-degenerate distribution of assets in the infinite horizon problem.

In the quantitative analysis, we calibrate our model to the U.S. economy and analyze the welfare effect of an income tax financed UI policy. Not only does the usual conflict of interest persist between the unemployed who receive the benefits and the employed who pay for those benefits, a conflict emerges between the workers with and without assets. Both receive benefits, but the rich can rely more on their savings for insurance. When we aggregate the welfare losses and gains over the distribution, we show that the welfare function is inverted U-shaped in benefits. At lower levels of benefits, the insurance effect of UI dominates the incentives. However, when UI increases, workers tend to care more about the negative effect of insurance on job finding and their welfare gains start to diminish.

A novel feature of our model captures the impact of UI benefits on workers' productivity. UI affects the average productivity of workers through (1) the allocation of workers to jobs of different productivities; and (2) through the entry decision of firms. We show that for low UI benefit levels, an increase in benefits has no effect on average productivity per worker. However, for high enough benefit levels, a rise in benefits pushes up wages which in turn reduces the entry of firms with lower productivities. This increases average worker productivity, while at the same time lowering total output.

APPENDIX

A. Proofs and derivations

A.1. *Partial derivatives of U and a′*

From Equation (11), we calculate the derivates:

$$
\begin{aligned}
U_y(a_1) &= \beta m u'(c_{e,2}) + U_{a_2}\frac{\partial a_2}{\partial y} + U_{\theta_1}\frac{\partial \theta_1}{\partial y} = \beta m u'(c_{e,2}) \\
U_a(a_1) &= R u'(Ra_1 - a_2) + U_{a_2}\frac{\partial a_2}{\partial a_1} + U_{\theta}\frac{\partial \theta_1}{\partial a_1} = R u'(a_1 - a_2) \\
U_V(a_1) &= \beta m u'(c_{e,2})\frac{-1}{\beta q} + U_{a_2}\frac{\partial a_2}{\partial V} + U_{\theta_1}\frac{\partial \theta_1}{\partial V} = \beta u'(c_{e,2})\frac{-\theta}{\beta} \\
U_{ay}(a_1) &= -R u''(Ra_1 - a_2)\frac{\partial a_2}{\partial y} \\
U_{aV}(a_1) &= -R u''(Ra_1 - a_2)\frac{\partial a_2}{\partial V}
\end{aligned}
$$

where $c_{e,2} = Ra_2 + y - \frac{V}{\beta q(\theta_1)}$ and where $U_{a_2} = 0$ and $U_{\theta_1} = 0$ from the envelope theorem.

Denote the maximand of U by $\phi(a_2, \theta_1) = u(Ra_1 - a_2) + \beta[mu(c_{e,2}) + (1-m)u(Ra_2)]$, *i.e.* the objective function that is maximized with respect to a_{2,θ_1}. We calculate the derivative of a_2 using the implicit function theorem. For the problem to have a maximum, we require that the Hessian of the maximand is positive $|\mathbf{H}| > 0$ (recall that $\phi_{\theta_1\theta_1}$ is assumed negative), where:

$$
|\mathbf{H}| = \begin{vmatrix} \phi_{a_2 a_2} & \phi_{a_2\theta_1} \\ \phi_{\theta_1 a_2} & \phi_{\theta_1\theta_1} \end{vmatrix}
$$

Applying the implicit function theorem,

$$\frac{\partial a_2}{\partial y} = -\frac{\begin{vmatrix}\phi_{a_2 y} & \phi_{a_2\theta_1}\\ \phi_{\theta_1 y} & \phi_{\theta_1\theta_1}\end{vmatrix}}{|\mathbf{H}|} = \frac{\phi_{a_2 y}\phi_{\theta_1\theta_1} - \phi_{\theta_1 y}\phi_{a_2\theta_1}}{|\mathbf{H}|} \quad \text{and} \quad \frac{\partial a_2}{\partial V} = -\frac{\begin{vmatrix}\phi_{a_2 V} & \phi_{a_2\theta_1}\\ \phi_{\theta_1 V} & \phi_{\theta_1\theta_1}\end{vmatrix}}{|\mathbf{H}|}$$
$$= \frac{\phi_{a_2 V}\phi_{\theta_1\theta_1} - \phi_{\theta_1 V}\phi_{a_2\theta_1}}{|\mathbf{H}|}$$

A.2. *Proof of Proposition 1*

Proof. $U_{a_1 y} > \frac{U_y}{U_V} U_{a_1 V}$ provided (where the partial derivatives of U are derived in the Appendix):

$$-Ru''(Ra_1 - a_2)\frac{\partial a_2}{\partial y} > \frac{\beta m u'(c_{e,2})}{\beta u'(c_{e,2})\frac{-\theta_1}{\beta}}\left(-Ru''(Ra_1 - a_2)\frac{\partial a_2}{\partial V}\right)$$
$$\frac{\partial a_2}{\partial y} > -\beta q\frac{\partial a_2}{\partial V}$$

We obtain the expressions for $\frac{\partial a_2}{\partial y}$ and $\frac{\partial a_2}{\partial V}$ from the first-order conditions (above). Then the condition for positive sorting of a on y becomes:

$$\left(\phi_{a_2 y} + \beta q \phi_{a_2 V}\right)\phi_{\theta_1\theta_1} < \left(\phi_{\theta_1 y} + \beta q \phi_{\theta_1 V}\right)\phi_{a_2\theta_1}$$

Observe that from the first-order conditions to the maximization problem, we obtain the cross partials on ϕ. First, note that $\phi_{a_2 y} = -\beta q \phi_{a_2 V} = \beta R m u''(c_{e,2})$ so that the left-hand side is zero. This follows from the envelope theorem since ϕ is maximized with respect to a_2 and θ_1. Then we derive the following:

$$\phi_{\theta_1 y} = \beta m' u'(c_{e,2}) + \beta u''(c_{e,2})\frac{\theta_1 q' V}{\beta q}$$
$$\phi_{\theta_1 V} = \beta m' u'(c_{e,2})\frac{-1}{\beta q} + \beta u'(c_{e,2})\frac{\theta_1 q'}{\beta q} + \beta u''(c_{e,2})\frac{-1}{\beta q}\frac{\theta_1 q' V}{\beta q}$$
$$= \frac{-1}{\beta q}\phi_{\theta_1 y} + \beta u'(c_{e,2})\frac{\theta_1 q'}{\beta q}$$

Therefore, the inequality can be written as:

$$0 < \beta u'(c_{e,2})\theta_1 q' \phi_{a_2\theta_1}$$

The term $\beta u'(c_{e,2})\theta_1 > 0$ but $q' < 0$, so the condition for positive sorting of a_1 on y is $\phi_{a_2\theta_1} < 0$. Equivalently:

$$\beta R\left(m'[u'(c_{e,2}) - u'(Ra_2)] + u''(c_{e,2})\frac{\theta_1 q' V}{\beta q}\right) < 0.$$

From the first-order condition $\phi_{\theta_1} = 0$ we obtain:

$$\frac{\theta_1 q' V}{\beta q} = -m'\frac{u(c_{e,2}) - u(Ra_2)}{u'(c_{e,2})}.$$

Substituting in the condition $\phi_{a_2\theta_1} < 0$:

$$m'[u'(c_{e,2}) - u'(Ra_2)] - u''(c_{e,2})m'\frac{u(c_{e,2}) - u(Ra_2)}{u'(c_{e,2})} < 0,$$

or, noting that $m' > 0$,

$$u'(c_{e,2})[u'(c_{e,2}) - u'(Ra_2)] < u''(c_{e,2})\left[u(c_{e,2}) - u(Ra_2)\right].$$

or alternatively

$$\frac{u'(c_{e,2}) - u'(Ra_2)}{u(c_{e,2}) - u(Ra_2)} < \frac{u''(c_{e,2})}{u'(c_{e,2})}.$$

This condition is equivalent to DARA by the mean value theorem: the left-hand side of condition (**U**) can be written as $u''(z)/u'(z)$ for some z in the interval $(c_{e,2}, Ra_2)$.

Finally, uniqueness of the equilibrium allocation follows from the fact that the inequality in condition (**U**) is strict and the fact that this is effectively a static problem with exogenous types. Legros and Newman (2007) establish uniqueness under condition (16), ensured by the measure-preserving market clearing condition, as long as types are exogenous. □

A.3. *Proof of Corollary 1*

Proof. We calculate the derivatives:

$$u'(c) = \alpha\left(\frac{\alpha c}{1-\gamma} + \beta\right)^{\gamma-1}$$
$$u''(c) = -\alpha^2\left(\frac{\alpha c}{1-\gamma} + \beta\right)^{\gamma-2}$$

and condition (**U**) becomes (where $c = Ra_2$):

$$\alpha\left(\frac{\alpha c_e}{1-\gamma} + \beta\right)^{\gamma-1}\left[\alpha\left(\frac{\alpha c_e}{1-\gamma} + \beta\right)^{\gamma-1} - \alpha\left(\frac{\alpha c}{1-\gamma} + \beta\right)^{\gamma-1}\right]$$
$$< -\alpha^2\left(\frac{\alpha c_e}{1-\gamma} + \beta\right)^{\gamma-2}\left[\frac{1-\gamma}{\gamma}\left(\frac{\alpha c_e}{1-\gamma} + \beta\right)^{\gamma} - \frac{1-\gamma}{\gamma}\left(\frac{\alpha c}{1-\gamma} + \beta\right)^{\gamma}\right]$$

and after dividing by α^2 and by $(\frac{\alpha c_e}{1-\gamma} + \beta)^{2\gamma-2}$, which under our assumptions are both positive, this implies:

$$1 - \left(\frac{\frac{\alpha c}{1-\gamma} + \beta}{\frac{\alpha c_e}{1-\gamma} + \beta}\right)^{\gamma-1} < -\frac{1-\gamma}{\gamma}\left[1 - \left(\frac{\frac{\alpha c}{1-\gamma} + \beta}{\frac{\alpha c_e}{1-\gamma} + \beta}\right)^{\gamma}\right],$$

or

$$1 - x^{\gamma-1} < -\frac{1-\gamma}{\gamma}\left[1 - x^{\gamma}\right] \text{ where } x = \frac{\frac{\alpha c}{1-\gamma} + \beta}{\frac{\alpha c_e}{1-\gamma} + \beta} \in (0, 1).$$

First consider $\gamma > 0$. After rearranging and multiplying by $\gamma x^{1-\gamma}$, which is positive for $\gamma > 0$:

$$x^{1-\gamma} - (\gamma + (1-\gamma)x) < 0$$
$$G(\gamma) - H(\gamma) < 0.$$

At $\gamma = 0$ and $\gamma = 1$ the expression is exactly zero, *i.e.* G and H cross at 0 and 1. Now, $G'(\gamma) = -x^{1-\gamma}\log x$, $H'(\gamma) = 1 - x$, and $G''(\gamma) = x^{1-\gamma}(\log x)^2 > 0$, $H''(\gamma) = 0$. Observe that $G(\gamma)$ is convex, $G''(\gamma) = x^{1-\gamma}(\log x)^2 > 0$, while $H(\gamma)$ is linear. As a result, for $\gamma \in (0, 1)$ condition (**U**) holds with strict inequality. For $\gamma = 1$, (**U**) holds with equality and for $\gamma > 1$ it holds with opposite inequality.

Now consider $\gamma < 0$. Since we multiplied by $\gamma < 0$, condition (**U**) now implies that $G(\gamma) - H(\gamma) > 0$. Using the same logic, we establish that condition (**U**) holds for $\gamma < 0$.

This establishes that for a risk averse worker with HARA utility function, condition (**U**) holds strictly if and only if $\gamma < 1$, *i.e.* there is DARA.

All the other cases can immediately be verified from the logic above, except for the case of CARA. There, $u'(c) = \alpha e^{-\alpha c}$, $u''(c) = -\alpha^2 e^{-\alpha c}$, so that condition (**U**) becomes:

$$\alpha e^{-\alpha c_e}\left(\alpha e^{-\alpha c_e} - \alpha e^{-\alpha c}\right) \le -\alpha^2 e^{-\alpha c_e}\left(1 - e^{-\alpha c_e} - 1 + e^{-\alpha c}\right)$$
$$e^{-\alpha c_e} - e^{-\alpha c} \le -\left(-e^{-\alpha c_e} + e^{-\alpha c}\right)$$

which holds with equality. □

A.4. *Infinite horizon: special case when $\beta R = 1$ and $\lambda = 0$*

These assumptions imply that asset levels when employed are invariant in steady-state equilibrium, since the employed workers consume a share of their assets exactly equal to the dividend. In that case, $a_{t+1} = \frac{a_t}{R} = \beta a_t$ and the value for employment is independent of $U(a_t)$. As a result, the employed worker's problem can be solved explicitly. The first-order condition of the employed worker is $u'(w + a_t - a_{t+1}) = \beta R E'(R a_{t+1})$. With $\beta R = 1$ and $\lambda = 0$ the solution is $a_{t+1} = \frac{a_t}{R} = \beta a_t$ and we can explicitly write the value for employment:

$$E(a_t) = \frac{1}{1-\beta} u\left(w_t + (1-\beta)a_t\right).$$

We can then write the problem of the unemployed as:

$$U(a) = \max_{a_{t+1},\theta}\left\{u(a_t - a_{t+1}) + \beta\left[m\frac{1}{1-\beta}u\left(w_t + (1-\beta)Ra_{t+1}\right) + (1-m)U(Ra_{t+1})\right]\right\}$$

subject to the firm's value:

$$V(y) = \max_{w_t}\{q\,(y - w_t) + \beta(1-q)V(y)\}$$
$$= \max_{w_t}\left\{\frac{q}{1-\beta(1-q)}\left[y - w_t\right]\right\}.$$

Using the standard technique in directed search, and similar to what we did in the two-period model, we substitute the wage and rewrite the problem as

$$U(a_t, y, V) = \max_{a_{t+1},\theta_t}\left\{u(a_t - a_{t+1}) + \beta\left[m\frac{1}{1-\beta}u\left((1-\beta)Ra_{t+1} + y - V\left[-\beta\lambda + \frac{1-\beta(1-q)}{q}\left(\lambda + \frac{1}{\beta} - 1\right)\right]\right) + (1-m)U(Ra_{t+1})\right]\right\}. \quad \text{(A.1)}$$

A.5. *Proof of Proposition 2*

Proof. The (interior) solution $a_{t+1}(a_t, y, V), \theta_t(a_t, y, V)$ to the maximization problem satisfies:

$$-u'(c_{u,t}) + \beta[mE_{a_{t+1}}(a_{t+1}, w_{t+1}) + (1-m)U_{a_{t+1}}(a_{t+1})] = 0$$
$$m'[E(a_{t+1}, w_{t+1}) - U(a_{t+1})] + mE_{w_{t+1}}(a_{t+1}, w_{t+1})\frac{\partial w_{t+1}}{\partial \theta_t} = 0.$$

Now we have monotone matching of a_t in y provided: $U_{a_t y} > \frac{U_y}{U_V} U_{a_t V}$.

$$\begin{aligned}
U_y &= m\beta E_{w_{t+1}}\frac{\partial w_{t+1}}{\partial y} + U_{a_{t+1}}\frac{\partial a_{t+1}}{\partial y} + U_{\theta_t}\frac{\partial \theta_t}{\partial y} = m\beta E_{w_{t+1}} \\
U_{a_t} &= Ru'(c_{u,t}) + U_{a_{t+1}}\frac{\partial a_{t+1}}{\partial a_t} + U_{\theta_t}\frac{\partial \theta_t}{\partial a_t} = Ru'(c_{u,t}) \\
U_V &= m\beta E_{w_{t+1}}\frac{\partial w_{t+1}}{\partial V} + U_{a_{t+1}}\frac{\partial a_{t+1}}{\partial V} + U_{\theta_t}\frac{\partial \theta_t}{\partial V} \\
&= -m\beta E_{w_{t+1}}\left(-\beta\lambda + \frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{\beta q}\right) \\
U_{a_t y} &= -u''(Ra_t - a_{t+1})\frac{\partial a_{t+1}}{\partial y} \\
U_{a_t V} &= -u''(Ra_t - a_{t+1})\frac{\partial a_{t+1}}{\partial V}
\end{aligned}$$

where $U_{a_{t+1}} = 0$ and $U_{\theta_t} = 0$ from the envelope theorem. Then:

$$\begin{aligned}
U_{a_t y} &> \frac{U_y}{U_V} U_{a_t V} \\
-u''(Ra_t - a_{t+1})\frac{\partial a_{t+1}}{\partial y} &> \frac{m\beta E_w(a_{t+1}, y)}{-m\beta E_w(a_{t+1}, y)\left(-\beta\lambda + \frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{\beta q}\right)} \\
&\quad \times \left(-u''(Ra_t - a_{t+1})\frac{\partial a_{t+1}}{\partial V}\right) \\
\frac{\partial a_{t+1}}{\partial y} &> -\frac{1}{-\beta\lambda + \frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{\beta q}}\frac{\partial a_{t+1}}{\partial V}
\end{aligned}$$

Writing the Hessian $|\mathbf{H}| > 0$ as:

$$|\mathbf{H}| = \begin{vmatrix} U_{a_{t+1}a_{t+1}} & U_{a_{t+1}\theta_t} \\ U_{\theta_t a_{t+1}} & U_{\theta_t\theta_t} \end{vmatrix}$$

then

$$\frac{\partial a_{t+1}}{\partial y} = -\frac{\begin{vmatrix} U_{a_{t+1}y} & U_{a_{t+1}\theta_t} \\ U_{\theta_t y} & U_{\theta_t\theta_t} \end{vmatrix}}{|\mathbf{H}|} \text{ and } \frac{\partial a_{t+1}}{\partial V} = -\frac{\begin{vmatrix} U_{a_{t+1}V} & U_{a_{t+1}\theta_t} \\ U_{\theta_t V} & U_{\theta_t\theta_t} \end{vmatrix}}{|\mathbf{H}|}$$

$$\frac{\partial a_{t+1}}{\partial y} > -\frac{1}{-\beta\lambda + \frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{\beta q}}\frac{\partial a_{t+1}}{\partial V}$$

$$U_{a_{t+1}y}U_{\theta_t\theta_t} - U_{\theta_t y}U_{a_{t+1}\theta_t} < -\frac{1}{-\beta\lambda + \frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{\beta q}}\left(U_{a_{t+1}V}U_{\theta_t\theta_t} - U_{\theta_t V}U_{a_{t+1}\theta_t}\right)$$

$$\left(U_{a_{t+1}y} + \frac{1}{-\beta\lambda + \frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{\beta q}}U_{a_{t+1}V}\right)U_{\theta_t\theta_t}$$
$$< \left(U_{\theta_t y} + \frac{1}{-\beta\lambda + \frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{\beta q}}U_{\theta_t V}\right)U_{a_{t+1}\theta_t} \tag{A.2}$$

Observe that from the first-order conditions to the (interior) maximization problem, we obtain the cross partials on U. First, note that:

$$U_{a_{t+1}y} = -\frac{1}{-\beta\lambda + \frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{\beta q}}U_{a_{t+1}V}$$

so that the left-hand side is zero. Then we derive the expression for $U_{\theta_t y}$ and $U_{\theta_t V}$ while we note that $m'[E(a_{t+1}) - U(a_{t+1})] + qE_{w_{t+1}}\frac{\partial w_{t+1}}{\partial \theta_t}] = 0$, which implies:

$$U_{\theta_t y} = \beta m' E_{w_{t+1}}(a_{t+1}, y) + \beta E_{w_{t+1}w_{t+1}}(a_{t+1}, y)\frac{\partial w_{t+1}}{\partial y}\frac{(1-\beta)(1-\beta(1-\lambda))\theta_t q'}{\beta q}V$$

$$U_{\theta_t V} = \beta\frac{\partial w_{t+1}}{\partial V}\left(m'E_{w_{t+1}} + E_{w_{t+1}w_{t+1}}\frac{(1-\beta)(1-\beta(1-\lambda))\theta_t q'}{\beta q}V\right)$$
$$+ \beta E_{w_{t+1}}\frac{(1-\beta)(1-\beta(1-\lambda))\theta_t q'}{\beta q}$$
$$= \left(\beta\lambda - \frac{[1-\beta(1-q)][1-\beta(1-\lambda)]}{\beta q}\right)U_{\theta_t y} + \beta E_{w_{t+1}}\frac{(1-\beta)(1-\beta(1-\lambda))\theta_t q'}{\beta q}$$

the right-hand side reduces to:

$$\frac{(1-\beta)(1-\beta(1-\lambda))\theta_t q'}{q}E_{w_{t+1}}(a_{t+1}, y)U_{a_{t+1}\theta_t}$$

Therefore, inequality (A.2) is satisfied provided $U_{a_{t+1}\theta_t} < 0$, since $q' < 0$:

$$U_{a_{t+1}\theta_t} = \beta m'[E_{a_{t+1}}(a_{t+1}, y) - U_{a_{t+1}}(a_{t+1})] + \beta m E_{a_{t+1},w}(a_{t+1}, y)\frac{\partial w_{t+1}}{\partial \theta_t}$$
$$= \beta m'[E_{a_{t+1}}(a_{t+1}, y) - U_{a_{t+1}}(a_{t+1})]$$
$$+ \beta m E_{a_{t+1}}(a_{t+1}, y)\frac{-1}{mE_{w_{t+1}}(a_{t+1}, y)}m'[E(a_{t+1}, y) - U(a_{t+1})]$$

from the FOC for θ_t

$$\frac{\partial w_{t+1}}{\partial \theta_t} = \frac{-m'}{m E_{w_{t+1}}(a_{t+1}, y)}[E(a_{t+1}, y) - U(a_{t+1})]$$

Therefore, $U_{a_{t+1}\theta_t} < 0$ provided

$$\frac{E_{a_{t+1}}(a_{t+1}, y) - U_{a_{t+1}}(a_{t+1})}{E(a_{t+1}, y) - U(a_{t+1})} < \frac{E_{a_{t+1}}(a_{t+1}, y)}{E_{w_{t+1}}(a_{t+1}, y)}$$

□

A.6. *Proof of Proposition 3*

Proof. The proof consists of two steps:

Step 1. Workers deplete assets when unemployed: $a_{t+1} < a_t$;
Step 2. Workers with longer unemployment duration earn lower wages.

As unemployed workers draw down their assets (step 1), they match with low productivity, low wage jobs. Step 2 establishes the negative duration dependence over time for a given cohort of workers.

Step 1. We closely follow Lemma 1 in Huggett (1993) and adjust our notation to show that $a_{t+1}(a_t) < a_t$ for all $a \geq \underline{a}$ if a worker is unemployed. For the remainder of step 1 in the proof, denote $a' = a_{t+1}$ and $a = a_t$, and likewise for all other variables. In our setting, the unemployed corresponds to the low type in Huggett (1993) and the employed to the high type. Then we define $x = (a, s)$ where a is the level of assets and $s \in \{u, e\}$ indicates whether the workers with income $w(s)$ is employed (with income $w(e) = w$) or unemployed (with income $w(u) = b$, in which case the worker chooses where to search). Then rewriting the value functions (2) and (3) we define $v(x)$, which describes the agent's decision problem as:

$$v(x) = \max_{a'} \left\{ u(c_s) + \beta \sum_{s'} v(a'(x), w(s'))\pi(s'|s) \right\} \tag{A.3}$$

where $\pi(s'|s)$ denotes the probability of transitioning from one s to another s' with $\pi(e|u) = m(\theta)$ and $\pi(u|e) = \lambda$.[43] The mapping T implicitly determines the optimal choices and $(Tv)(x)$ corresponds to (2) and (3). We define functions $v_n(x)$ for $n = 0, 1, 2, \ldots$ where $v_0(x) = 0$ and where $v_{n+1}(x) = Tv_n(x)$. We show by induction that $v_n'(a, e) \leq v_n'(a, u)$, where $v'(a, s)$ is the derivative of v with respect to assets a. For $n = 0$, this holds trivially. Next, suppose this property holds for n. Then we show that it holds for $n + 1$. Equation (A.4) is the first-order condition for the maximization problem implicit in the mapping Tv_n. The value of a' that solves this first-order condition for fixed $x = (a, s)$ is $a_{n+1}(x)$ and satisfies:

$$u'(c_s) \geq \beta \sum_{s'} v_n'(a', s')\pi(s'|s) \tag{A.4}$$

and with equality when $a' > \underline{a}$.

43. With slight abuse of notation, we have left out the optimization decision of the unemployed worker who chooses where to search since we take the search decision as given.

Because of the induction assumption that $v_n'(a,e) \leq v_n'(a,u)$, the right-hand side of (A.4) evaluated at (a', u) is larger than or equal to the right-hand side at (a', e) provided $\pi(u|e) < \pi(u|u)$ or $\lambda < 1 - m(\theta)$. Likewise, the left-hand side evaluated at (a', u) is larger than evaluated at (a', e) since $b < (1-\tau)w$, therefore $u'(c_u) \geq u'(c_e)$. This completes the induction step since $v_{n+1}'(a,s) = u'(Ra - a_{n+1}(a,s) + w(s))$. Moreover, as in Huggett (1993), $v_n'(a,s)$ converges pointwise to the derivative of the value function $v'(a,s)$. Since $v'(a,u) = u'(Ra - a'(a,u) + b)$ and decreasing in a, it immediately follows that $a'(a,u) < a$, thus establishing that for the unemployed, assets are decreasing over time.

Step 2. In the cross-section, workers with lower assets apply to lower productivity jobs y and earn lower wages, from positive sorting of assets on wages and productivity. Therefore, for a given cohort of workers, as their assets decline over time ($a_{t+1} < a_t$, from Step 1), so will their wages ($w_{t+1} < w_t$) and the productivity of their jobs. Therefore, workers who have longer unemployment duration have lower wages, thus establishing negative duration dependence. □

B. Wealth data and regressions

B.1. *PSID*

We begin with a sample of all PSID reference persons[44] over the age of 25. First, we drop households from the Latino supplemental sample as they have a higher rate of missing information on hours and labor market compensation. Additionally, we drop all households where the reference person has incomplete information on their hours of work and labor market earnings. Next, we drop all households who are self-employed as this is outside the scope of our model. To maintain representativeness of the U.S. population we keep both married and unmarried individuals and weigh using cross-sectional family specific weights. To capture a household's ability to smooth consumption through self-insurance, we use a measure of liquid household assets, namely total household net worth excluding home equity and we use the PSID constructed value of this stock of wealth. Our measure of earnings uses the reference person and is the sum of several labor income components that includes both wages and salaries as well as bonuses, overtime and tips that form part of an individual's labor market compensation. We exclude farm and business income as well as the earnings of the self-employed who we drop from the sample. The PSID collects labor compensation at the annual frequency and we convert it to our monthly frequency by dividing by $\frac{4}{52}$. For each individual with positive labor market compensation, we calculate the ratio of assets to the reference person's monthly earnings and we calculate the median of these ratios across individuals.

B.2. *NLSY*

Using NLSY79, we first construct all EUE (employment to unemployment to employment) transitions for the cohort of 1979 (NLSY79). This sample covers the years 1979–2016. We use the CPI reported by the BLS to convert the market value of wages and assets to 2000 dollars. Next, we run the following regressions after controlling for a set of individual and aggregate controls, including age, labor market experience, race, gender, educational attainment, ability, occupation, industry, as well as year and month fixed effects. In regression (1), the left-hand side variable is

44. Referred to as the household head in earlier PSID documentation.

TABLE B.1
Duration dependence regression

	All	
	(1)	(2)
Unemp. Duration	0.0013 (0.0004)	0.0012 (0.0004)
Liq. Wealth		0.0089 (0.0016)
Observations	14381	14381

TABLE B.2
Regression of the job finding rate on assets

ω_0	0.27
	(0.00004)
ω_1	−0.71
	(0.00001)
R^2	0.82
N	49314

the reemployment wages, and the right-hand side variable is unemployment duration. In regression (2), we also add wealth as a covariate. Below are the coefficients (with standard deviations) of these regressions (Table B.1).

B.3. *Model regressions*

To assess the ability of the model to replicate the negative relationship between asset holding and job finding rate. We do this by simulating 24,000 workers for 1000 periods. To study the relationship between asset holdings and job finding rates in the steady state, we use observations in last 30 periods. We run the following regression, where UE_i is the monthly transition rate from unemployment to employment for a given worker i and where we apply a log-type transformation for wealth ($\tilde{a}$).[45] (following Burbidge *et al.*, 1988):

$$UE_i = \omega_0 + \omega_1 \tilde{a}_i + \nu_I,$$

and we obtain the following regression results (Table B.2).

Next, to study the impact of unemployment duration dependence of wages we isolate all EUE transitions (similar to our exercise in NLSY) in the simulation. Then, we run the regression with the reemployment wage as the dependent variable and the duration of unemployment as the independent variable for each individual i:

$$w_i = \zeta_0 + \zeta_1 \text{dur}_i + \psi_I,$$

and we obtain the following regression results (Table B.3).

45. In order to account for negative assets, we follow Burbidge *et al*. (1988) and Lise (2013) and transform liquid wealth a at the moment of falling into unemployment, by $\tilde{a} = \log(a + \sqrt{1 + a^2})$.

TABLE B.3
Regression of wages on duration

ζ_0	2.31
	(0.000007)
ζ_1	−0.0006
	(0.000001)
R^2	0.34
N	307257

C. Numerical algorithm

Here we describe the algorithm used to solve the model. We solve the model off-grid by using the Euler equation for consumption smoothing. In this procedure, we use 1600 initial points for asset and productivities. The decision for wages (market tightness) is solved on a grid point where the lower bond is the UI level and upper bond is the highest level of productivity. We discretize this wage space into 300 points. The algorithm consists of the following steps.

(1) Guess a tax rate, $\tau_{0,t}$.
(2) Guess a dividend pay, $d_{0,t}$.
(3) Guess the distribution of firms $G_0 = (G_{0,v}(y), G_{0,j}(y, w_t))$ with vacant and filled jobs, and guess the distribution of workers $F_0 = (F_{0,u}(a_t), F_{0,e}(a_t, w_t))$ who are either unemployed or employed.
(4) Guess initial values of employment and unemployment: $U_0(a_t)$, $E_0(a_t, w_t)$. Also, guess a policy function for consumption of unemployed and employed workers, $c_u^0(a)$, $c_e^0(a, w)$. This will automatically give a value for savings next period, a'.

- With this, we first construct the right-hand side of the Euler equation for employed worker. Next, we use the Euler equations to back out the new value for consumption, $c_u^1(a)$. Notice that this can be done in closed forms since we know the inverse of utility functions.

$$u'(c_e^1(a, w)) = \beta R \left[\lambda u'(c_u^0(a)) + (1 - \lambda)u'(c_e^0(a, w))\right] + \nu^e$$

- In above equations ν^e is the shadow price of being asset constrained (Lagrange multiplier) and is zero for unconstrained workers.
- We first solve the saving problem (consumption policy) of employed workers given our guess for the consumption policy of unemployed workers.
- Notice that, since the optimal savings implied by $c_e^1(a, w)$ is not necessarily on the asset grid, we make a two-dimensional interpolation over the latest guess on saving and wage space on the right-hand side of the Euler equation.[46]
- The optimal new value for $c_e^1(a, w)$ will be the one implied by the right-hand side if the budget constraint does not bind; if it does, we simply set $a' = \underline{a}$.
- We repeat this sequence and update the guess on $c_e^1(a, w)$ in a loop till $|c^1 - c^0| < \epsilon$.
- With $c_e^1(a, w)$ at hand, we can also get the new value of a' simply from the budget constraint.

46. The interpolation is done using interp2 function.

- Next, using the off-grid policy we commence another loop to solve for the value of employment, $E(a,w)$, using value function iteration (VFI). In this loop, we use the utility of consumption for employed, $c_e(a,w)$, solved above and the guess on the value of unemployment $U_0(a_t)$. To update the value function, we again use a two-dimensional interpolation and query points are saving and wages.

(5) Labor Market clearing Loop: guess the threshold of firms' entry $y^\star$, for firms as well as the measure of entrant firms.

(6) Sorting Loop: Given the guess on the entry level of firms, start sorting unemployed workers to the firm (asset to the productivity) from bottom to the top of distribution.Substitute Equation (4) into the value of a filled job, Equation (5), and rearranging we get

$$q(\theta_t) = \frac{[V(y)(1-\beta)+k][1-\beta(1-\lambda)]}{\beta(y_t - w_t - (1-\beta V))} \tag{C.1}$$

- The value of a vacancy at the threshold of firm entry is zero. This is $V(y^\star)=0$, where $y^\star$ is the productivity level below which firms do not enter the market.
- Knowing the value of a vacancy at the bottom of the distribution and having a guess for $y^\star$, we can find the relationship between θ and w.
- Once again, like employed worker problem above, we construct the right-hand side of the Euler equation for unemployed worker. Then we use the Euler equation to solve for the $c_u^1(a)$ using the closed form, for the inverse of utility function and the solution to $c_e^1(a,w)$ we got above.[47]

$$u'(c_u^1(a)) = \beta R\left[m(\theta)u'(c_e^0(a,w)) + (1-m(\theta))u'(c_u^0(a))\right] + \nu^u.$$

- Here we solve for the saving problem (consumption policy) of unemployed workers for every possible decision in the labor market (market tightness). Since the optimal savings implied by $c_u^1(a)$ is not necessarily on the asset grid, we make a two-dimensional interpolation over the latest guess on saving and market tightness space on the right-hand side of the Euler equation. Each market tightness corresponds to one level of wage.
- The optimal new value for $c_u^1(a)$ will be the one implied by the right-hand side if the budget constraint does not bind; if it does, we simply set $a' = \underline{a}$.
- We repeat this sequence and update the guess on $c_u^1(a)$ in a loop till $|c^1 - c^0| < \epsilon$
- With $c_u^1(a)$ at hand, we can also get the new value of a' simply from the budget constraint.
- Next, using the off-grid policy we commence another loop to solve for the value of Unemployment, $U(a_t)$, using value function iteration (VFI). In this loop, we use the utility of consumption for unemployed, $c_u(a_t)$, solved above and the solution on the value of Employment $E(a_t, w_t)$ from previous step. To update the value function of unemployment, we again use a two-dimensional interpolation and query points are saving and market tightnesses.
- Since we are sorting workers to the vacancies from the bottom to the top, this implies that the bottom unemployed workers in the asset distribution are constrained.[48] Therefore, for

47. Here also ν^u is the shadow price of being asset constrained (Lagrange multiplier) and is zero for unconstrained workers.

48. We check that above.

these workers $a_t = a_{t+1}$. So, we can re-write Equation (2) for constrained workers

$$U(a_t) = \frac{1}{1-\beta(1-m(\theta_t))}\left[u(c_{u,t}) + \beta m(\theta_t)E(a_{t+1}, w_{t+1})\right] \tag{C.2}$$

$$c_{u,t} = ra_t + b + d_t \tag{C.3}$$

- Given the menu of tightness-wage bundles and the value $E(a_t, w_t)$ we solved in the last step, a worker can find its optimal submarket to apply for. Given a level of asset, we have solved for all values of unemployment, $U(a_t)$, for each submarket. We had also solved for the value of employment corresponding to each of these submarkets (through the wages). The optimal decision in the labor market, is choosing the submarket that gives the worker the highest value of unemployment given her level of asset.
- Using the optimal allocation condition and given the current value of vacancy and optimal choice of market tightness, we can find the value of posting a vacancy in the next submarket where the optimal matched firm type solves:

$$\beta m(\theta_t)\frac{\partial E(a_{t+1}, w_{t+1}(y))}{\partial w_{t+1}}\frac{\partial w_{t+1}}{\partial y} = \tag{C.4}$$

$$\beta m(\theta_t)\frac{\partial E(a_{t+1}, w_{t+1}(y))}{\partial w_{t+1}} \times \left[1 - \left(\frac{(1-\beta(1-q(\theta_t)))(1-\beta(1-\lambda))}{\beta q(\theta_t)} - \beta\right)\frac{\partial V(y)}{\partial y}\right] = 0 \tag{C.5}$$

- Given the optimal market tightness chosen by unemployed worker, we allocate workers to market tightnesses from the distribution and construct $\theta_t(a_t, y) = \frac{v_{t,y}}{u_{t,a}}$. Here we keep track of the distribution.
- In the next iteration of the sorting loop, using Equation (C.1) and the value of vacancy we obtained through previous iteration using optimal allocation condition, we again solve for the optimal market tightness for unemployed worker with higher levels of asset holdings.
- When we move away from budget constrained, workers simultaneously choose optimal labor market decision ($\theta_t(a_t, y)$) as well as saving for next period (a_{t+1}). Since we started solving this problem from bottom to the top of distribution, for any level of a_t above the constraint we know the value of a_{t+1} which are below a_t. Therefore, the workers knows the value of depleting asset to a_{t+1} which is $U(a_{t+1})$.

(7) By sorting workers to the firms from bottom to the top of distribution, three scenarios may happen

(a) An unemployed worker with a level of asset holding below the highest gets sorted to the highest productivity level. In this case, not all unemployed workers are allocated to the submarkets. Therefore, we update first by increasing the measure of entrant firms at the same level of entry and then by lowering the threshold of firm entry $y^\star$ (go back to Step 5).

(b) A vacancy with a level of productivity below the highest gets sorted to the highest asset holding unemployed worker. In this case, high productivity firms do not get allocated to any vacancies. Therefore, we update first by decreasing the measure of entrant firms at the same level of entry and then by increasing the threshold of firm entry $y^\star$ to make sure firms with high productivities all get allocated (go back to Step 5).

(c) The unemployed workers with highest levels of asset holdings gets sorted to the highest productivity vacancies. In this case, the allocation of workers to the firms is such that the labor market is cleared.

(8) Check the convergence of $U_0(a_t)$. If not converged go back to step 4 and update $U_0(a_t)$.

(9) Using the policy functions for workers (job finding and saving for unemployed workers and saving for employed workers) and firms (job filling rates for firms with a vacancy), we update the distribution of workers and firms. Since the saving decision of workers are not necessarily on grid points, we follow Ríos-Rull (1999) in the following way

- Suppose the distribution has only mass at the grid points. Let $p_{e,i,t}$ denote the mass of agents with employment status e and asset stock a_i in the beginning of period t.
- We also suppose agents who are located at a certain gridpoint chose a_{t+1} in between the grid points, *i.e.* $a_j < a_{t+1} < a_{j+1}$, then they are split up over the two grid points, with weights determined according to the distance of their choice to the nodes.
- Let $f_{e,i,t}$ denote the mass of agents with employment status e and asset a_i at the end of period t. Then we have:

$$f_{e,j,t} = \sum_{i=1}^{N} p_{e,i,t}\alpha_{e,j,i,t}$$

with

$$\alpha_{e,j,i,t} = \begin{cases} 0 & \text{if} \quad a'(e,a_i;a_t) < a_{j-1} \\ \frac{a'(e,a_i;a_t)-a_{j-1}}{a_j-a_{j-1}} & \text{if} \quad a_{j-1} < a'(e,a_i;a_t) < a_j \\ 1 & \text{if} \quad a'(e,a_i;a_t) = a_j \\ \frac{a_{j+1}-a'(e,a_i;a_t)}{a_{j+1}-a_j} & \text{if} \quad a_j < a'(e,a_i;a_t) < a_{j+1} \\ 0 & \text{if} \quad a'(e,a_i;a_t) \geq a_{j+1} \end{cases}$$

- After updating the distribution, we check the convergence of the distribution of workers and firms. If not converged, go back to step 3 and update the distributions.

(10) Using the converged distribution of firms with filled $H_f(y)$, compute the total dividend paid by firms and compare it with previous guess.

$$d_t = \int [(y - w_t)h_f(y, w_t) - v_t(y)k]dy.$$

If they are not similar, go back to step 2 and update the guess for dividend pay-out.

(11) Use the distributions to compute the mass of workers with unemployment benefit entitlements and tax paid by employed workers to check if the government budget is balanced.

$$u_t b = \tau \int w_t(a_t) f_e(a_t) da.$$

If total benefit is higher than tax, go back to step 1 and increase τ_t, if total benefit is less than total tax, decrease τ_t.

(12) Check ex-post if condition $\mathbf{U}_\infty$ holds.

D. Alternative distributions of profits

Dividends as a wage subsidy. In the benchmark model the firms' dividends are equally distributed among all workers. In other words, we have implicitly assumed that all workers own an equal share of all firms. In this Section, we depart from this assumption and propose a new way of redistribution of dividends. We assume that the firm's dividend is fully taxed and redistributed among unemployed workers to finance their benefits. Therefore, dividends are now a form of wage subsidy, because the tax burden is shifted away from wages in detriment of a tax on the profits of the firms. If the total dividend is not enough to finance a given level of UI, then we use a proportional tax on wages to cover what is lacking. B is the total benefit allocated to unemployed workers, $B = ub$. D is the total dividend of firms, $D = \int[y - w(y) - v(y)k]dy$. As a result, b and d are the level of benefit and dividend an unemployed worker receives, where we assume that the left over dividends (if any) are distributed equally among employed and unemployed workers. Therefore, there are two possible scenarios

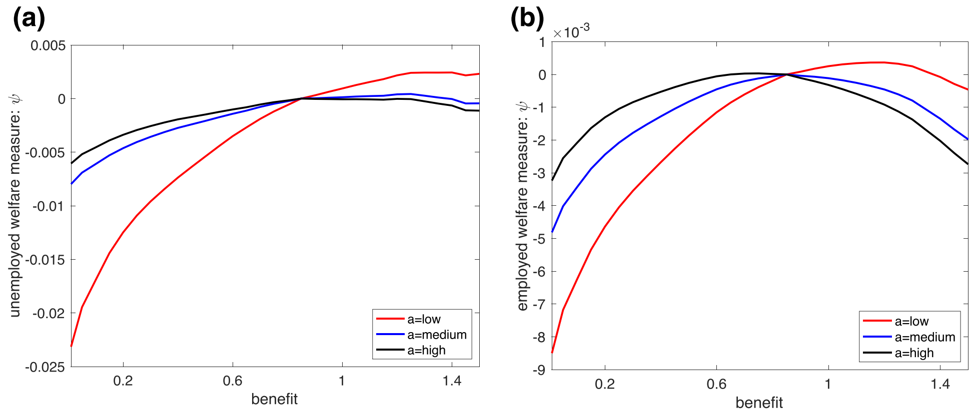
(1) if $D \geq B$: $c_{u,t} + a_{t+1} = b + Ra_t + D - B$ and $\tau = 0$.
$c_{e,t} + a_{t+1} = w + Ra_t + D - B$

(2) if $D < B$: $c_{u,t} + a_{t+1} = b + Ra_t$ and $\tau > 0$ to finance $\frac{B-D}{u}$ for each unemployed
$c_{e,t} + a_{t+1} = (1 - \tau)w + Ra_t$

Using the benchmark calibration, up to $b = 1.25$ the total amount of dividend D under all counterfactual economies are higher than total amount of benefits distributed B. This implies that within this range, higher benefits are not associated with higher taxes for employed workers in this case. This is because the whole UI is now financed with dividends. The excess of dividend $(D - B)$ will also be distributed equally among all workers. Even though this excess is diminishing in benefits, it is always positive. When UI benefits increase, the net transfer to unemployed workers—consisting of benefits and excess dividends—increases. At the same time, the excess transfer to employed workers falls but they do not pay any taxes.

In Figure D.1, we plot the welfare for unemployed and employed workers of different assets, and in Figure D.2 we plot the average welfare. For benefits higher than 1.25, the total amount of dividend D is not sufficient enough to cover unemployment benefits. This implies that from this threshold of UI onward a proportional tax to wages is applied to cover what is lacking for financing benefits: $D - B < 0$. Interestingly that is exactly where the welfare function is maximized for employed workers and high asset unemployed ones. Still, asset poor unemployed workers prefer higher levels of UI no matter that this comes at the expense of higher taxes when they become employed. For asset rich unemployed and all employed workers, the negative effect of taxation on wages in their welfare function kicks in as soon as dividends are not enough to cover UI. These workers need less insurance than asset poor unemployed workers and therefore gain higher utility from higher levels of UI only when it is financed via dividends only.

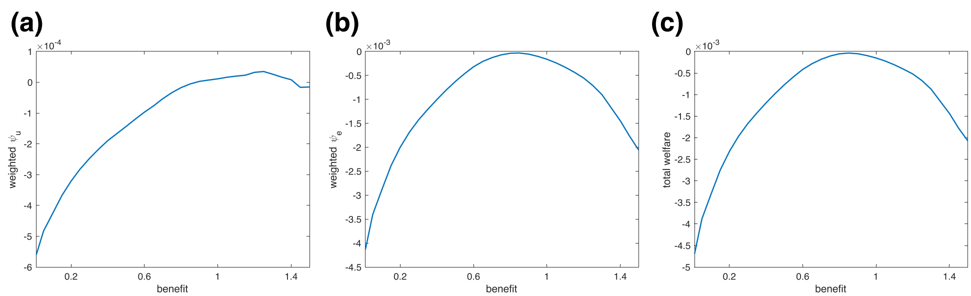
Dividends proportional to asset holdings. In this Section, we consider yet another scheme to distribute profits: workers receive a share of the firms' dividend in proportion of their asset holdings. The idea is that assets are invested and that the return is proportional to the amount invested. We assume that workers with negative asset holdings do not receive any dividend while others depending on their position in the wealth distribution receive a share of the dividend. This complicates the numerical solution of the model further as we need to make a guess both on total amount of dividends distributed in the economy as well as on the share of each worker which is a function of the pdf of asset distribution. We keep the same benchmark economy as our main analysis in 4.1 where $b = 0.8$. The welfare results are presented in Figures D.3 and D.4.



unemployed workers employed workers

Figure D.1

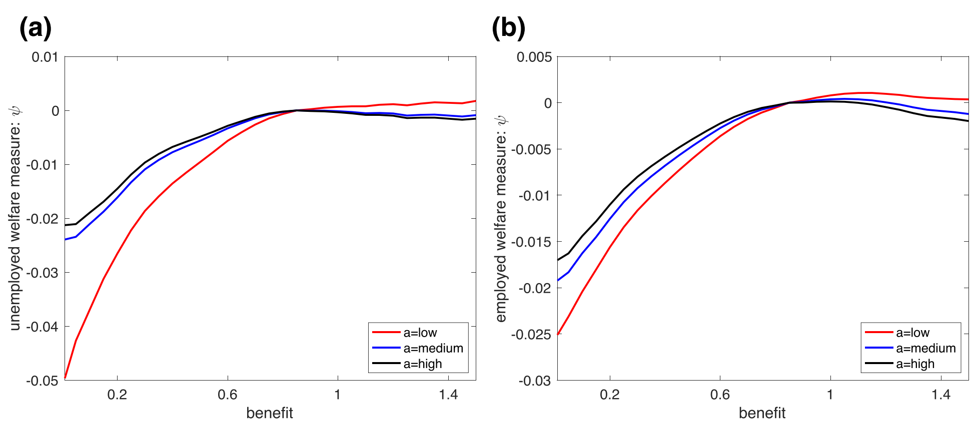
Welfare measure: ψ



Average value unemployed Average value employed Total welfare

Figure D.2

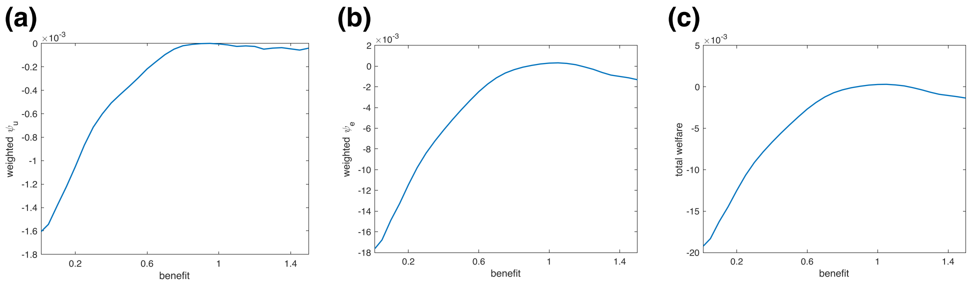
Welfare



unemployed workers employed workers

Figure D.3

Welfare measure: ψ



Average value unemployed

Average value employed

Total welfare

FIGURE D.4
Welfare

The welfare schedules are still inverted U-shape in benefits, but interestingly the workers' welfare is maximized at a much higher level of UI compared with the regime where all workers are assumed to own an equal share of total dividend. Higher benefits mean lower levels of dividends and also lower levels of savings. However, the new dividend regime incentivizes workers to save more to be able to get a higher share of dividends. For higher UI benefits, workers reduce their savings, but they do so less than in the constant dividend regime. Workers receive a higher share of dividends while they need less self-insurance because of higher benefits. As a result, relatively higher savings of workers increase their consumption which is welfare improving. Therefore, the negative effect of lower job finding rates kicks in at higher values of UI benefits.

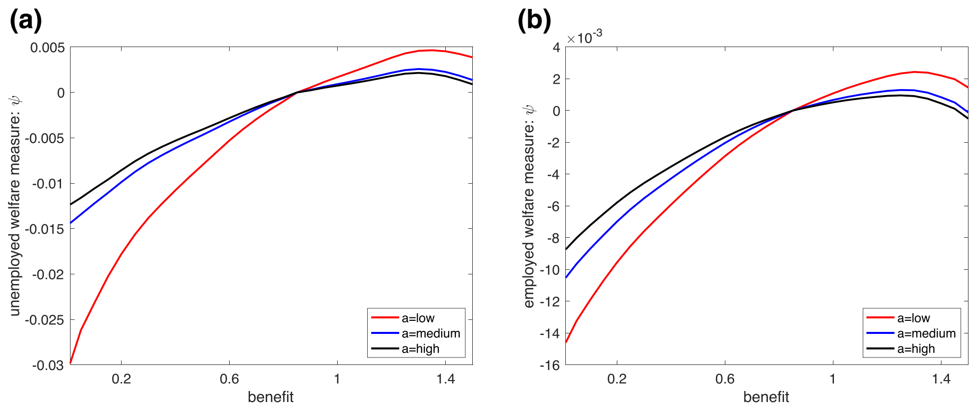
E. Capital and endogenous interest rate

In this Section, we assume that each employed worker produces $yf(\kappa)$ where y is the firm specific productivity, $f(\cdot)$ is an increasing and strictly concave production function and κ is the capital stock supplied by workers. We assume that $yf(\kappa) = y\kappa^{\alpha}$. To be able to compare our results with the benchmark analysis we use the same value of parameters (with the exception of endogenized interest rate, r) as the ones in Section 4.1. Moreover, we choose α such that the range of output in this economy to be similar to that in the benchmark economy. We assume for this part that the borrowing constraint is zero and workers use their savings to lend them to firms for production. The firm first-order condition implies that $r = yf'(\kappa)$ and for the equilibrium capital $\kappa^{\star}$, the firm's per period dividend is equal to $d = zf(\kappa^{\star}) - r\kappa^{\star} - w$, where r is the interest rate. We re-write the value of a filled job as

$$J(y, w) = \max_{\kappa} \; y\kappa^{\alpha} - r\kappa - w + \beta[\lambda V(y) + (1-\lambda)J(y, w)]. \tag{E.1}$$

We repeat the welfare exercise from Section 4.4. The only difference here is that the interest rate changes in the counterfactual economies (Figure E.1).

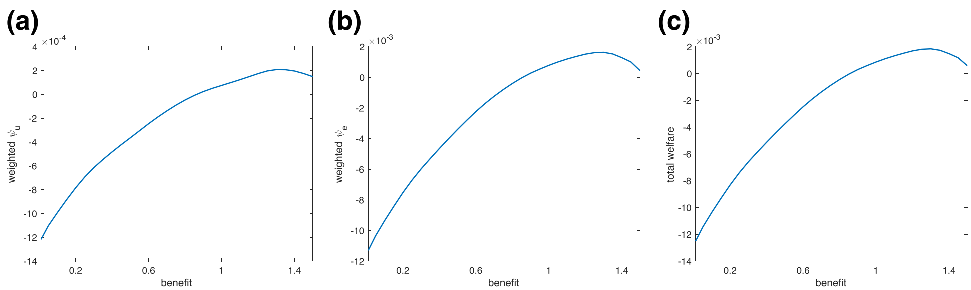
Qualitatively, the welfare results with an endogenous interest rate are similar to the benchmark economy. The lower interest rate at each benefit level compared to the benchmark analysis results in lower levels of saving. This magnifies the importance of external insurance and that is why the maximum welfare for both employed and unemployed workers are achieved at higher



unemployed workers employed workers

FIGURE E.1

Welfare measure: ψ when capital is endogenous



Average value unemployed Average value employed Total welfare

FIGURE E.2

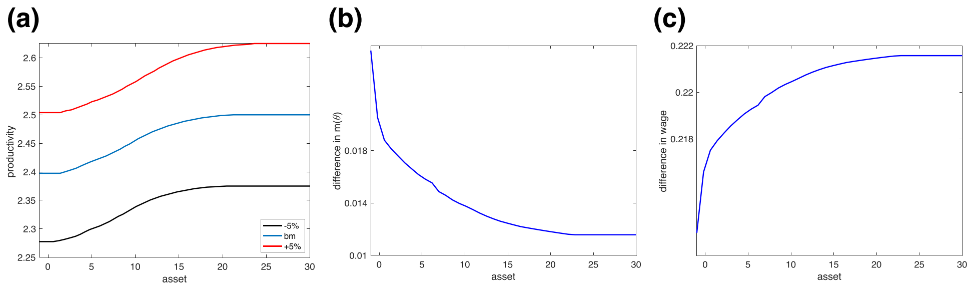
Welfare when capital is endogenous

levels. But apart from that the shape of welfare function for all employed and unemployed workers remain qualitatively the same to our main exercise where we assumed a small open economy (Figure E.2).

F. Change in productivity

In this Section, we evaluate the impact of a change in the productivity distribution. We shift the distribution of productivities 5% to the right and left relative to the benchmark economy. Figure F.1(a) shows the change in allocation of workers to firms. When the productivities rise, the entire allocation moves up, including the threshold. There are some changes in the endogenous outcomes, in particular the probability of job finding and wages, all of which are mediated through the sorting of workers to firms.

Higher productivities means a higher probability of job finding and higher wages, as depicted in Figure F.1(b) and (c). They shows the difference in policy functions when the distribution of productivities shift 10% (5% in each direction). Although both the job finding probability and wages are higher when the distribution of productivities shifts up, the effect is heterogenous



Allocation Difference in prob. of job finding Difference in wages

FIGURE F.1
Change in productivity distribution

across the distribution of assets. The change in probability of job finding is least for high asset holders compared to asset poor unemployed workers. In contrast, the change in wages is positively correlated with asset holdings. This implies that when productivities increase, asset rich workers are more willing to take higher risks and apply for disproportionally higher wages while their probability of job finding does not increase as much. In contrast, poor workers apply for jobs they can get with a considerably higher probability but the wage of those jobs do not increase only a little.

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Supplementary Data

Supplementary data are available at *Review of Economic Studies* online.

Data Availability Statement

The data and code underlying this research is available on Zenodo at https://doi.org/10.5281/zenodo.7661130.

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