Stochastic Sorting

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Motivation

- 2-sided matching: labor as an inspection good (Nelson 1970)
- Matching problem (Becker 1973) with stochastic types:
  1. match $\rightarrow$ ex ante characteristics x, y
  2. output $\rightarrow$ ex post realizations ω, σ
- Realistic + can confront model with data:
  1. Attributes change
  2. Account for mismatch
  3. Noise is part of model

Examples

$$x \to \omega$$

$$y \to \sigma$$

	x, y	ω, σ
Marriage	x : man's education	ω : income
	y : woman's education	σ : income

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	y : woman's education	σ : income
Job Market	x : MBA degree	ω : worker productivity
	y : job level/position	σ : realized demand/technology

Examples

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	x, y	ω, σ
Marriage	x : man's education	ω : income
	y : woman's education	σ : income
Job Market	x : MBA degree	ω : worker productivity
	y : job level/position	σ : realized demand/technology
Executives	x : past experience	ω : CEO performance
	y : initial market value	σ : stock price change

Applications

1. Mismatched CEOs
   - Ex post randomness in match
   - mismatch + selection:
     $\rightarrow$ many CEOs are the wrong (wo)man for the job

   $\Rightarrow$ Technology: output and distributions
2. Household Income Inequality: stochastic vs. marital sorting

Related Work

Mismatch: confronting matching models with reality

- Search Frictions: Shimer and Smith (2000), Cheremukhin, Restrepo, Tutino (2016)
- Learning: Anderson-Smith (2011)
- Matching under uncertainty (Het. pref.): Chiappori-Reny (2005), Legros-Newman (2007) (no mismatch); Chade (2006)
- Unobserved heterogeneity + multidimensional types: Choo-Siow (2006) Galichon-Salanié (2011), Lindenlaub (2012)

Main Features of Model

- Endowments when matching: heterogeneous and stochastic
- Who matches with whom?
  - Matching based on ex ante attributes
    $\Rightarrow$ Ex ante: no mismatch (Becker)
  - Match value and payoff depend on ex post realization of types
    $\Rightarrow$ Ex post: mismatch
- Key assumption: no rematching (same logic if cost)

The Model Setup

General Framework

- Agents

$$\begin{aligned} \text{Workers:} \quad & x \to \omega \sim F(\omega|x) \\ \text{Firms:} \quad & y \to \sigma \sim G(\sigma|y) \end{aligned}$$

  $\to$ joint distribution $K(\omega, \sigma|x, y)$
- Output:

$$q(\omega, \sigma)$$

- Competitive equilibrium/stability/efficient matching $\mu(x)$
- Remark:
  - Special Case: Independence $K(\omega, \sigma|x, y) = F(\omega|x)G(\sigma|y)$
  - Assume continuous variables with K, F, G, q smooth

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Transferable Utility (TU)

- The expected surplus of a match between a type x and y:

$$V(x,y) = \int_{\underline{\omega}}^{\overline{\omega}} \int_{\underline{\sigma}}^{\overline{\sigma}} q(\omega,\sigma) k(\omega,\sigma|x,y) d\omega d\sigma$$

  where k is the density of K
- Determinants of equilibrium allocation:
  1. Complementarity of match output $q(\omega,\sigma)$
  2. Distributions $K(\omega,\sigma|x,y) \rightarrow$ stochastic dominance

Transferable Utility (TU)

Proposition (Sorting)

Optimal sorting patterns are as follows:

1. *PAM if K is spm (sbm) in (x, y) for each (ω, σ) and q is spm (sbm) in (ω, σ);*
2. *PAM if $\int_{\underline{\omega}}^{\omega} \int_{\underline{\sigma}}^{\sigma} K(s, t|x, y) d\omega d\sigma$ is spm (sbm) in (x, y) (and marginals independent), if $q_{\omega\sigma}$ is spm (sbm) in (ω, σ);*

The conditions on q are also necessary if it is to hold for all K.

$\rightarrow$ Condition on q is necessary if result to hold *for all* K

$\rightarrow$ Proof: applying integration by parts iteratively sketch proof

Transferable Utility (TU)

Proposition (Sorting)

Optimal sorting patterns are as follows:

1. *NAM if K is spm (sbm) in (x, y) for each (ω, σ) and q is sbm (spm) in (ω, σ);*
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→ Condition on q is necessary if result to hold *for all* K

→ Proof: applying integration by parts iteratively sketch proof

Transferable Utility (TU)

- Special case: cond. independence: $K = F(\omega|x)G(\sigma|y)$ and FOSD ($F_x < 0, G_y < 0$)
  - If F and G degenerate, then we recover Becker

Transferable Utility (TU)

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  - If F and G degenerate, then we recover Becker

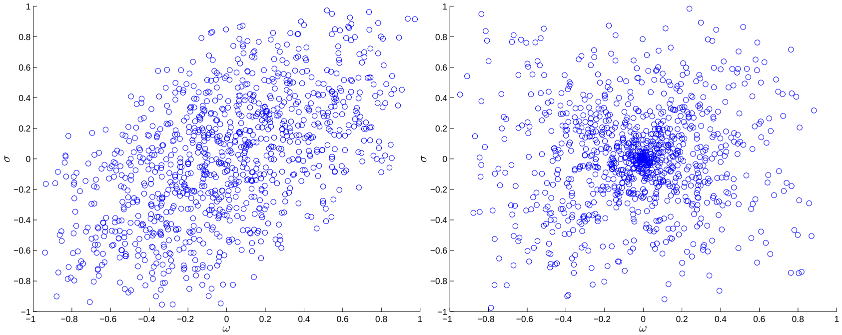
- Selection from matching: even if no correlation in K, there is correlation in observed outcomes across matches of ω, σ

$$\begin{aligned}
\text{Cov}[\omega,\sigma] &= \int_{\underline{\omega}}^{\overline{\omega}}\int_{\underline{\sigma}}^{\overline{\sigma}} \omega\sigma k(\omega,\sigma)d\omega d\sigma - \int_{\underline{\omega}}^{\overline{\omega}} \omega k(\omega)d\omega \int_{\underline{\sigma}}^{\overline{\sigma}} \sigma k(\sigma)d\sigma \\
&= \int_{\underline{x}}^{\overline{x}} \mathbb{E}[\omega|x]\mathbb{E}[\sigma|\mu(x)]d\Gamma(x) - \int_{\underline{x}}^{\overline{x}} \mathbb{E}[\omega|x]d\Gamma(x) \int_{\underline{x}}^{\overline{x}} \mathbb{E}[\sigma|\mu(x)]d\Gamma(x) \\
&= \text{Cov}\left[\mathbb{E}[\omega|x], \mathbb{E}[\sigma|y]\right]
\end{aligned}$$

$\rightarrow$ exploit this in decomposition married household inequality

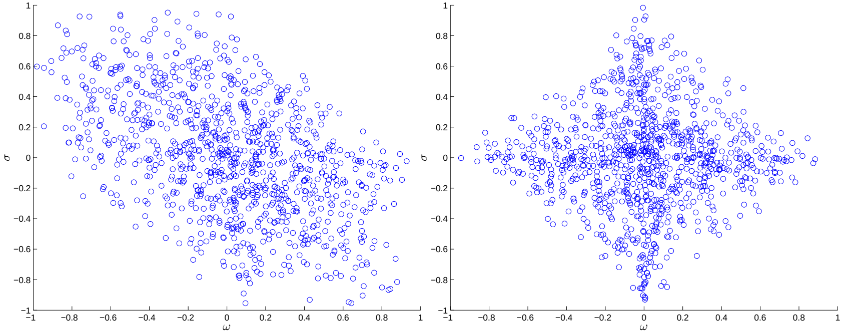
Transferable Utility (TU)

Examples of observed (ω, σ) – PAM: FOSD vs. MPS



Transferable Utility (TU)

Examples of observed (ω, σ) – NAM: FOSD vs. MPS



▸ Technology and Distribution

Transferable Utility (TU)

Example: supermodularity of $q(\omega, \sigma)$ is not sufficient

- Theorem 1: sorting depends on q and distributions
- Technology:
  - $q = \omega\sigma^2$ supermodular
  - Distributions: F, G conditionally independent:
    - $F_x < 0$ (FOSD)
    - $G = \mathcal{N}(\mu(y), s^2(y))$ with $\mu'(y) > 0$ and $s'(y) < 0$
- Then:

$$\begin{aligned} V(x,y) &= \mathbb{E}[\omega|x]\mathbb{E}[\sigma^2|y] \\ &= \mathbb{E}[\omega|x]\left(s^2(y) + \mu^2(y)\right) \end{aligned}$$

- The cross partial is:

$$V_{xy} = \mathbb{E}_x[\omega|x]\left(\frac{ds^2}{dy} + \frac{d\mu}{dy}2\mu\right)$$

$\therefore\ V_{xy} < 0 \iff \frac{ds^2}{dy} < -\frac{d\mu}{dy}2\mu$: NAM with q supermodular

Transferable Utility (TU)

Some Observations

- TU: simple and tractable
- But: ex post payoffs not pinned down ($\exists$ continuum of splits)
- Most applications: information on ex post payoffs
- Non-linear preferences: pins down ex post payoffs
    1. Risk Sharing
    2. Contracting under moral hazard

Non Transferable Utility (NTU)

Risk Sharing

- Stochastic characteristics $\Rightarrow$ Uncertainty $\Rightarrow$ Risk sharing

$$
\begin{aligned}
\Phi(x,y,v) \quad &= \quad \max_{c_x,c_y} \int_{\underline{\omega}}^{\overline{\omega}} \int_{\underline{\sigma}}^{\overline{\sigma}} u(c_y(\omega,\sigma))k(\omega,\sigma|x,y)d\omega d\sigma \\
\text{s.t.} \quad & \quad c_x(\omega,\sigma) + c_y(\omega,\sigma) = q(\omega,\sigma) \;\; \forall \;\; (\omega,\sigma) \\
& \quad \int_{\underline{\omega}}^{\overline{\omega}} \int_{\underline{\sigma}}^{\overline{\sigma}} u(c_x(\omega,\sigma))k(\omega,\sigma|x,y)d\omega d\sigma \geq v
\end{aligned}
$$

- Pins down consumption and thus ex post payoffs

Non Transferable Utility (NTU)

Risk Sharing

- NTU matching problem $\Rightarrow$ Legros and Newman (2007)
- PAM (NAM) $\Leftrightarrow$ Generalized Increasing (Decr.) Differences
- Differential version of their condition (Spence-Mirrlees):
  - PAM if and only if

$$\Phi_{xy} > \frac{\Phi_x}{\Phi_v}\Phi_{vy}$$

- Focus on $K(\omega, \sigma|x, y) = F(\omega|x)G(\sigma|y)$ and FOSD

Non Transferable Utility (NTU)

Risk Sharing: Overview Main Results

1. Sorting pattern only depends on q, not on distributions
    - if one side is risk neutral (e.g. firm); or

$$\rightarrow \text{ PAM if } \ q_{\omega\sigma} > 0$$

Non Transferable Utility (NTU)

Risk Sharing: Overview Main Results

2. If u is HARA (CRRA, log, CARA, quadratic,...)
   - $\rightarrow$ PAM if $V = \int\int \hat{q}_{\omega,\sigma} k d\omega d\sigma$ is spm where $\hat{q}$ is a transformation of q: apply Proposition TU
   - $\rightarrow$ FOSD and independence, we verify conditions on $\hat{q}$

CRRA $u = \frac{c^\alpha}{\alpha}$ then

$$\hat{q} = \frac{q^\alpha}{\alpha(1-\alpha)^{1-\alpha}}$$

α-Root-SPM: can have NAM even if $q_{\omega\sigma} > 0$

LOG $\alpha = 0$: $\hat{q}$ spm or q log-spm

CARA : $\lim \alpha \rightarrow 1$. If $q_{\omega\sigma} = 0$ then matching is independent of distribution (as in one side risk neutral)

Matching with Agency

- Executives match with firms
- NTU $\Rightarrow$ pins down ex post payoffs
- Matching + moral hazard problem
- Variation Holmström-Milgrom linear contracting model
- Objective: sorting due to?
  - match value
  - distribution
- Key assumptions: No rematching, no separation

Holmström-Milgrom with Matching

- Large number of risk averse CEOs, risk neutral firms
- CEO-firm pair (x, y) match. Timing:
  1. Firm offers output-contingent (q) contract
  2. CEO type ω realized (public); CEO chooses effort e
  3. Firm type σ realized (not observed) $\rightarrow$ output q observed
  4. Payments as specified in the contract
- Technology: output and distributions

$$\begin{aligned} q &= \omega \cdot (e + \sigma) \\ \omega &\sim \mathcal{LN}(k(x), u(x)^2) \\ \sigma &\sim \mathcal{N}(t(y), s(y)^2) \end{aligned}$$

- Linear contracts (α, β): $w(q, \omega) = \beta(\omega) + \alpha(\omega) q$
- CEO: CARA preferences $-e^{-r\left(w - \frac{e^2}{2}\right)}$; Reservation wage $a(x)$

Optimal Contracting Problem

- Principal's problem is (where β, α, e depend on ω):

$$\begin{aligned}
\max_{\beta,\alpha,e} \quad & \int \left(\mathbb{E}[q|e] - (\beta + \alpha\mathbb{E}[q|e])\right) dF(\omega|x) \\
\text{s.t.} \quad & \int \left(\mathbb{E}\left[-e^{-r\left(\beta+\alpha q-\frac{e^2}{2}\right)}\right]\right) dF(\omega|x) \geq -e^{-ra} \quad (\textbf{PC}) \\
& e \in \arg\max_{\hat{e}} \int -e^{-r\left(\beta+\alpha q-\frac{\hat{e}^2}{2}\right)} dG(\sigma|y), \forall\omega \quad (\textbf{IC})
\end{aligned}$$

  where $q = q(\omega, \sigma, y), \alpha(\omega), \beta(\omega), e(\omega)$
- **Remark:** (**PC**) is ex ante, before ω is revealed, while (**IC**) must hold for each realization of ω

Sketch Derivation and Optimal Contract

- (**IC**) $\Rightarrow \alpha(\omega) = e(\omega)/\omega$ for all ω
- Insert into objective function and (**PC**)
- Optimal Contract $(\alpha(\cdot), \beta(\cdot), e(\cdot))$ is

$$\begin{aligned}
\alpha(\omega) &= \frac{1}{1 + rs^2(y)} \\
\beta(\omega) &= a(x) - \frac{\omega t(y)}{1 + rs^2(y)} + \frac{\omega^2}{2\left(1 + rs^2(y)\right)^2}\left(rs^2(y) - 1\right) \\
e(\omega) &= \frac{\omega}{1 + rs^2(y)}
\end{aligned}$$

Optimal Contract

- Equilibrium:

$$\begin{aligned} w &= a + \frac{\omega^2}{2\left(1+rs^2\right)} + \frac{\omega}{1+rs^2}\sigma \\ \pi &= \omega t - a + \frac{\omega^2}{2\left(1+rs^2\right)} + \frac{rs^2}{1+rs^2}\omega\sigma \\ q &= \omega(t+\sigma) + \frac{\omega^2}{1+rs^2} \end{aligned}$$

- Ex ante Match Surplus:

$$\begin{aligned} V(x,y) &= \int\int q(\omega,\sigma,x,y)dF(\omega|x)dG(\sigma|y) \\ &= \int\left[\omega t + \frac{\omega^2}{1+rs^2}\right]dF \end{aligned}$$

Optimal Contract

With ω lognormal:

$$\begin{aligned}
\mathbb{E}w(x) &= a(x) + \frac{\mathrm{e}^{2(k+u^2)}}{2(1+rs^2)} \\
\mathbb{E}\pi(y) &= \mathrm{e}^{k+\frac{u^2}{2}}t - a(x) + \frac{\mathrm{e}^{2(k+u^2)}}{2(1+rs^2)} \\
V(x,y) &= \mathrm{e}^{k+\frac{u^2}{2}}t + \frac{\mathrm{e}^{2(k+u^2)}}{1+rs^2}.
\end{aligned}$$

Endogenous Outside Option $a(x)$

- Ex post wages w: pinned down by optimal contract
- Ex ante compensation determines $a(x)$
- From FOC:

$$\max_x V(x,y) - a(x) \quad \Rightarrow \quad a'(x) = V_x(x,x)$$

and therefore $a(x) = a(\underline{x}) + \int_{\underline{x}}^{x} V_x(\tau,\tau)d\tau$ or:

$$a(x) = a(\underline{x}) + \int_{\underline{x}}^{x} \Bigg(\mathrm{e}^{k(z)+\frac{u(z)^2}{2}}\ t(z)\ \big(k'(z) + u(z)\,u'(z)\big) + \frac{\mathrm{e}^{2\,(k(z)+u(z)^2)}\ (k'(z) + 2\,u(z)\,u'(z))}{1 + r\,s(z)^2} \Bigg) dz + a(\underline{x})$$

where $a(\underline{x}) \in [0, V(\underline{x},\underline{x})]$.

PAM

- Match Value is separable

$$\Phi(x, y, \overline{v}) = \int \int \left(\frac{\omega^2}{1 + rs^2} + \omega(t + \sigma) \right) dFdG - \frac{1}{r} \log(-\overline{v}(x))$$

$\rightarrow$ from CARA, quadratic cost, normal distribution

$\Rightarrow$ If FOSD: PAM $\iff$ $\frac{\partial^2}{\partial x \partial y} \left(\frac{\omega^2}{1+rs^2} + \omega t \right) > 0$
(from Proposition 1, TU)

Application 1. Mismatched CEOs

Empirical Exercise

Objective: illustrate the model can be applied to analyze mismatch

1. Use US data CEO compensation and firm profits to estimate:
   - Match value function
   - CEO and firm type distributions
2. Quantify mismatch in market for CEOs
3. Decompose value loss due to mismatch
   - Forgone complementarities
   - Changes in effort (incentives)

Mismatched CEOs

Data

- Data sources:
  - Wages: Execucomp (Compustat) – total compensation: TDC1
  - Profits: Compustat: change in MkVal
- Constructing the variables:
  1. Newly hired 2010 (4 separations, 53 missing obs.)
  2. Rank firms by 2010 market value: $y =$log(MkVal)
  3. Rank workers: $x = y$
  4. w: TDC1(2011)$+$TDC1(2012)
  5. π: MkVal(2012)-MkVal(2010)

Mismatched CEOs

Implementing the Model

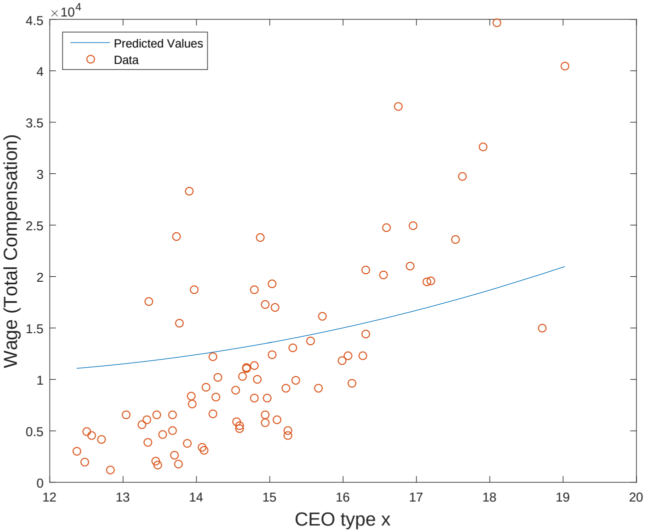
- Parametric form for k, u, t, s: e.g. $k(x) = k_0 + k_1 x + k_2 x^2$
- Calculate $a(x)$ from integral expression
- Distributions

$$\begin{aligned} F(\omega|x) &= \mathcal{LN}\left(k(x), u(x)^2\right) \\ G(\sigma|y) &= \mathcal{N}\left(t(y), s(y)^2\right) \end{aligned}$$

- Solve for ω, σ from w, π expressions
- Estimate $k_0, k_1, k_2, \ldots$ with MLE

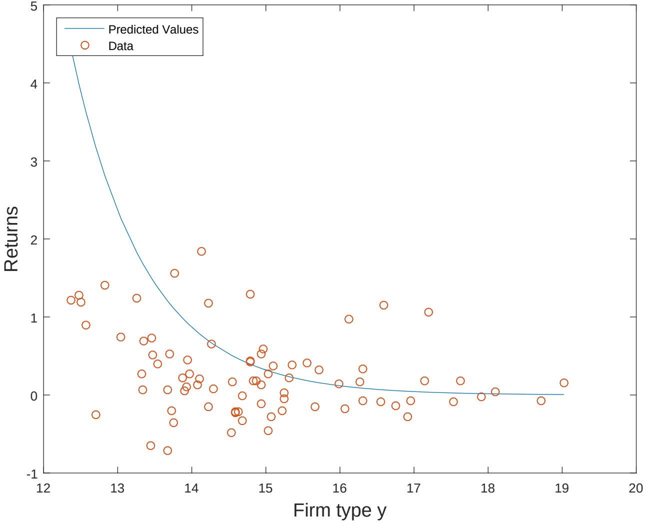
Mismatched CEOs

Wages



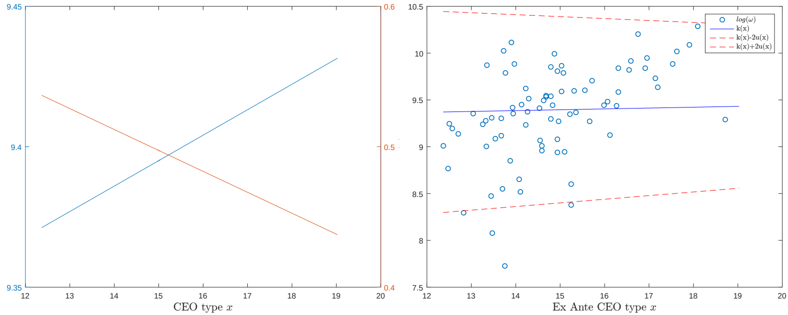
Mismatched CEOs

Return $= \pi / V_0$



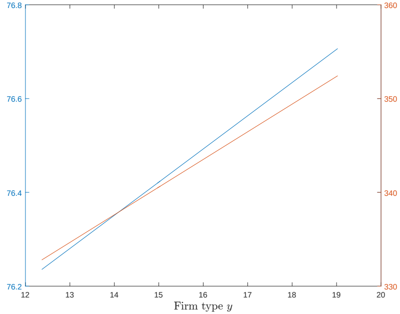
Mismatched CEOs

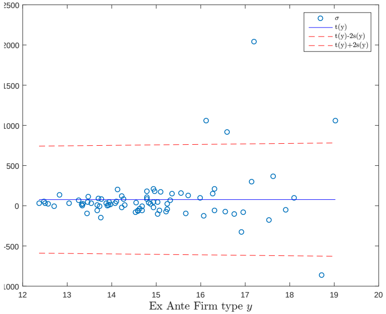
Estimated Parameters k, u



Mismatched CEOs

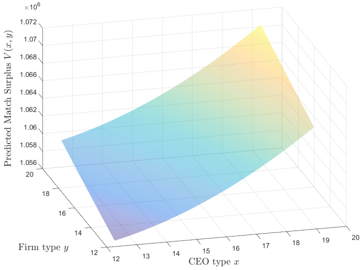
Estimated Parameters t, s

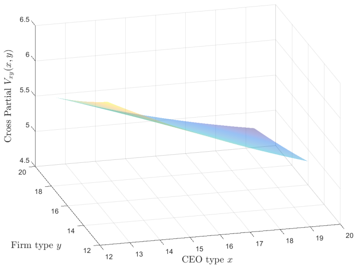




Mismatched CEOs

Match Value: $V(x,y)$ and V_{xy}

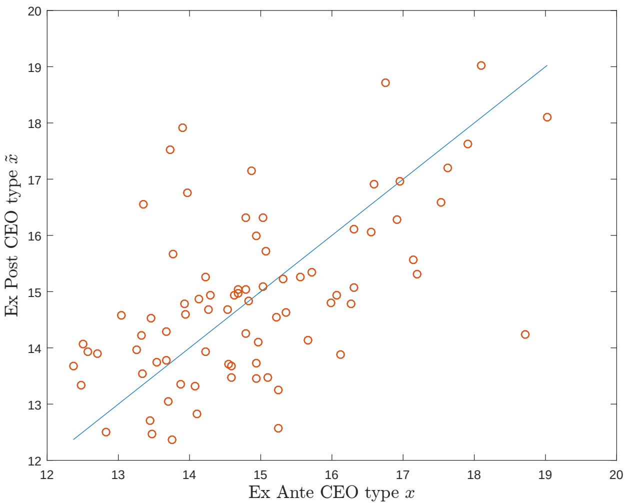




$\Rightarrow$ Justifies identifying assumption of PAM!

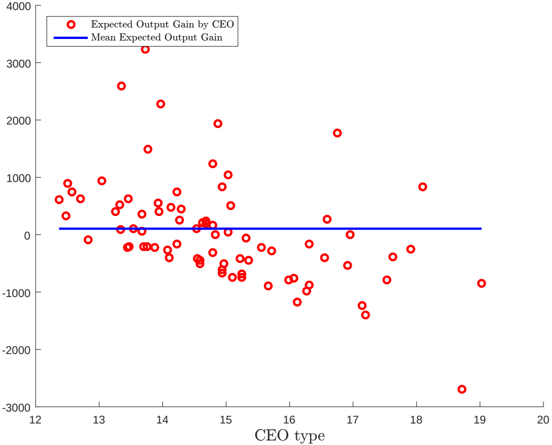
Mismatched CEOs

Mismatch



An Experiment: reassign CEOs after ω

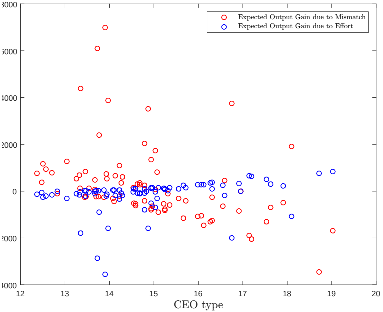
Output Gain

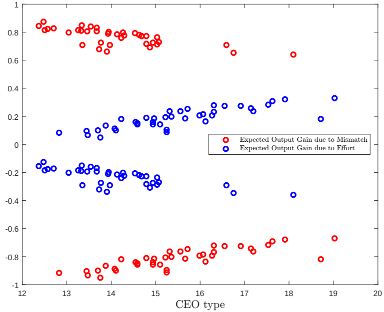


- Given $V_{xy} > 0$, aggregate output gain positive

An Experiment: reassign CEOs after ω

Decomposition: Mismatch and Effort





- Mainly mismatch, not effort
- Mismatch biggest at bottom

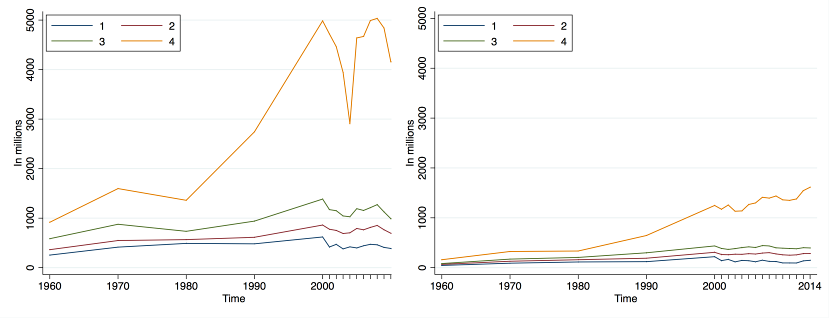
Application 2. Household Income Inequality

Marital vs. Stochastic Sorting

- Household income inequality $\uparrow$ between 1960-2014: Var $\times$ 8
- Census + ACS data on education, earnings of married couples
- 4 education categories
- No data on ex post consumption
- Accounting exercise (like Greenwood e.a. (2015); Lam (1997))
- Inequality due to:
  1. Marital Sorting: ex ante $\rightarrow$ education
  2. Stochastic Sorting: ex post $\rightarrow$ earnings

Household Income Inequality

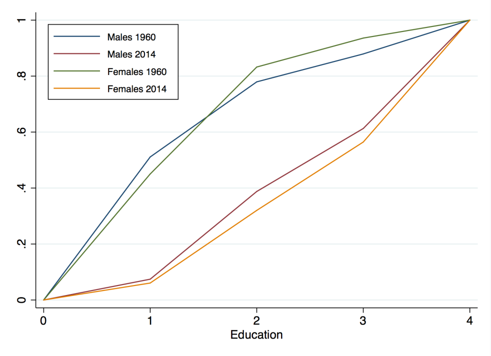
Variance of Male and Female Earnings



- Correlation from 13.4% in 1960 to 23.4% in 2014

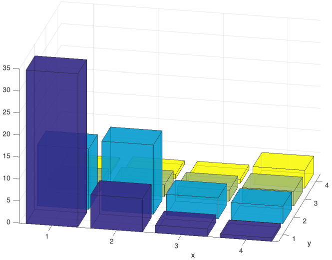
Household Income Inequality

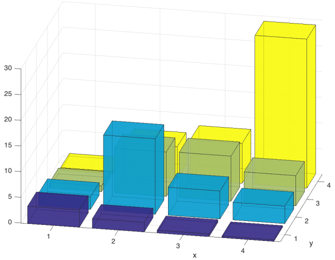
Education 1960-2014

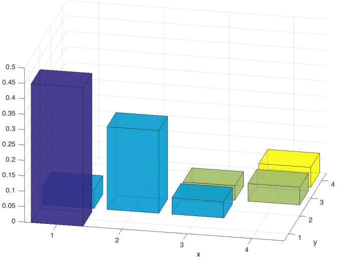


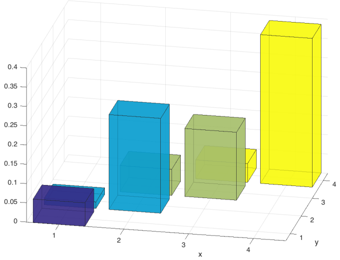
1. Huge right-shift for males and females
2. More for females

Household Income Inequality









Decomposition

Baseline	Sample	Normal Model*		
$\mathrm{Var}_{2014}(\omega+\sigma)$	4.01×10^9	3.92×10^9		
$\mathrm{Var}_{1960}(\omega+\sigma)$	0.49×10^9	0.42×10^9		
$\frac{\mathrm{Var}_{2014}(\omega+\sigma)}{\mathrm{Var}_{1960}(\omega+\sigma)}$	8.18	9.33		

	$X_{1960}[x, y]$	$\mathrm{Var}_{2014}(\omega+\sigma; X_{1960})$	$\frac{\mathrm{Var}_{2014}(\omega+\sigma; X_{1960})}{\mathrm{Var}_{1960}(\omega+\sigma)}$	X_{2014} explains
A. Stochastic Sorting (ex post)				
1. Marginals Earnings*	F_{1960}, G_{1960}	0.75×10^9	1.78	81%
2. Correlation Earnings*	ρ_{1960}	3.50×10^9	8.33	11%
Total (Normal model)*	$F_{1960}, G_{1960}, \rho_{1960}$	0.68×10^9	1.39	85%
Total	K_{1960}	0.79×10^9	1.61	80%
B. Marital Sorting (ex ante)				
1. Marginals Education	$\Gamma_{1960}, \Psi_{1960}$	1.70×10^9	3.47	58%
Allocation: Gender Equal.	$\Gamma_{2014}, \tilde{\Psi}_{2014}, \mu^f_{1960}$	3.49×10^9	7.12	13%
All more educated	$\Gamma_{1960}, \tilde{\Psi}_{1960}, \mu^f_{2014}$	2.29×10^9	4.67	43%
2. Assortativeness Education	d_{1960}	3.97×10^9	8.10	1%
Total	M_{1960}	1.69×10^9	3.45	58%

* Assumes (ω, σ) is normally distributed.

Concluding Remarks

- Stochastic Sorting: Becker with realistic types
- Appealing:
    1. Characteristics change
    2. Mismatch in data
    3. “Noise” is integral part

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- Appealing:
  1. Characteristics change
  2. Mismatch in data
  3. "Noise" is integral part
- Applications
  1. Mismatched CEO's: loss driven by by mismatch, not effort provision $\Rightarrow$ focus on *selection*, rather than incentives
  2. Household Ineq.: 80+% stochastic sorting; little marital sorting

Stochastic Sorting

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NBER Summer Institute
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Transferable Utility (TU)

- Sketch of part (i):
  - Integration by parts and marginals function of one type yield

$$V_{xy} = \int_{\underline{\sigma}}^{\overline{\sigma}} \int_{\underline{\omega}}^{\overline{\omega}} q_{\omega\sigma}(\omega, \sigma) K_{xy}(\omega, \sigma | x, y) d\sigma d\omega$$

  - If K is supermodular in (x, y) then so is V if and only if q is supermodular. Similar for K submodular
- Sketch for part (ii):
  - More integration by parts and marginals, functions of one type, yield

$$V_{xy} = \int_{\underline{\sigma}}^{\overline{\sigma}} \int_{\underline{\omega}}^{\overline{\omega}} q_{\omega\omega\sigma\sigma}(\omega, \sigma) \left(\int_{\underline{\omega}}^{\omega} \int_{\underline{\sigma}}^{\sigma} K_{xy}(s, t | x, y) ds dt \right) d\sigma d\omega$$

  - If $\int_{\underline{\omega}}^{\omega} \int_{\underline{\sigma}}^{\sigma} K$ is supermodular in (x, y) then so is V if and only if q is supermodular. Similar for K submodular Return

Transferable Utility (TU)

Examples of observed (ω, σ) – NAM: FOSD vs. MPS

Distribution of matched ω, σ realizations under PAM/NAM and with FOSD and MPS. Simulations with 1000 types. Under FOSD (both for PAM and NAM), x, y uniform on $[-.5, .5]$, and $\omega = x + \varepsilon, \sigma = y + \varepsilon$ where $\varepsilon_\omega, \varepsilon_\sigma$ are conditionally independent uniform draws on $[-.5, .5]$. Under MPS, (both for PAM and NAM), x, y uniform on $[0, 1]$, and $\omega = x \cdot \varepsilon, \sigma = y \cdot \varepsilon$ where $\varepsilon_\omega, \varepsilon_\sigma$ are conditionally independent uniform draws on $[-1, 1]$.

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